Lesson 5

Airspeed and Wind

$1 \text{ Knot} = 1 \frac{\text{nautical mile}}{\text{hour}}$

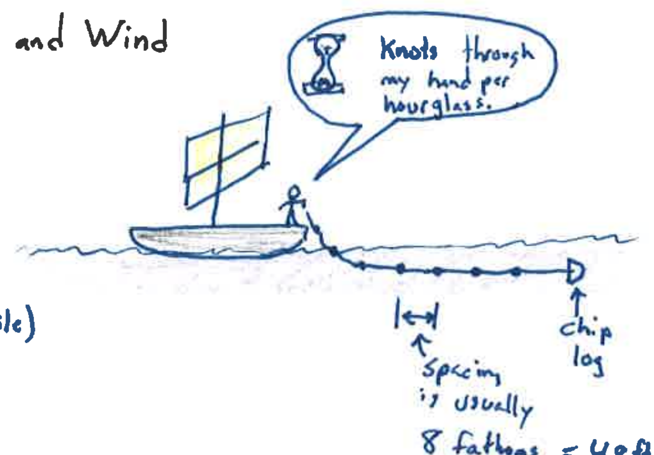
$= 1.151$ mph (statute mile)

$= 20.254$ in/s

$= 0.514$ m/s

nautical mile = 6076 ft

statute mile = 5280 ft

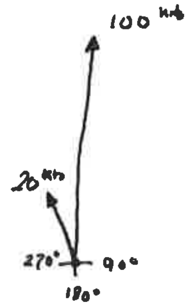


$c = f\lambda$

So 1 knot every 28 seconds is 1 knot

The atmosphere is not static. Wind greatly influences the operation and design of aircraft and rockets.

- Given an aircraft with a heading of 0° (North) and an airspeed of 100 kts, what is the bearing and ground speed given a 20 kt wind from 165°?



This is just a vector addition problem.

$$\vec{V} = (0, 100) + (20\sin(-15°),\ 20\cos(15°))$$
$$= (-5.17,\ 119.3)$$

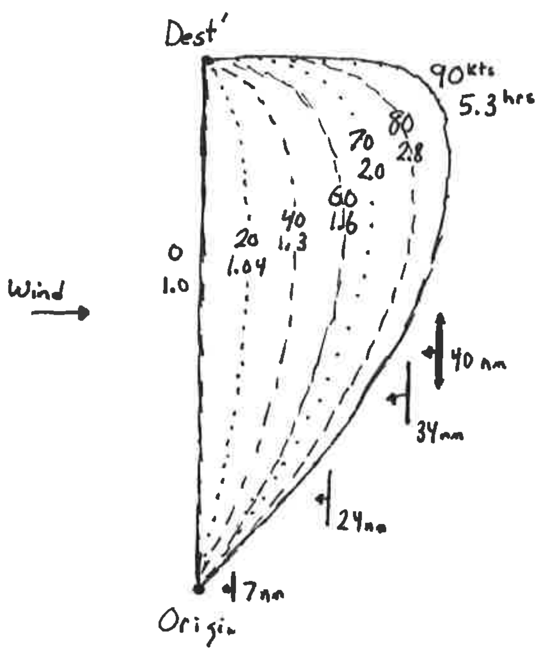
$$|\vec{V}| = \sqrt{5.17^2 + 119.3^2} \approx \boxed{119.4 \text{ kts}}$$

$$\text{bearing} = \arg(\vec{V}) = \angle\vec{V} = \operatorname{atan}\left(\frac{119.31}{5.17}\right)$$

↑ for numerical calculations, use atan2(x, y)

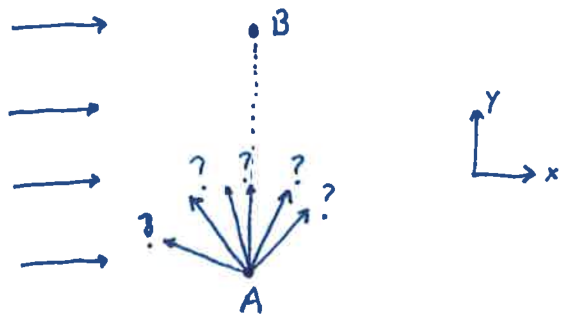
$$= -2.48° = \boxed{357°}$$

- Given two airports on <u>exactly</u> the same longitude and 100 nm apart, a naive pilot might always point exactly towards the destination. What happens?



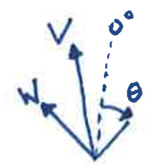
This is a poor way to plan (or fly) a flight.

- Given a 20 kt crosswind from 270°, what heading should a 100 knot aircraft steer to have a ~~heading~~ course of 360°?



Ground Velocity = Air Velocity + Wind Velocity

$$|G|(d_x \ d_y) = |V|(n_x \ n_y) + |W|(w_x \ w_y)$$



$n_x = \sin\theta$

$n_y = \cos\theta$

divide by flight velocity and rearrange (substitute n_x, n_y)

$$\sin\theta_h = \frac{G}{V} d_x - \frac{W}{V} w_x$$

$$\cos\theta_h = \frac{G}{V} d_y - \frac{W}{V} w_y$$

two equations

two unknowns: heading θ, ground speed G

For this problem,

$d_x = 0$, $d_y = 1$ $\quad w_x = 1$ $\quad w_y = 0$

Solve

$$\sin\theta_h = \frac{G}{V} \cdot 0 - \frac{W}{V} \cdot 1 \Rightarrow \theta_h = \operatorname{asin}\left(-\frac{W}{V}\right) = \operatorname{asin}\left(-\frac{20}{100}\right)$$

