Lesson 6

Propulsion

APD - Chap 3

Derivation of drag equation for a control volume approach.

- Momentum Gov Equ.

$$\iiint_V \cancelto{0}{\frac{\partial}{\partial t}(\rho \vec{V})\, dV} + \iint_S \rho(\vec{V}\cdot n)\vec{V}\, dS = \iiint_V \cancelto{0}{\rho f\, dV} + \iint_S -p\hat{n}\, dS + \iint_S \cancelto{0}{\bar{\tau}\cdot\hat{n}\, dS} + B$$

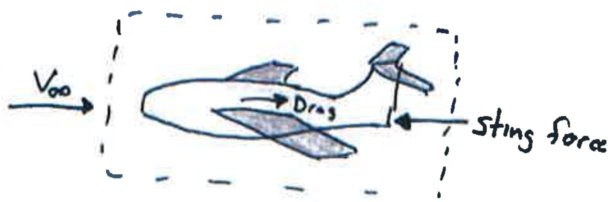
Steady state

distributed No body force

Careful C.V

- Drag

To maintain the model in the wind tunnel, a force must be constantly applied to the model



- This force goes through the control volume.
- Or consider a body force B applied to the model such that the C.V is in equilibrium

$$B_x = -\text{Drag}$$

- Simplify Equ

$$\iint_S \rho(\vec{V}\cdot n)\vec{V}\, dS = -\iint_S p\,\hat{n}\cdot dS + \cancel{B}\begin{pmatrix}-\text{Drag}\\ -\text{Sideforce}\\ -\text{Lift}\end{pmatrix}$$

- Consider only the drag (x component) on the left and right faces

$\hat{n} \leftarrow$ $\quad$ $dS \rightarrow \hat{n}$

$dS = dl$

$$\text{Drag} = -\iint_S \rho(\vec{V}\cdot n)\vec{V}_x\, dS - \iint_S p\,\hat{n}\cdot dS$$

$$= -\iint_S \rho\left[\begin{pmatrix}V_x\\V_y\\V_z\end{pmatrix}\cdot\begin{pmatrix}n_x\\n_y\\n_z\end{pmatrix}\right] V_x\, dS - \iint p\,\hat{n}_x\, dS$$

- Upstream face (left)

$\hat{n} \leftarrow$ $\quad \hat{n} = (-1,0,0)^T$

- Downstream face (right)

$\rightarrow \hat{n}$ $\quad \hat{n} = (1,0,0)^T$

- Constant pressure Integral

Q: What is $\iint_S p_\infty \hat{n}\, dS$?

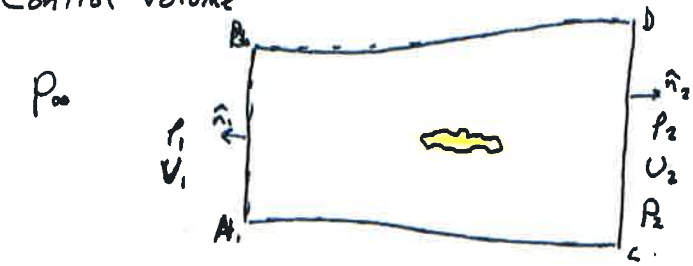
A: 0

So, we can add an arbitrary pressure offset to the pressure term.

- Drag

$$\text{Drag} = -\iint_S \rho V_x n_x V_x \, d\ell + \iint_S (p_\infty - p) n_x \, d\ell$$

- Control Volume



- Drag

$$\text{Drag} = -\iint_S \rho_1 U_1 \cancelto{-1}{\hat{n}_{1x}} U_1 \, d\ell - \iint_S \rho_2 U_2 \cancelto{+1}{\hat{n}_{2x}} U_2 \, d\ell + \iint_S (p_\infty - p_1) \cancelto{-1}{n_{1x}} \, d\ell + \iint_S (p_\infty - p_2) \cancelto{+1}{n_{2x}} \, d\ell$$

- Mass Continuity

$$\cancelto{\text{Steady}}{\iiint_V \frac{\partial}{\partial t}(\rho)\, dV} + \underbrace{\iint_S \rho V \cdot \hat{n} \, dS}_{\text{in x dir only and applied to C.V.}} = 0$$

in x dir only and applied to C.V. $\Rightarrow \iint -\rho_1 \cancelto{U_1}{V_x} \, d\ell + \iint \rho_2 \cancelto{U_2}{V_x} \, d\ell = 0$

Multiply by U_1 (a constant!)

$$\underbrace{\int_A^B\!\!\int -\rho_1 U_1 U_1 \, d\ell}_{\text{Familiar term from mom' eqn!!}} + \int_C^D\!\!\int \rho_2 U_1 U_2 \, d\ell = 0$$

