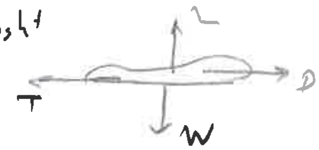
Lesson 8 part 3

Power required

Power Required

SLF: Steady Level flight

$L = W \qquad T = D$



$$P = \underbrace{T V_\infty}_{\text{thrust Velocity}} = D V_\infty = \frac{D}{L} L V_\infty = \frac{C_D}{C_L} W V_\infty$$

For steady level flight, the velocity is

$$\frac{1}{2} \rho V_\infty^2 S C_L = L \quad \Rightarrow \quad V_\infty = \sqrt{\frac{2W}{\rho S C_L}}$$

Substitute into power

$$P = \underbrace{\left(\frac{C_D}{C_L}\right) W}_{\text{Thrust}} \underbrace{\sqrt{\frac{2W}{\rho S C_L}}}_{V_\infty} = \sqrt{\frac{2 C_D^2 W^3}{\rho S C_L^3}} = \underbrace{\sqrt{\frac{2 W^3}{\rho S}} \left(\frac{C_D}{C_L^{3/2}}\right)}_{\text{book eq 5.57 wrong}}$$

So to minimize power, minimize $\frac{C_D}{C_L^{3/2}}$ or maximize $\frac{C_L^{3/2}}{C_D}$.

Compare to ~~[illegible]~~ thrust where T is:

$$T = W\left(\frac{D}{L}\right) = W\left(\frac{C_D}{C_L}\right)$$

Velocity to maximize $\frac{C_L^{3/2}}{C_D}$ (that is, minimizing power)

$$\frac{C_L^{3/2}}{C_D} = \frac{C_L^{3/2}}{C_{D_0} + K C_L^2}$$

Take the derivative wrt C_L

$$\frac{d\left(\frac{C_L^{3/2}}{C_D}\right)}{dC_L} = \underbrace{\frac{d}{dC_L}\left(\frac{C_L^{3/2}}{C_{D_0} + K C_L^2}\right)}_{} = 0$$

$$= \frac{-\sqrt{C_L}\left(C_L^2 K - 3C_{D_0}\right)}{\underbrace{2\left(C_L^2 \cdot K + C_{D_0}\right)^2}_{\text{always } +}} = 0$$

$$\frac{\frac{3}{2} C_L^{1/2}\left(C_{D_0} + K C_L^2\right) - 2KC_L\left(C_L^{3/2}\right)}{\left(C_{D_0} + K C_L^2\right)^2}$$

$$\frac{\frac{3}{2} C_L^{1/2} C_{D_0} + \frac{3}{2} K C_L^{5/2} - 2KC_L^{5/2}}{\left(\frac{3}{2} C_L^{1/2} C_{D_0} - \frac{1}{2} K C_L^{5/2}\right) / \left(C_{D_0} + K C_L^2\right)^2}$$

Thus

$$C_L^2 K - 3C_{D_0} = 0 \Rightarrow C_{D_0} = \frac{1}{3} K C_L^2 \quad \text{OR} \quad C_L = \sqrt{\frac{3C_{D_0}}{K}}$$

For minimum power, the { induced drag is $\frac{2}{3}$ of the total drag
zero-lift drag is $\frac{1}{3}$ of the total drag }

Compare to min thrust where $C_{D_0} = C_{D_i}$ at T_{min}.

Compute the min power V_∞

$$L = \frac{1}{2}\rho V_\infty^2 S C_L \Rightarrow V_\infty = \sqrt{\frac{2W}{\rho S C_L}} = \sqrt{\frac{2W}{\rho S \sqrt{\frac{3C_{D_0}}{K}}}}$$

$$\boxed{V_{\infty_{P_{min}}} = \sqrt{\frac{2W}{\rho S}}\sqrt[4]{\frac{1}{3}\frac{K}{C_{D_0}}}}$$

where did we see a similar form? $V_{T_{min}} = \sqrt{\frac{2W}{\rho S}}\sqrt[4]{\frac{K}{C_{D_0}}}$

Minimum-Thrust Velocity

There is a factor of $\left(\frac{1}{3}\right)^{1/4} \approx 0.7598 \approx 0.76$

$$\boxed{V_{P_{min}} = 0.76\, V_{T_{min}}}$$

Min power is slower than min thrust

why? $P = T \cdot V$

And $\left(\frac{L}{D}\right)_{P_{min}} = 0.866\left(\frac{L}{D}\right)_{T_{min}}$

Ex: Estimate the minimum power speed of a Bonanza at SSL.

$$\frac{W}{S} = \frac{3650 \text{ lbf}}{177 \text{ ft}^2}, \quad \rho = 0.00237 \frac{\text{slug}}{\text{ft}^3}, \quad C_{D_0} \approx 0.0192, \quad AR \approx 5.8, \quad e \approx 0.8$$

Aerodynamics:

$$K \approx \frac{1}{\pi A\!R e} = \frac{1}{\pi \cdot 5.8 \cdot 0.8} = 0.0686$$

Apply to $V_{P_{min}}$ equ:

$$V_{P_{min}} = \underbrace{\sqrt{2 \cdot \frac{3650 \cancel{\text{lbf}}}{177 \cancel{\text{ft}^2}} \cdot \frac{\cancel{\text{slug}} \text{ ft}}{\cancel{\text{lbf}} \text{ s}^2} \cdot \frac{\cancel{\text{ft}^3} \text{ ft}^2}{0.00237 \cancel{\text{slug}}}}}\ \underbrace{\sqrt[4]{\frac{1}{3} \frac{0.0686}{0.0192}}}$$

$$= 131.9 \frac{\text{ft}}{\text{s}} \cdot 1.044 = 138 \text{ ft/s} \approx 94 \text{ mph} \approx 81 \text{ kt}$$

$$\boxed{V_{P_{min}} = 138 \text{ ft/s}}$$

What is the minimum thrust Velocity?

$$V_{T_{min}} = \underbrace{\sqrt{\cdots \sqrt{\cdots}}}_{\text{hard}} = \underbrace{\frac{V_{P_{min}}}{0.76}}_{\text{easy}} = 181 \text{ ft/s} \approx 107 \text{ kt}$$

$$\boxed{V_{T_{min}} = 181 \text{ ft/s}}$$