$$= -11.5°$$

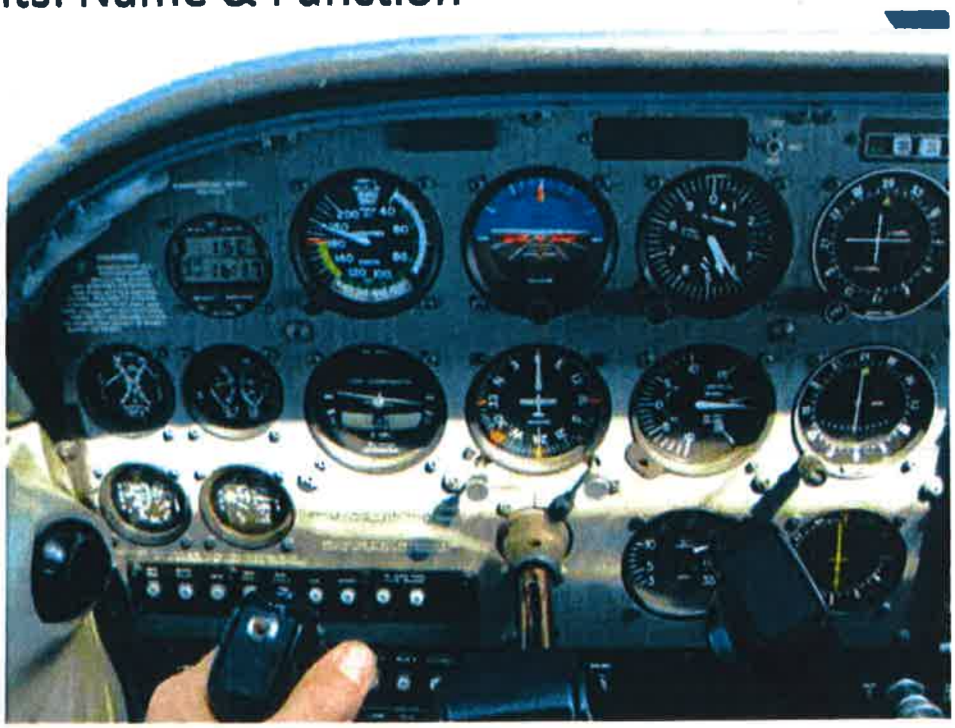
$$\cos(\theta_h) = \frac{G}{V} \cdot 1 - \frac{W}{V} \cdot 0 \Rightarrow \frac{G}{V} = \cos(-11.5°) = 0.979$$

This is simple since the solution was decoupled.

EGB verifies

348° & 98 kts

Instruments: Name & Function



40	60	100
43	63	100

Airspeed

- Indicated airspeed (KIAS). What the pilot sees.
- Calibrated airspeed (KCAS). Remove indicator bias and error.
- True airspeed (KTAS). Actual velocity through air.
- Equivalent airspeed (KEAS). Constant dynamic pressure

Aircraft Operation Speeds

- Vs – Stall
- Vmo – Maximum operating
- Vmc – Minimum controllable airspeed on grnd. (Twin engine a/c -> off runway)
- Vmca – Minimum controllable airspeed in air.
- Vd – Maximum Dive speed
- Vne – Never exceed
- Vx – Speed for best angle of climb
- ....
- Mmo – Maximum operating Mach number
- Md – Dive Mach number
- ....

Compressible Flow

- Speed of Sound

$$a = \sqrt{\gamma R T}$$

- Mach number

$$M = \frac{V}{a}$$

1117 ft/s SSL

- Isentropic Ideal Gas Process (relate stagnation to static pressure)

$$p_t = p\left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}}$$

$\gamma_{air} = 1.4$

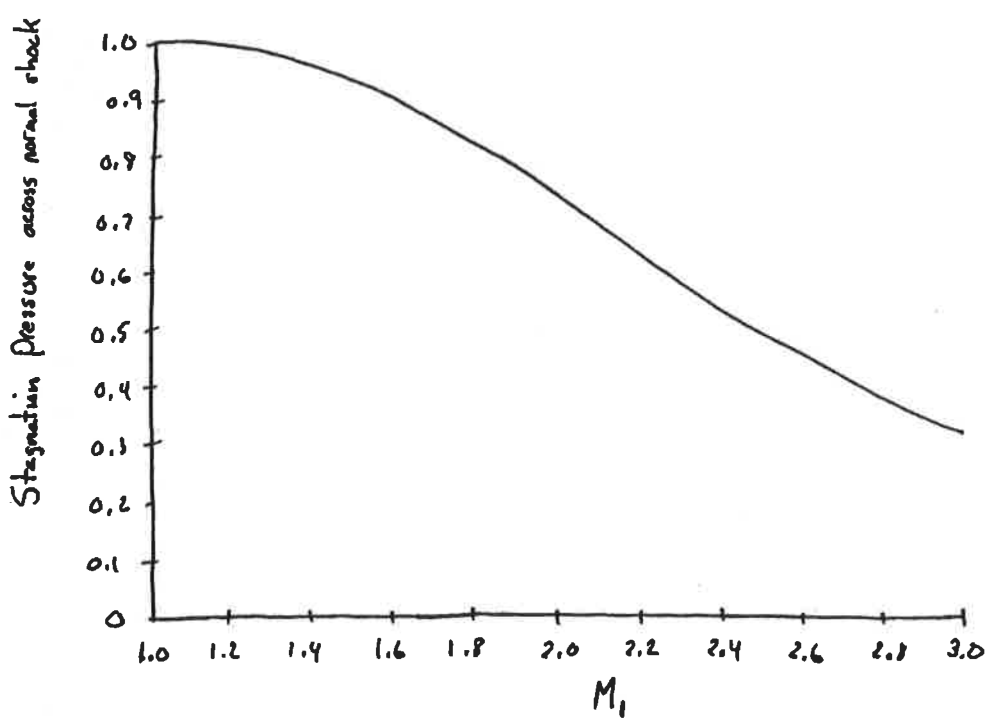
p_t ↑ pressure at V = 0

p ↑ static pressure at M

Compressible Flow

- Normal Shock (Stagnation pressure ratio **drops** across shock)

$$\left(\frac{p_{t_3}}{p_{t_1}}\right)=\left(\frac{\frac{\gamma+1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_1^2}\right)^{\frac{\gamma}{\gamma-1}}\left(\frac{2\gamma}{\gamma+1}M_1^2-\frac{\gamma-1}{\gamma+1}\right)^{\frac{-1}{\gamma-1}}$$



Dynamic Pressure

An aerodynamic force scaling term resulting from incompressible theory.