- Drag

$$\text{Drag} = \underbrace{+\int_C^D\!\!\int \rho_2 U_1 U_2 \, d\ell_2 - \int_C^D\!\!\int \rho_2 U_2 U_2 \, d\ell_2}_{\text{Combine to give}} - \iint_S \cancelto{0}{(p_\infty - p_1)} \, d\ell_1 + \iint \cancelto{0}{(p_\infty - p_2)} \, d\ell_2$$

$p_1 = p_\infty$ Far enough upstream

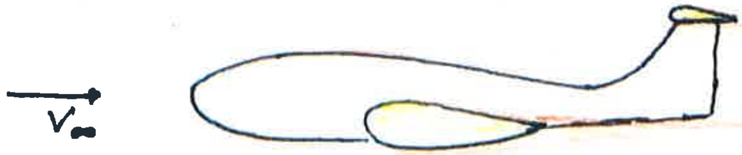
Far enough downstream

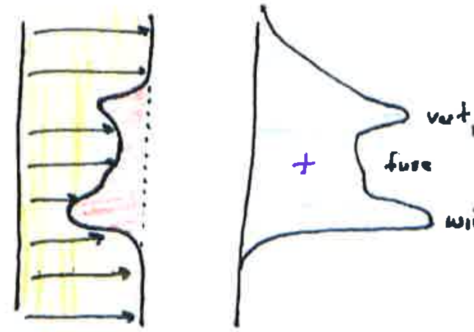
$$\boxed{D = \int_C^D\!\!\int \underbrace{\rho_2 U_2}_{\text{Mass flux}} \underbrace{(U_1 - U_2)}_{\text{This is a velocity deficit}} \, d\ell}$$

Generic result for a C.V away from shape being tested.

Near the shape/vehicle, you <u>must</u> include the pressure terms.

Full aircraft in steady level flight





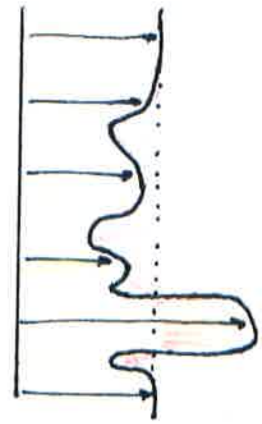
U_2 Momentum deficit

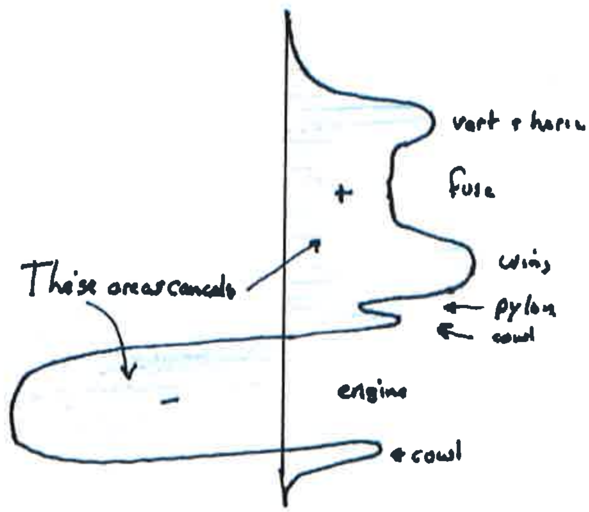
$$C_D \geq 0$$

$D > 0$

$\cancel{F} = ma \Rightarrow$ the aircraft decelerates

Propulsion!

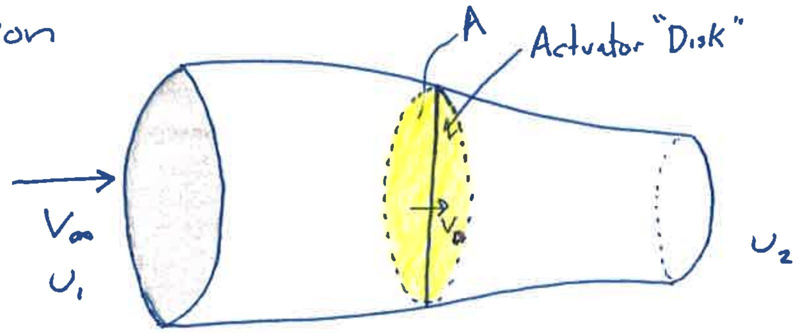




Momentum Deficit

$$C_D = \int \frac{\rho_2 U_2 (U_1 - U_2)\, d\ell}{qS} = 0$$

Propulsion



From the CV,

$$T = -D = -\iint_S \underbrace{\rho_2 U_2}_{\dot{m}/A} \underbrace{(U_1 - U_2)}_{V_\infty - U_2} \, dS$$

