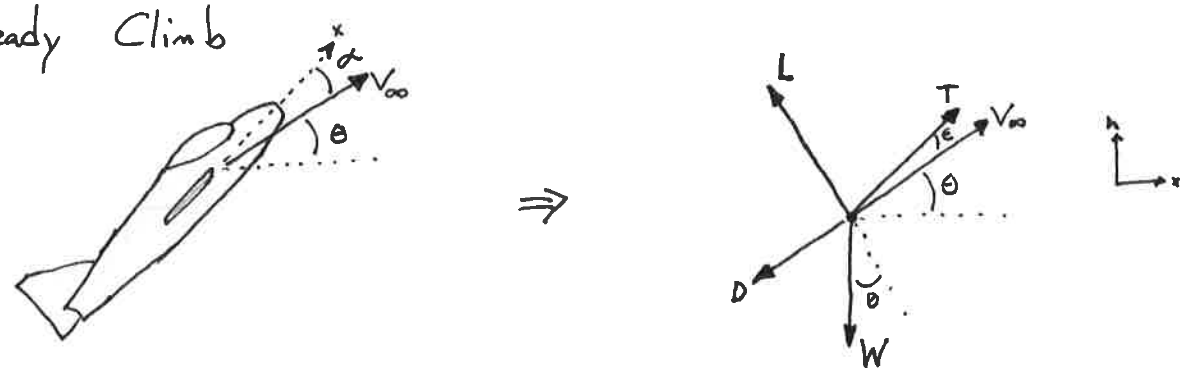
Lesson 9

Climb + Descend

Steady Climb



Free body diagram

Assume ϵ is relatively small ($\cos\epsilon \approx 1$, $\sin\epsilon \approx 0$)

In the direction of flight ($\parallel V_\infty$)

$$T - D - W\sin\theta = 0 \qquad \Rightarrow \sin\theta = \frac{T-D}{W}$$

The angle depends on the excess thrust and weight.

In the orthogonal "lift" direction, ($\perp V_\infty$)

$$L - W\cos\theta = 0$$

The rate of climb is the h component of V_∞

$$\dot{h} = V_\infty \sin\theta \equiv V_v$$

← Engineers would say ft/s, pilots say ft/min (fpm)

and the horizontal component is

$$\dot{x} = V_\infty \cos\theta \equiv V_H$$

Take the rate of climb ($\dot{h} = V_\infty \sin\theta$) and substitute in the FBD ~~angle~~ ($\sin\theta = \frac{T-D}{W}$)

$$\dot{h} = V_\infty \frac{T-D}{W} = \frac{V_\infty T - V_\infty D}{W} = \frac{\text{Propulsive Power} - \text{Drag Power}}{\text{Weight}}$$

AKA Required Power

$$= \frac{\text{Excess Power}}{\text{Weight}}$$

The rate of climb depends on the excess power.

Invoking algebra elves (to do the work),

$$\dot{h} = V_\infty \left(\underbrace{\frac{T}{W}}_{\text{thrust to weight ratio}} - \frac{1}{2}\rho V_\infty^2 \underbrace{\left(\frac{W}{S}\right)^{-1}}_{\text{Inverse wing loading}} C_{D_0} - \underbrace{\left(\frac{W}{S}\right)}_{\text{wing loading}} \underbrace{\frac{2K\cos^2\theta}{\rho V_\infty^2}}_{\text{notice that } \cos^2\theta \text{ means that induced drag is zero when going straight up!}} \right)$$

- Increasing $\frac{T}{W}$ always increases $\dot{h}$
- Increasing $\frac{W}{S}$ can increase or decrease $\dot{h}$ depending on geometry and flight conditions

$$\dot{h} = \frac{V_\infty T - V_\infty D}{W} = V_\infty \left(\frac{T}{W} - \frac{D}{W} \right)$$

$$D = \frac{1}{2} \rho V_\infty^2 S \left(C_{D_0} + K C_L^2 \right)$$

$$L = W \cos\theta$$

$$\frac{1}{2} \rho V_\infty^2 S \, C_L = W \cos\theta$$

$$C_L = \frac{W \cos\theta}{\frac{1}{2} \rho V_\infty^2 S}$$

$$V_\infty \left(\frac{T}{W} - \frac{D}{W} \right) = V_\infty \left(\frac{T}{W} - \frac{\frac{1}{2} \rho V_\infty^2 S}{W} \left(C_{D_0} \right) \right.$$