- **Buckingham Π:**

The Force depends on some function of density, Velocity, Cross sectional area, and an unknown non-dimensional parameter.

Force = mass · acceleration = $m \cdot \ell \cdot t^{-2}$

Density = $m \cdot \ell^{-3}$

Velocity = ℓt^{-1}

Area = ℓ^2

Π = unitless

$$F = (\rho)^a (V)^b (A)^c \Pi$$

$$m \ell t^{-2} = (m\ell^{-3})^a (\ell t^{-1})^b (\ell^2)^c \cancel{\Pi}^{\,1}$$

$a = 1$

$b = 2$

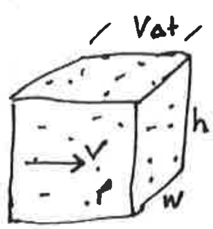
$\Rightarrow c = 1$

Thus, $F = \rho V^2 A \cdot \text{Constant}$

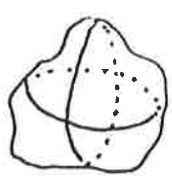
$\frac{1}{2}\rho V^2 = q$ Dynamic Pressure

- **Physics**

Impulse is $\Delta m V$, Force is $\frac{\text{Impulse}}{\Delta t}$



Blob of air moving at V velocity with density ρ.
Cross sectional area is $h \cdot w = A$



Interacts with a solid surface of exactly $h \cdot w$ cross section

$$J = (\text{mass} \cdot \text{velocity})_{\text{start}} - \cancel{(\text{mass} \cdot \text{velocity})_{\text{end}}} = C \cdot \text{mass} \cdot \text{velocity}$$

(Ignore this and just put a constant)

$= \underbrace{\Delta t \rho V h w}_{\text{mass}} \cdot V \cdot C \Rightarrow \text{Force} = \frac{J}{\Delta t} = \rho V^2 A \cdot C$

Example
Lift $= \frac{1}{2}\rho V^2 S_{wing} \cdot C_L$

Dynamic Pressure $= q = \frac{1}{2}\rho V^2$

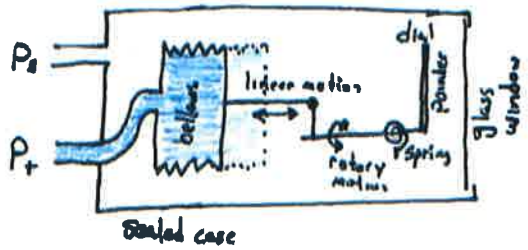
- Q: Is dynamic pressure (q) equal to stagnation pressure minus static pressure (p_t-p)?
- A: **No**. But it is a reasonable approximation for incompressible flows.

$$p_t - p = p\left(\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}} - 1\right)$$

$$\begin{aligned} q &= \frac{1}{2}\rho V^2 \\ &= \frac{1}{2}\frac{p}{RT}M^2 a^2 \\ &= \frac{1}{2}\gamma p M^2 \end{aligned}$$

Airspeed Indicator

Measures the difference in total pressure and static pressure.



The airspeed indicator is calibrated for SSL.

Using isentropic compressible flow theory, the pressure difference $P_t - P_s = \Delta P$ is calibrated to a SSL velocity.

$$\Delta P_{cal} = P_{ssl}\left[\left(\frac{\gamma-1}{2}\left(\frac{V_{cal}}{a_{ssl}}\right)^2 + 1\right)^{\frac{\gamma}{\gamma-1}} - 1\right]$$

Consider an ASI as a function that converts ΔP_{cal} to a V_{cal} velocity reading.