mass flow per area

$$\boxed{T = \dot{m}\,(U_2 - U_\infty)}$$

The thrust is also the pressure difference across the disk · disk area

$$T = \Delta p_{disk} \cdot A_{disk}$$

Applying Bernoulli's eqn, gives

$$V_D = \frac{1}{2}(U_1 + U_2)$$ the average of upstream and downstream

rearranged, this is

$$V_D = U_1 + w$$

half the increment in velocity across the disk

$$T = 2\rho A (U_1 + w) w = \underbrace{\rho A (U_1 + w)}_{\text{mass flux through disk}} \cdot \underbrace{2w}_{\text{Velocity increment through disk}}$$

Power.

$$P = 2w \cdot \rho A (U_1 + w)^2 = \underbrace{2w}_{V\text{ incr' disk}} \cdot \underbrace{\rho A (U_1 + w)}_{\text{mass flux}} \cdot \underbrace{U_1 + w}_{\text{velocity disk}}$$

$$= 2w\rho A (U_1 + w)^2$$

Useful Power

$$P_{useful} = T U_1$$

Induced power

$$P_i = T w$$

Efficiency

$$\eta = \frac{P_{useful}}{P_{total}} = \frac{T U_1}{T U_1 + T w} = \frac{U_1}{U_1 + w}$$

Rearrange to

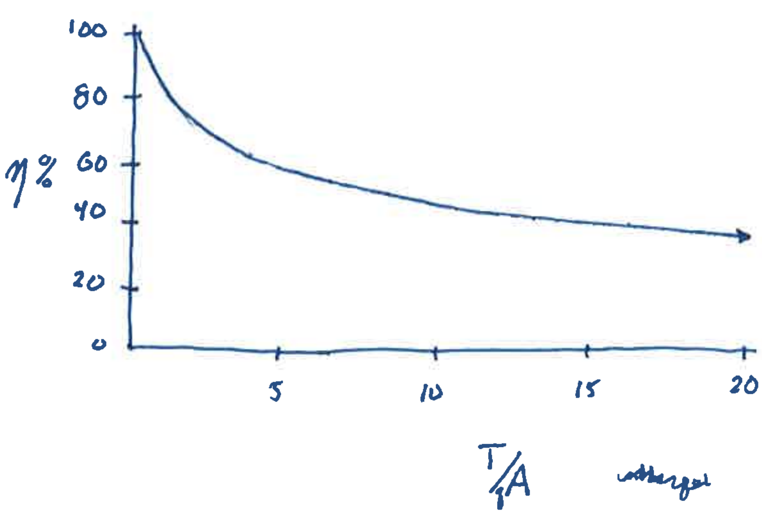
$$\eta = \frac{2}{1 + \frac{U_2}{U_\infty}} \qquad \text{also} \qquad \eta = \frac{1}{1 + \frac{w}{V}}$$