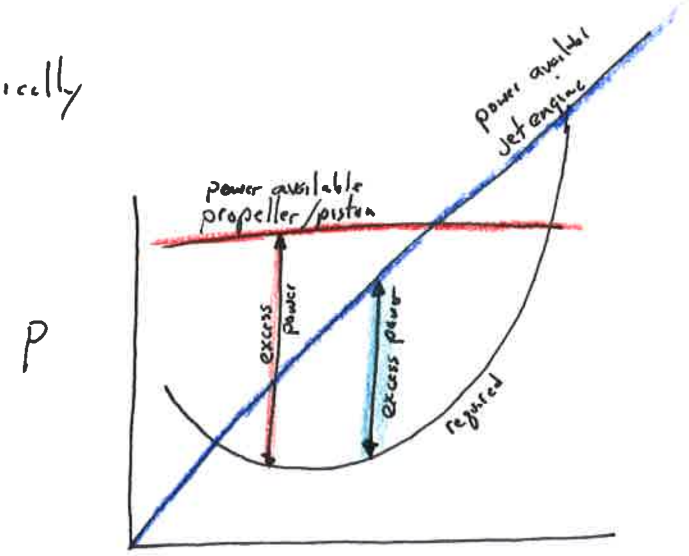
$$\left. - \frac{1}{2} \frac{\rho V_\infty^2 S}{W} K \frac{W^2 \cos^2\theta}{\left(\frac{1}{2} \rho V_\infty^2 S \right)^2} \right)$$

$$= V_\infty \left(\frac{T}{W} - \frac{1}{2} \rho V_\infty^2 \left(\frac{W}{S} \right)^{-1} C_{D_0} \right.$$

$$\left. - \frac{K W \cos^2\theta}{\frac{1}{2} \rho V_\infty^2 S} \right)$$

$$\dot{h} = V_\infty \left(\frac{T}{W} - \frac{1}{2} \rho V_\infty^2 \left(\frac{W}{S} \right)^{-1} C_{D_0} - \left(\frac{W}{S} \right) \left(\frac{2 K \cos^2\theta}{\rho V_\infty^2} \right) \right)$$

Graphically

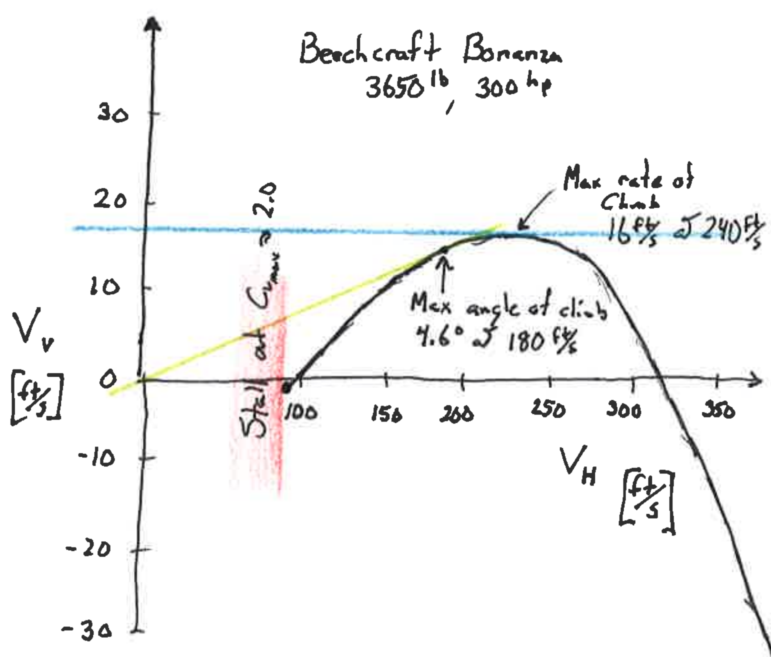


- Notice that the piston/propeller aircraft has a maximum excess power at a lower velocity than the completely identical jet powered aircraft.

- The jet aircraft maximizes rate of climb at a higher speed than the equivalent prop powered aircraft. (within the bounds of ~~[illegible]~~ feasible engines).