C-170

Cessna

ILLUSTRATED PARTS CATALOG

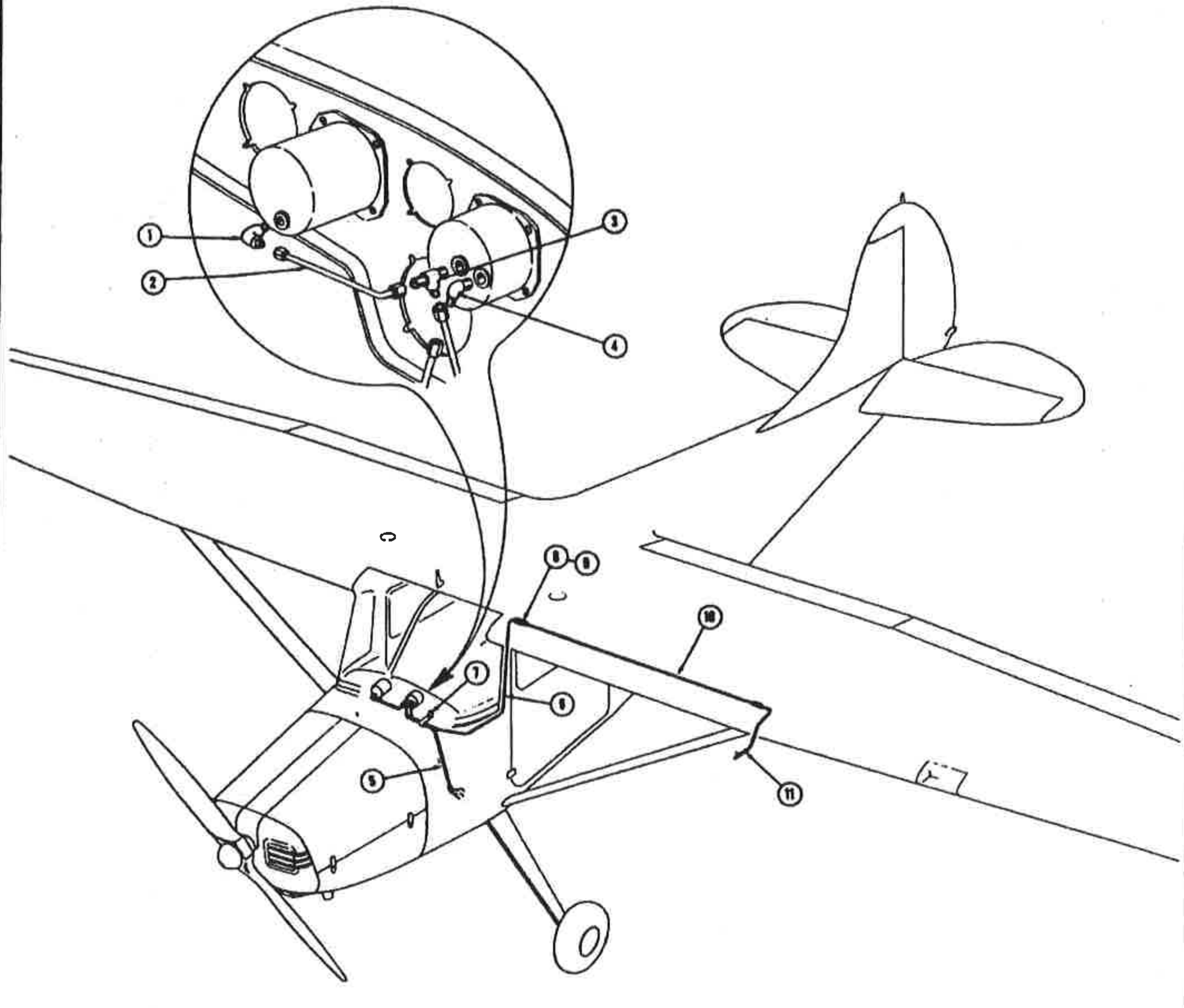


FIGURE 44 - PITOT SYSTEM

FIGURE AND REF. NO.	PART NUMBER	PART NAME (1 2 3 4 5 6)	UNITS REQ'D ON MODEL
	0500205	Pitot System Installation (Reference Only)	1
44-1	AN822-4D	Elbow	1
44-2	0400106-61	Pitot Line - Altimeter to Airspeed Indicator	1
44-3	AN826-4D	Tee	1
44-4	AN823-4D	Elbow	1
44-5	0500106-19	Pitot Line - Static - Air Speed to Static Source	1
44-6	0500106-51	Pitot Line - Airspeed to Wing Root	1
44-7	0413263	Clamp Assembly	1
44-8	NAS397-10	Clamp - (Tinnerman A3122-10-59)	2
44-9	AN884-4-12	Hose	1
44-10	See Figure 3	Pitot Line - Wing Root to Pitot Tube	
44-11	See Figure 3	Tube - Pitot	