Canonically

$$\eta = \frac{2}{1 + \sqrt{1 + T_c}}$$

T_c = a thrust coefficient

$$= \frac{1}{8} \underbrace{\frac{T}{A}}_{\text{disk loading}}$$



Efficient engines have a low $\frac{T}{qA}$
Fan jet designs

High thrust engines have low η
military

Piston Engine w̄ Propeller

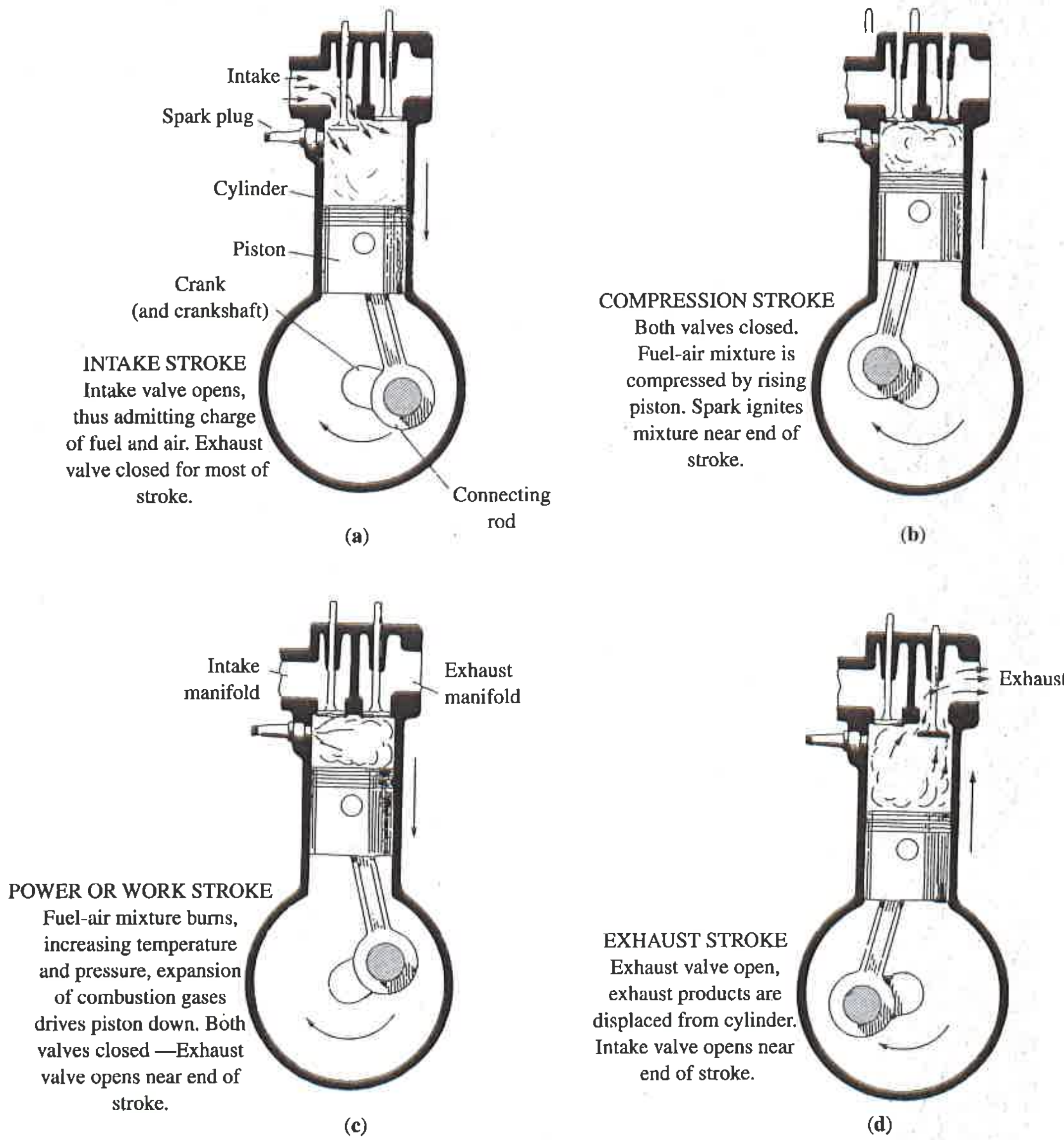


Figure 3.3 Diagram of the four-stroke Otto cycle for internal combustion spark-ignition engines. (*After Edward F. Obert, Internal Combustion Engines and Air Pollution, Intext, 1973.*)

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Aircraft Fuel Systems

Avgas: Aviation Gasoline (see ASTM D910-11) Piston Engines

The most common is 100LL ("One hundred low lead" or "hundred low lead")

The # represents the octane rating.

Low lead is only relative to other older products. (as 100 with 1 g/L) 100LL has 0.5 g/L of tetraethyl lead

Some fuel grades have 2 #s. (Ex: 80/87.) The 1st number represents the typical octane rating (aka aviation lean). The 2nd number represents an aviation rich octane rating. Aircraft engines have the capability to vary the mixture; a rich mixture is less susceptible to detonation (i.e. higher octane). 100LL is 130 w/ rich mixtures.

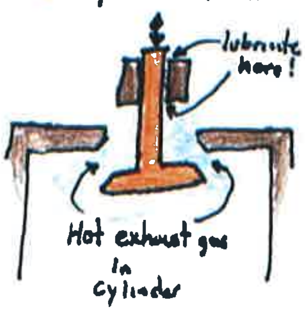
Each fuel grade is dyed a specific color. These colors did not scan properly.

- 100LL is blue ← common
- 80/87 is red ← very rare now
- 115/145 is purple ← Reno, NV why? air-racers w/ high compression WW2 engi

Q: Why is there lead in fuel?

A: Lubrication: of valves in a high temperature burning environment.

Teledyne-Continental "Current aircraft engines feature valve gear components which are designed for compatibility with the leaded ASTM D910 fuels. In such fuels, the lead acts as a lubricant, coating the contact areas between the valve guide, and seat. The use of unleaded auto fuels with engines designed for leaded fuels can result in excessive exhaust valve seat wear due to the lack of lead. The result can be remarkable, with cylinder performance deterioration to unacceptable levels in under 10 hours."