Hodograph (Greek: "hod" meaning "way" + graph)

plot vertical and horizontal velocities



$$\dot{h} = V_\infty \left(\frac{T}{W} - \frac{1}{2}\rho V^2 \left(\frac{W}{S}\right)^{-1} C_{D_0} - \left(\frac{W}{S}\right) \frac{2K\cos^2\theta}{\rho V^2} \right.$$

$$\dot{x} = V_\infty \cos\theta$$

$$\theta = \operatorname{asin}\left(\frac{T-D}{W}\right) \Rightarrow \frac{\dot{h}}{V} = \sin\theta$$

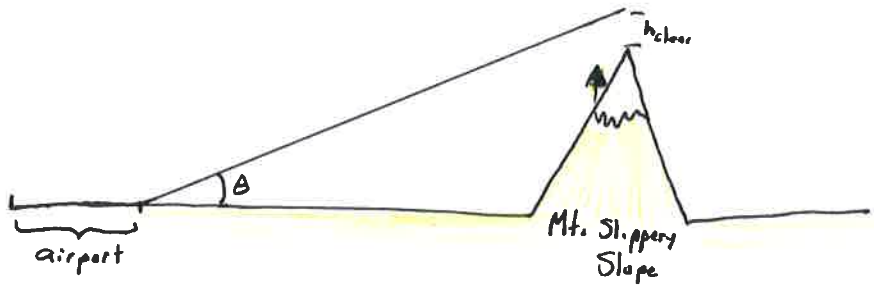
$$L = W\cos\theta$$

← These speeds are only reached in a dive.

In FAA/pilot notation, the best <u>rate</u> of climb is V_y "Vee why"

the best angle of climb is V_x "Vee ex"

Ex: In your Bonanza, you wish to clear a mountain ridge by the most altitude. Which speed do you choose?



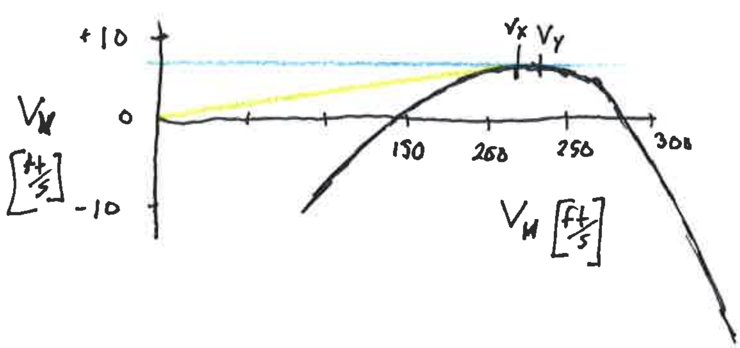
$\boxed{V_x}$ best angle of climb

Ex: In your Bonanza, you wish to get to 5000 ft as <u>soon</u> as possible. Which speed do you choose?

$\boxed{V_y}$ best rate of climb

Ex: In your Bonanza, you are on final approach and encounter a nasty downburst (10 ft/s down), which speed should you fly to survive?

Shift hodograph down by 10 ft/s



- For a stronger downburst, would you pick a maximum ~~minimum~~ time to impact or a minimum angle of descent?
- V_x changes with wind (both horizontal and vertical wind)

Gliding $(T=0)$

From earlier with $T=0$

$$\dot{h} = -\frac{V_\infty D}{W} = -\frac{1}{2}\rho V^3 \left(\frac{W}{S}\right)^{-1} C_{D_0} - \left(\frac{W}{S}\right)\frac{2k\cos^2\theta}{\rho V}$$

And

$$L = W\cos\theta \quad \text{and} \quad D = W\sin\theta$$

$$\frac{L}{D} = \frac{W\cos\theta}{W\sin\theta} = \frac{1}{\tan\theta}$$

$$\theta = \operatorname{atan}\left(\left(\frac{L}{D}\right)^{-1}\right) = \operatorname{atan}\left(\left(\frac{C_L}{C_D}\right)^{-1}\right)$$

Since $L = \frac{1}{2}\rho V^2 S C_L$, the velocity of a glide is

$$V = \sqrt{\frac{2\cos\theta}{\rho C_L}\cdot\frac{W}{S}} \qquad V = \sqrt{\frac{2L}{\rho S C_L}} = \sqrt{\frac{2\,W\cos\theta}{\rho S C_L}}$$