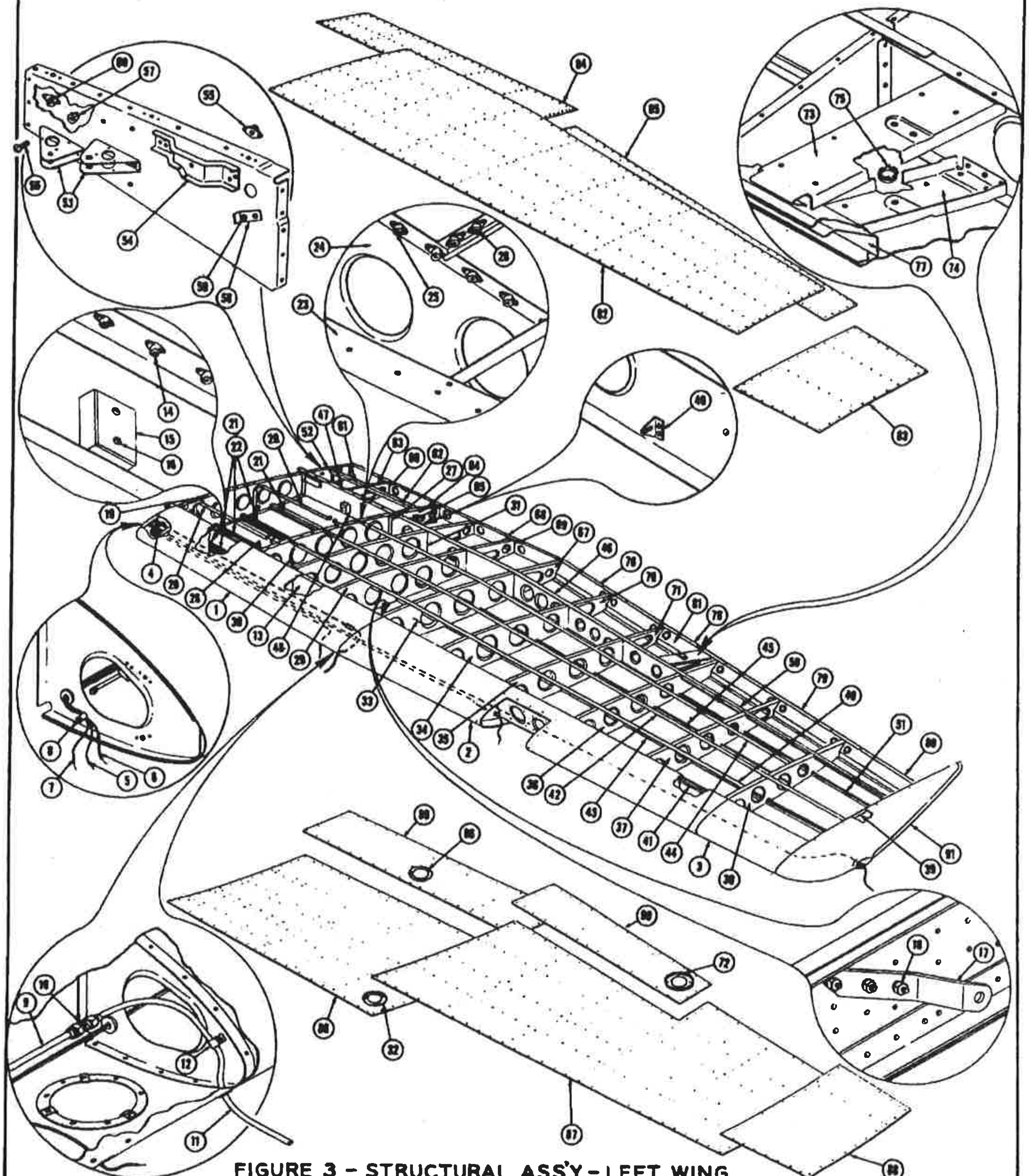


FIGURE 3 - STRUCTURAL ASS'Y - LEFT WING

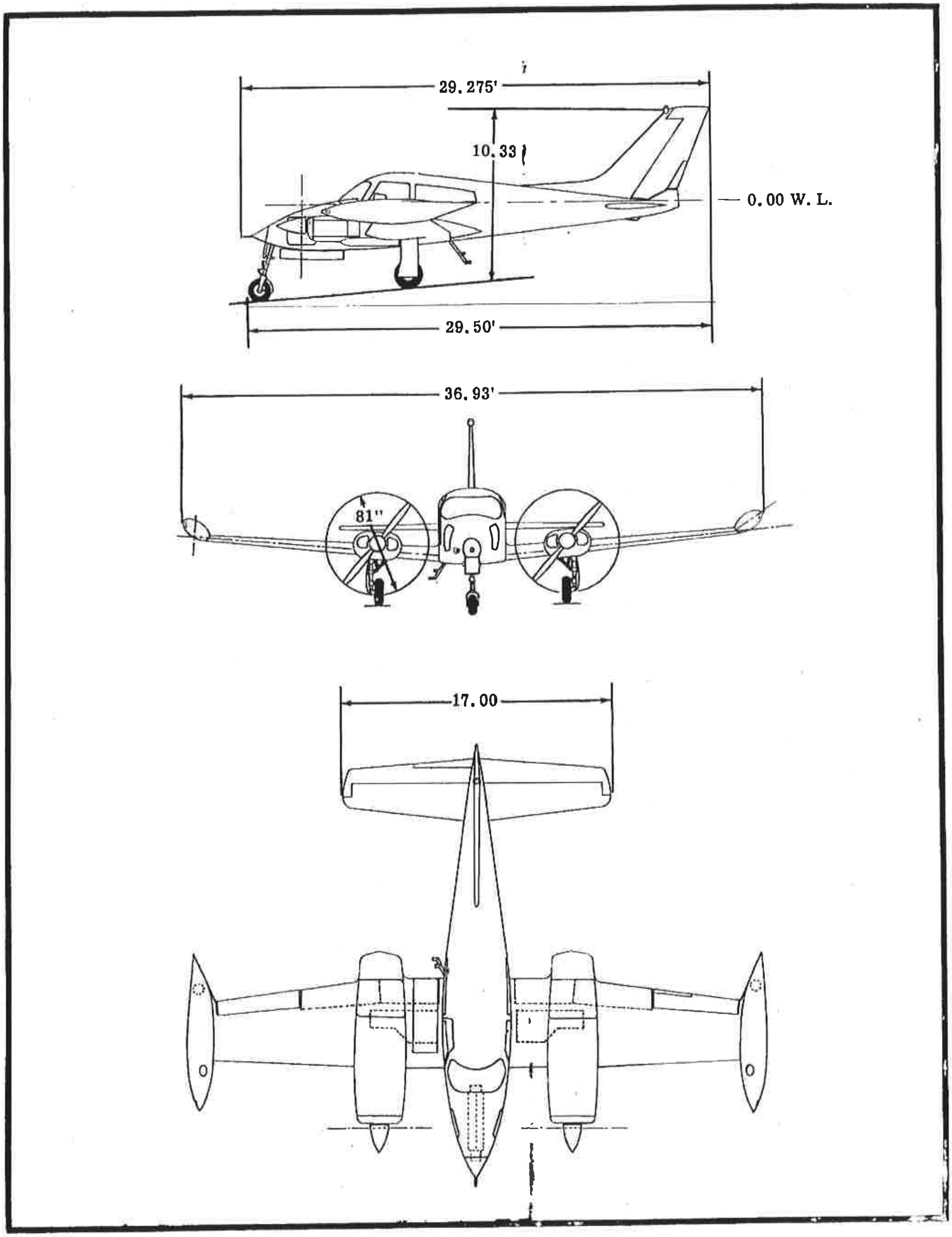
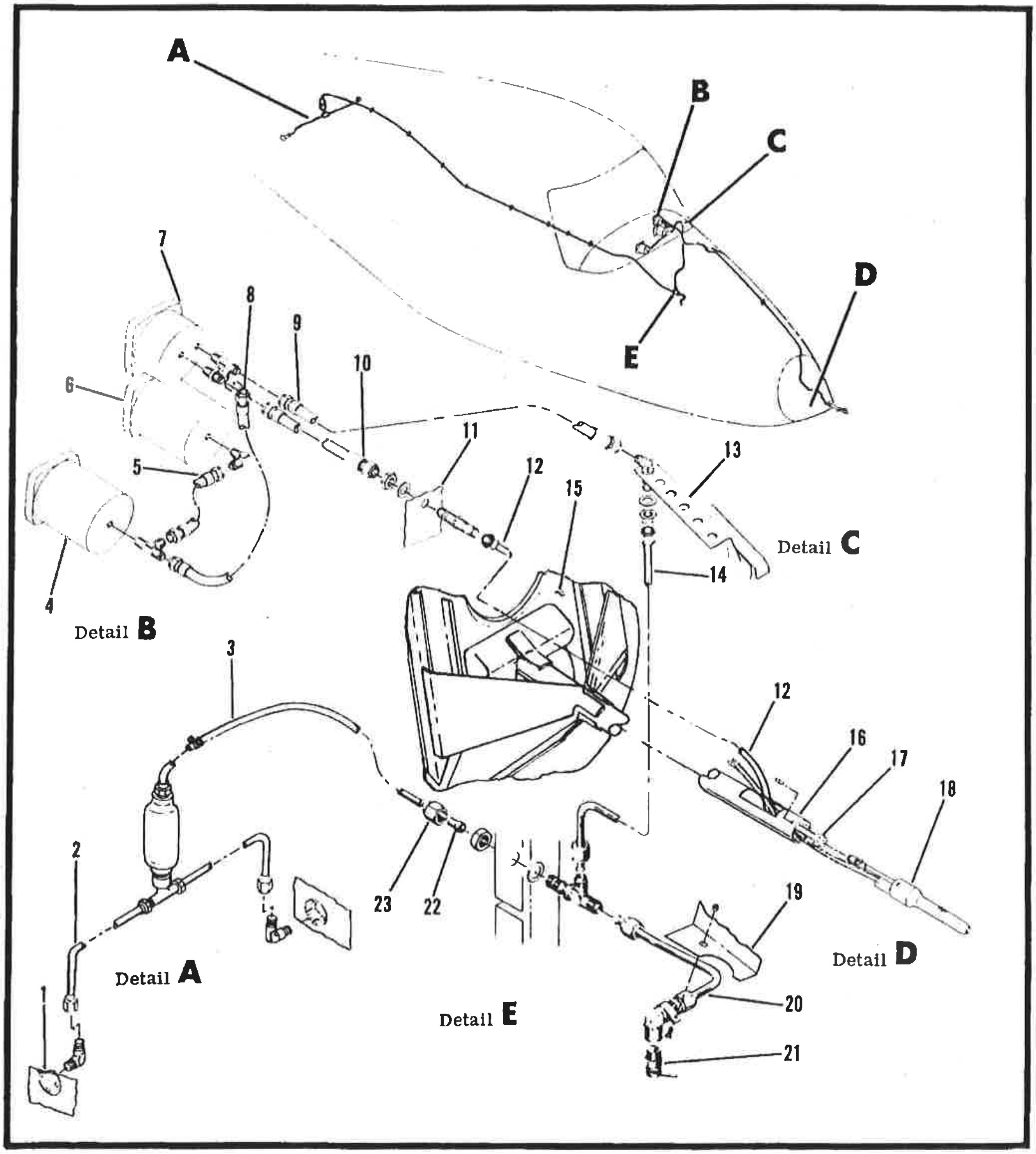