Properties:

Density: $6.01 \frac{lbf}{U.S.\,gal}$ at 59°F to $6.41 \frac{lbf}{U.S.\,gal}$ at −40°F

Freeze: $T < -58°C \approx -72°F$

Conductivity: $< 450 \frac{pS}{m}$

Jet Fuel: (ASTM D1655) Turbine Engines + Diesel Piston Engines

The most common ~~is~~ are Jet A. (civilian) and since the 90s JP-8 (military).

Similar in properties to Kerosene (i.e. lighter less viscous than diesel)

Color is "straw colored" This color did not scan properly.

3 main civilian types:

- Jet A: freezes at −40°F ← common
- Jet A1: freezes at −53°F
- Jet B: Freezes at −76°F ← mixture of 30% Kerosene + 70% gasoline ~~diesel~~.

Cold, eh?

Several military types

- JP 8: similar to Jet A (MIL-DTL 83133)
- JP 5: Navy version of JP8 with a higher flashpoint for carrier safety.
- JP 4: Older jet fuel (phased out in 1970s to 1990s)

The exotics:

- JP 7: SR71 fuel w a very high flashpoint (140°F)
  Trivia: additive to apparently reduce radar signature of exhaust!

Properties

Density: 6.4 − 7.0 lbs/gal

Unfortunately, different refineries can produce jet fuel with different densities. Plus, the density changes with temperature.

Piston Engine w/ Propeller

Specific Fuel Consumption

$$[SFC] = \frac{\text{weigh of fuel burn per time}}{\text{power output}} = \left[\frac{lb}{hp \cdot h}\right]$$

Ex: A C-172 burns 9 gal/hr during cruise, what is the SFC if the power output is 130 hp?

$$SFC = \frac{9\ \cancel{gal}}{hr}\cdot\frac{6.01\ lbf}{\cancel{gal}}\cdot\frac{1}{130\ hp} \approx \boxed{0.416\left[\frac{lb}{hp\,h}\right]}$$

For a piston engine,

- Power is relatively constant with V_∞
- SFC is constant with V_∞ and altitude
- Power ratio is a function of the density ratio

$$\frac{P}{P_0} = \frac{\rho}{\rho_0}$$

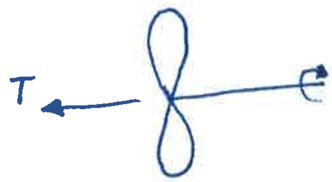
Ex: A particular C-172 has about 140 hp at SSL. What is the power at 10000 ft on a standard day?

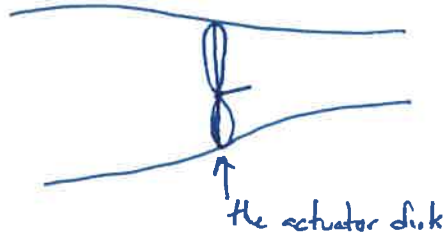
$$P = P_0\frac{\rho}{\rho_0} = 140^{hp}\cdot\frac{1.7556\times10^{-3}\ slug/ft^3}{0.00237\ slug/ft^3}$$

$$\boxed{P = 103^{hp}}$$

Propeller

Converts rotational power into linear power





Efficiency

$$\eta = \frac{P_{available}}{P_{input}}$$

Advance Ratio

$$J = \frac{V_\infty}{ND} = \frac{\text{Forward velocity}}{\text{revolutions per second} \cdot \text{diameter}}$$

Ex: A 60 inch propeller is operating at 60 mph at 2500 rpm. What is J?

$$J = \frac{V_\infty}{ND} = \frac{60 \text{ miles}}{\text{hr}} \cdot \frac{\text{min}}{2500 \text{ rev}} \cdot \frac{1}{60 \text{ in}} \cdot \frac{\text{hr}}{60 \text{ min}} \cdot \frac{5280 \text{ ft}}{1 \text{ mile}} \cdot \frac{12 \text{ in}}{1 \text{ ft}}$$

$$= \boxed{0.42}$$

Thrust Coefficient

$$C_T = \frac{T}{\rho n^2 D^4} \qquad C_T = f(J)$$

Power Coefficient

$$C_P = \frac{P}{\rho n^3 D^5} \qquad C_P = f(J)$$

Efficiency

$$\eta = \frac{TV}{P} = \frac{C_T \rho n^2 D^4 V}{C_P \rho n^3 D^5} = \frac{C_T}{C_P} \underbrace{\frac{V}{nD}}_{J} = \boxed{\frac{C_T}{C_P} J}$$