To ~~maximize~~ minimize θ, the L/D ratio should be maximized. For small θ

$$\left(\frac{L}{D}\right)_{max} = \sqrt{\frac{1}{4C_{D_0}k}} \approx \sqrt{\frac{\pi AR e}{4C_{D_0}}}$$

with a velocity of

$$V_{L/D\,max} = \sqrt{\frac{2}{\rho}\frac{W}{S}}\;\sqrt[4]{\frac{k}{C_{D_0}}}$$

To minimize the sink rate $\dot{h}$,

$$\dot{h}_{\substack{min\\sink\\rate}} = \sqrt{\frac{2}{\rho\,(C_L^3/C_D^2)}\cdot\frac{W}{S}}$$

at a velocity of

$$V_{\substack{min\\sink\\rate}} = \sqrt{\frac{2}{\rho}\cdot\sqrt{\frac{K}{3C_{D_0}}}\,\frac{W}{S}} \qquad V_{\substack{min\\sink\\rate}} = \sqrt{\frac{2}{\rho}\sqrt{\frac{K}{3C_{D_0}}\,\frac{W}{S}}}$$

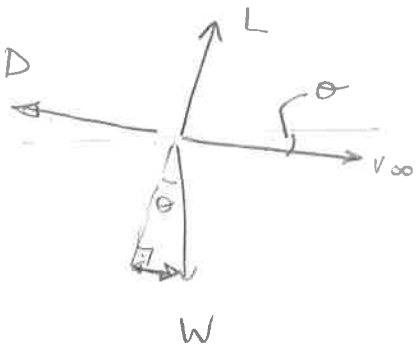
11a
OR
12a

GLIDING

Again using

$$\cancel{T}-D+W\sin\theta=0$$

$$L-W\cos\theta=0$$



$\rightarrow T=0$

$$\sin\theta=+\frac{D}{W}$$

& $L=W\cos\theta$
$D=W\sin\theta$

$$\dot{h}=V_\infty\sin\theta=+V_\infty\frac{D}{W}$$

$$\therefore\ \dot{h}=+\frac{1}{2}\rho V_\infty^3\left(\frac{W}{S}\right)^{-1}C_{D_0}+\frac{W}{S}\left(\frac{2K\cos^2\theta}{\rho V_\infty}\right)$$

$$\left.\begin{aligned}L&=W\cos\theta\\D&=W\sin\theta\end{aligned}\right\}\tan\theta=\frac{D}{L}=\frac{C_D}{C_L}$$

$$L=\frac{1}{2}\rho V^2SC_L$$

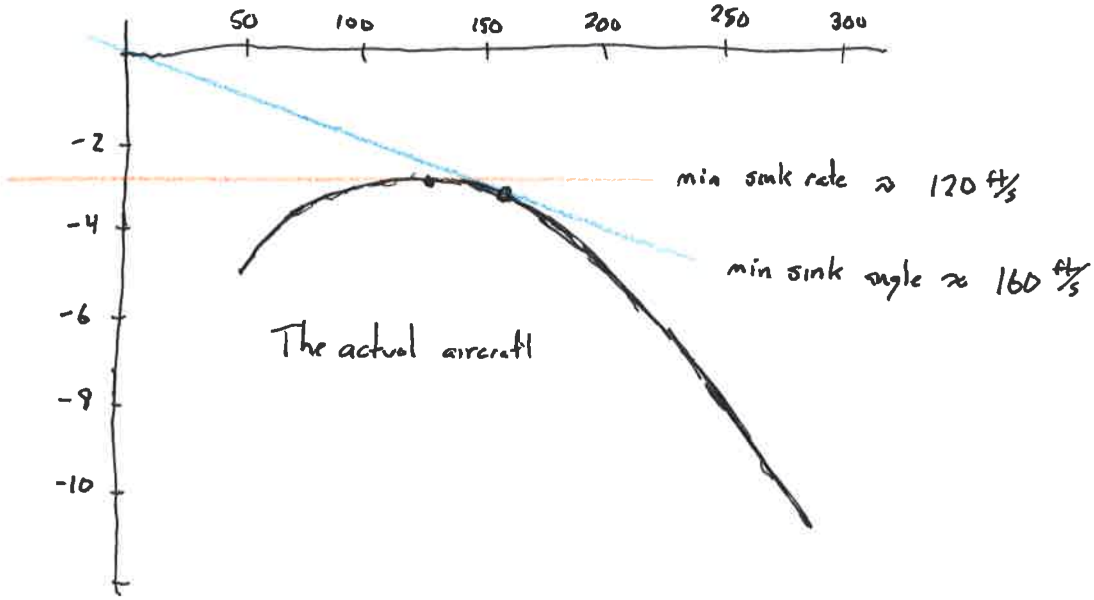
$$\therefore V=\sqrt{\frac{2L}{\rho SC_L}}=\sqrt{\frac{2W\cos\theta}{\rho SC_L}}$$