Figure 1-1. Three View 310L and 310N Aircraft



1. Static Opening
2. Static Crossover Line
3. Static Line
4. Vertical Velocity Indicator
5. Hose (Vertical Velocity to Altimeter)
6. Altimeter
7. Airspeed Indicator
8. Hose (Vertical Velocity to Airspeed)
9. Hose (Airspeed to Bracket)
10. Hose (Airspeed to Bulkhead)
11. Forward Cabin Bulkhead
12. Pitot Pressure Line
13. Tube Support Bracket
14. Forward Static Line
15. Nose Bulkhead
16. Mount Tube
17. Pitot Extension Line
18. Pitot Tube
19. Parking Brake Bracket
20. Static Drain Line
21. Drain Valve
22. Sleeve
23. Nut

Figure 12-8. Pitot-Static System Installation

Calculation

- How much pressure differential does a calibrated airspeed reading generate? Apply isentropic compressible theory.

$$\Delta p = p_t - p_{static}$$

$$\Delta p_{cal} = p_{ssl}\left[\left(\frac{\gamma-1}{2}\left(\frac{V_{cal}}{a_{ssl}}\right)^2+1\right)^{\frac{\gamma}{\gamma-1}}-1\right]$$

- Notice that we use SSL conditions for everything other than delta p and Vcal. The ASI is **designed** with a reference at SSL.

- The objective is to find the true airspeed. You already know the pressure ratio from the calibrated airspeed.

$$\frac{\Delta p_{cal}}{p_{ssl}}$$

- Given an altitude, you can convert ssl to local pressures

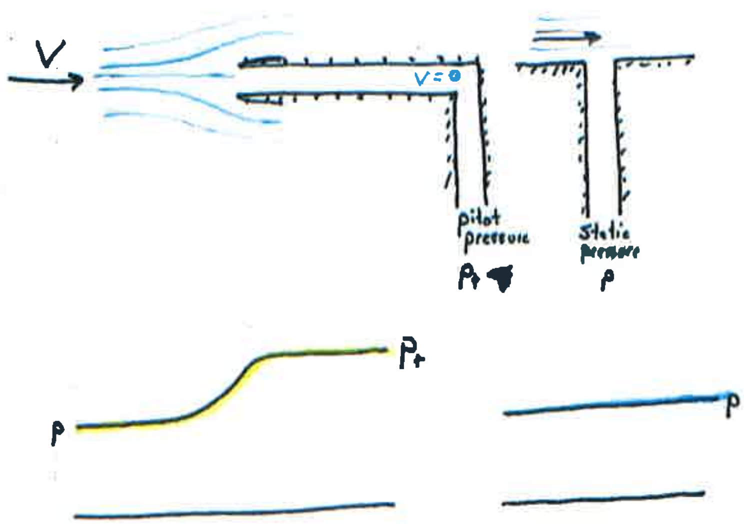
$$\delta = \frac{p}{p_{ssl}}$$

- Substitute to give the true airspeed

$$V_{true} = \sqrt{\frac{2a^2}{\gamma - 1}\left[\left(\frac{\Delta p_{cal}}{p_{ssl}}\frac{1}{\delta} + 1\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}$$

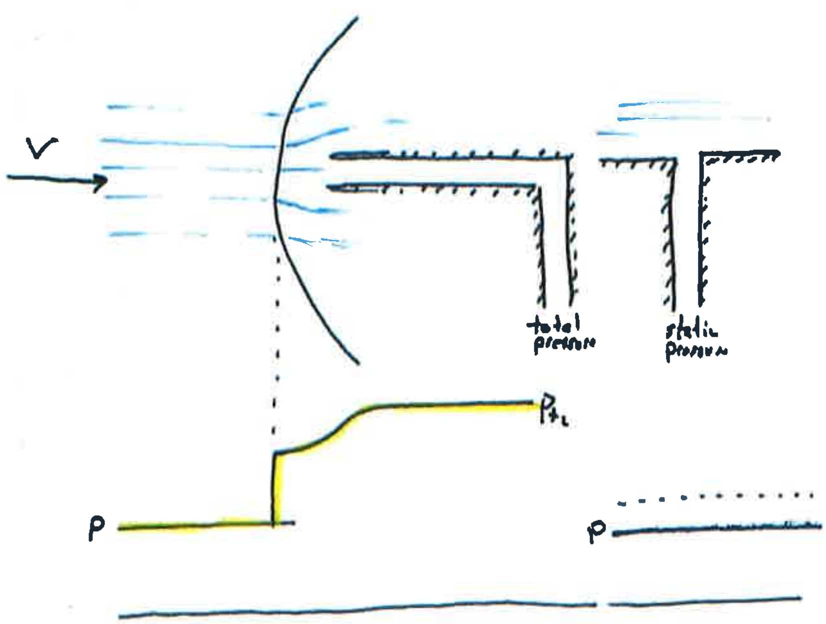
Subsonic

- Region from M=0 to M=1
- This is NOT the same as incompressible
- You can not expect to use incompressible formulas beyond M=0.3.



Supersonic

- M>1
- Now have a normal shock in front of pitot tube
- Assume a weak oblique shock and fan for static port gives almost isentropic flow. Thus the static pressure is almost equal to the freestream static pressure.



Notice for a strong shock, the static pressure port will begin to read slightly higher. This analysis ignores this issue!!

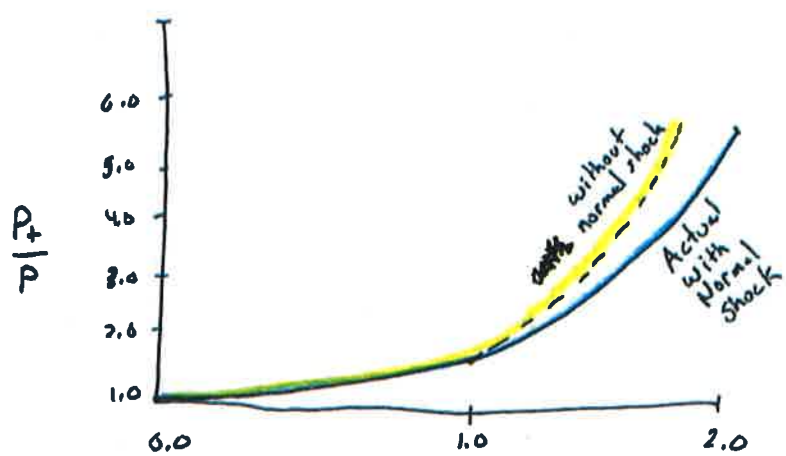
- Pitot pressure ratio given known pressure ratio (Pt2/p1) and shock

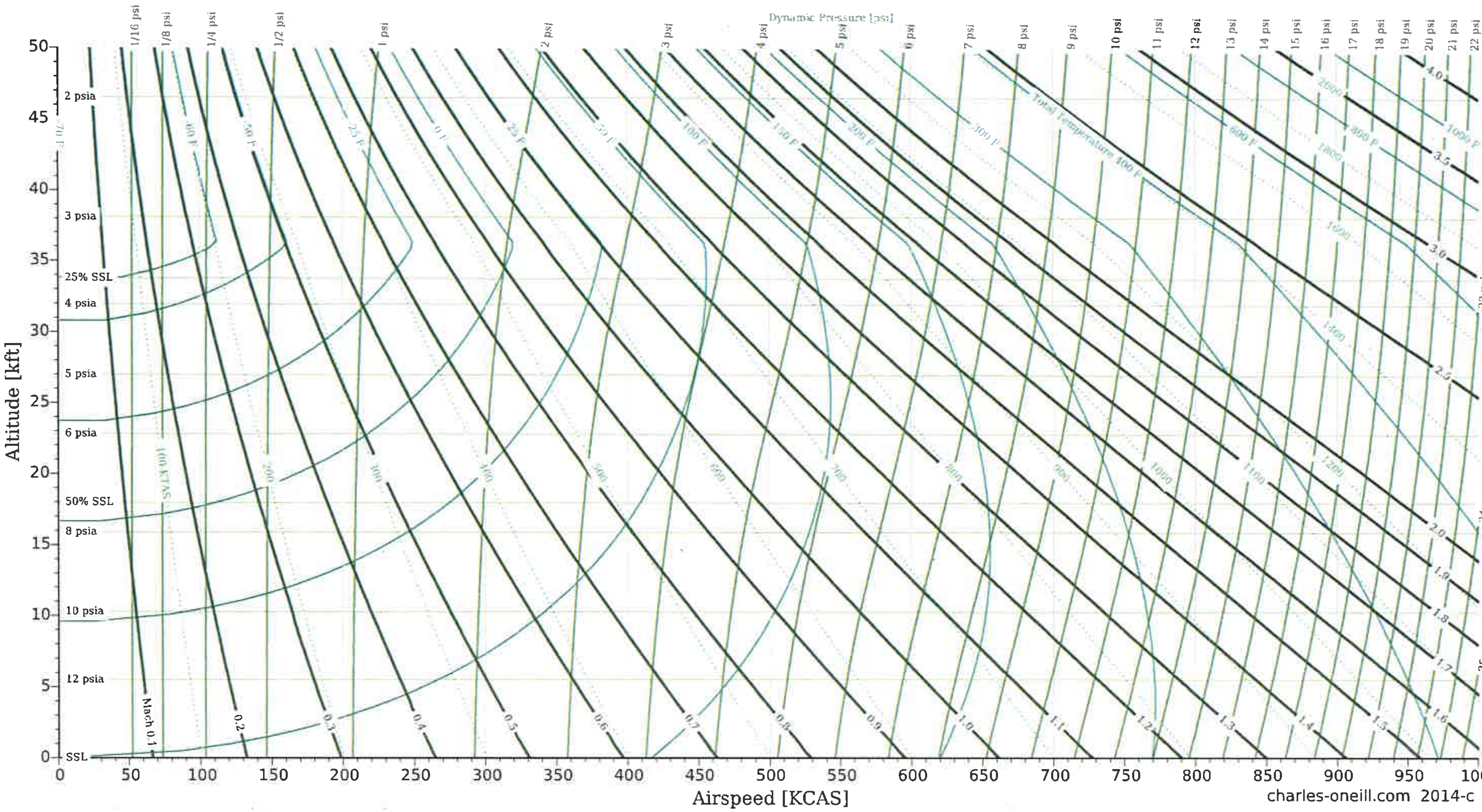
$$\frac{p_{t_1}}{p_1} = \left(\frac{p_{t_2}}{p_1}\right)\left(\frac{p_{t_1}}{p_{t_2}}\right)$$

- Substitute compressible flow equations and solve

$$\left(\frac{p_{t_2}}{p_1}\right) = \left(\frac{\gamma+1}{2}M_1^2\right)^{\frac{\gamma}{\gamma-1}}\left(\frac{2\gamma}{\gamma+1}M_1^2 - \frac{\gamma-1}{\gamma+1}\right)^{\frac{-1}{\gamma-1}}$$

- Now, given the upstream Mach number, you can find the pitot pressure ratio. Typically you will need to iterate to solve for M given a known pitot pressure ratio (see subsonic for finding pressure ratio).