To minimize θ, $\frac{L}{D}$ should be maximized

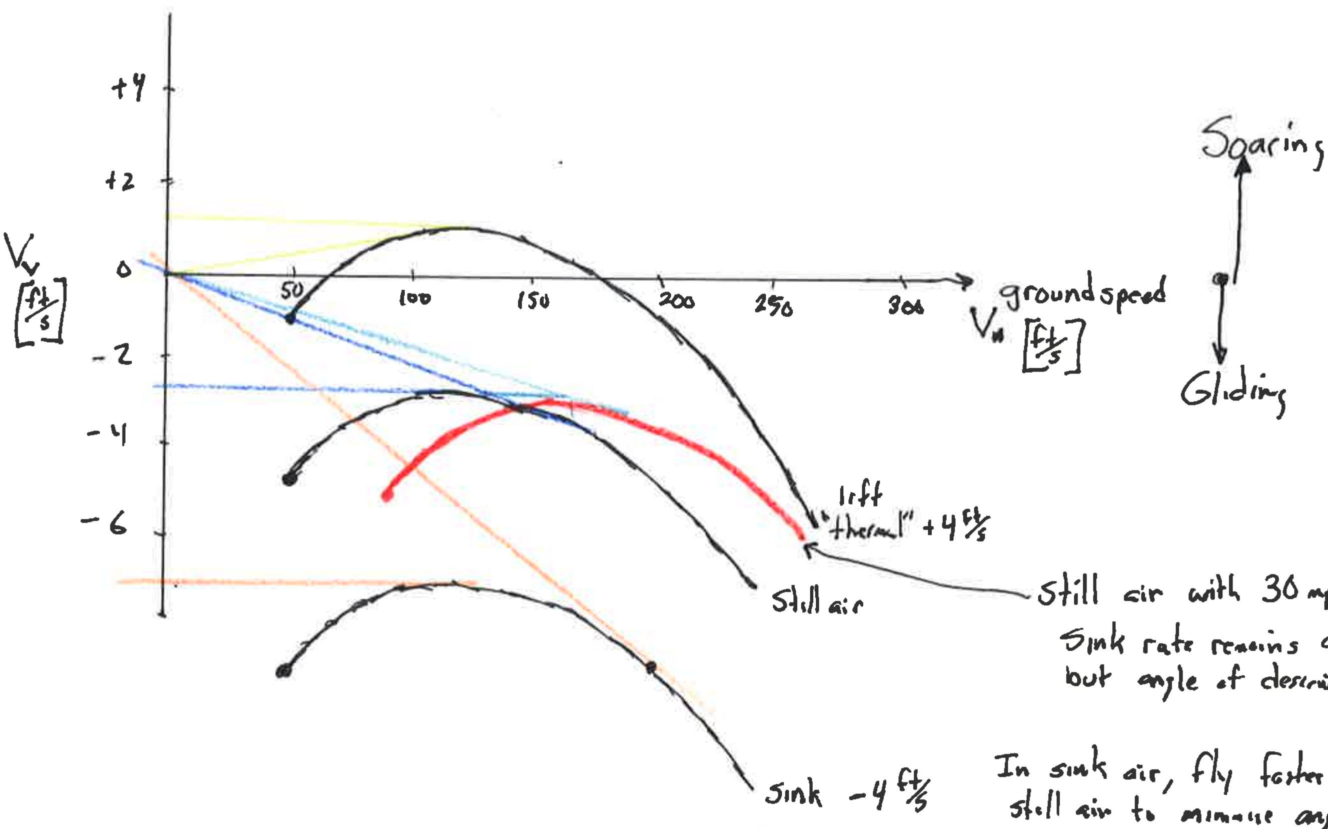
For a Nimbus 2 glider (approximate)

$AR = 28$ $W = 1200^{lb}$ $b = 66.5^{ft}$

$C_{D_0} \approx 90$ counts



Add wind (both vertical and horizontal)



Still air with 30 mph ~~headwind~~ tail wind
Sink rate remains constant but angle of descent is less.

In sink air, fly faster than in still air to minimize angle of descent