Dynamic Pressure [psi]
1/16 psi
1/8 psi
1/4 psi
1/2 psi
1 psi
2 psi
3 psi
4 psi
5 psi
6 psi
7 psi
8 psi
9 psi
10 psi
11 psi
12 psi
13 psi
14 psi
15 psi
16 psi
17 psi
18 psi
19 psi
20 psi
21 psi
22 psi
23 psi
24 psi
25 psi
Total Temperature 100 F
2 psia
3 psia
25% SSL
4 psia
5 psia
6 psia
50% SSL
8 psia
10 psia
12 psia
SSL
100 KTAS
Mach 0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0
2.5
3.0
3.5
4.0
Altitude [kft]
Airspeed [KCAS]
0
5
10
15
20
25
30
35
40
45
50
0
50
100
150
200
250
300
350
400
450
500
550
600
650
700
750
800
850
900
950
1000
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Quiz #1:

- What altitude? FL45
- What KTAS? 440

Quiz #2:

- What altitude?
- What KTAS?

Quiz #3:

- What altitude?
- What KTAS?

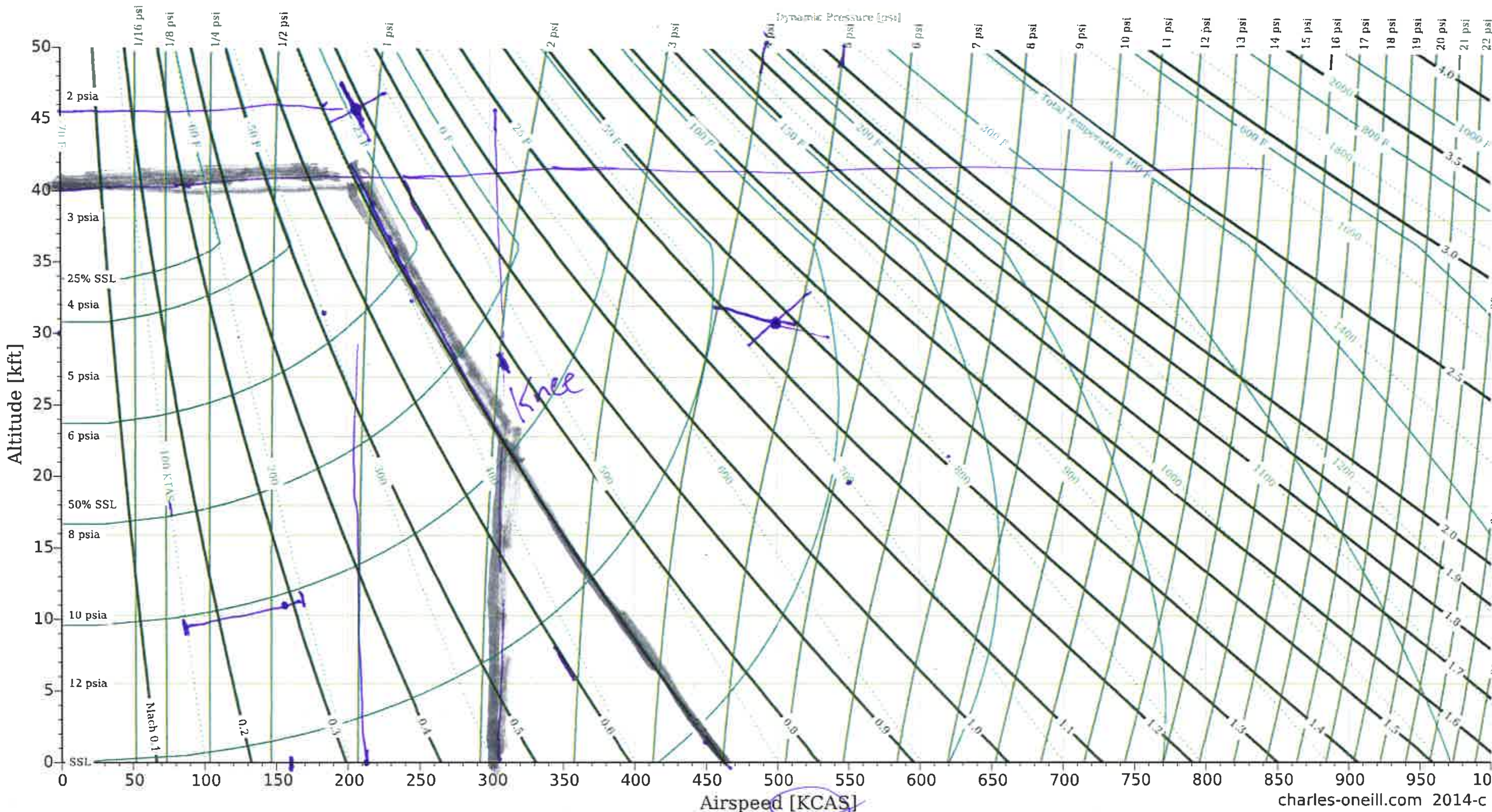
Quiz #4: Piper PA-34

h= 11040

160

185 KTAS

Lucky for us, set at pressure altitude (29.92 inHg). SSL!



pilot sees - bias

500 KCAS @ 30k
740 KTAS
q · 4.7 psi
T = 70°