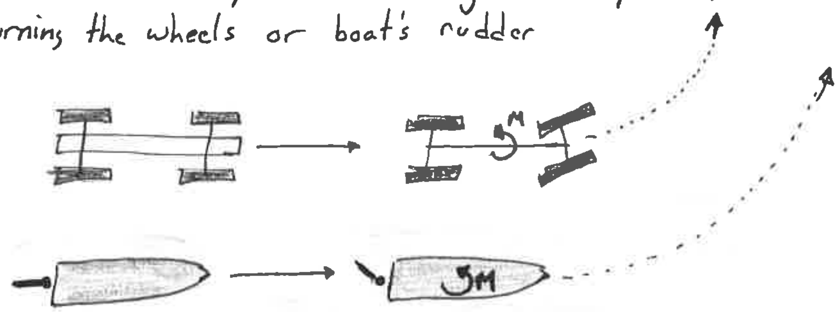
Lesson 11

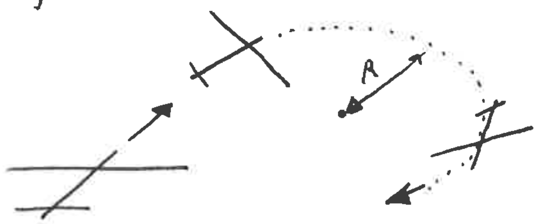
Accelerated Flight: Level turns

Turns

In cars and boats, a turn is generated by a yaw moment generated by turning the wheels or boat's rudder



The aircraft is different. A proper coordinated turn is established by banking the aircraft.

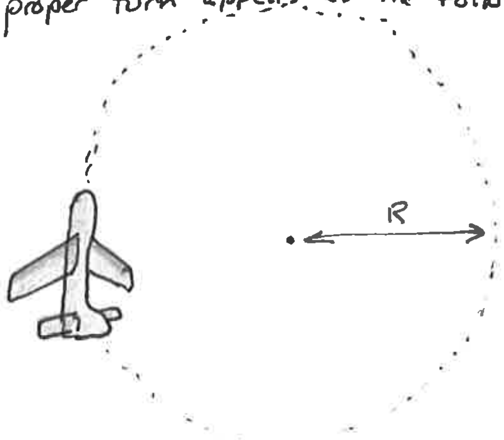


1) Apply aileron deflection until bank angle reached.
2) Return aileron deflection to <u>zero</u>
3) Slight increase in α with elevator

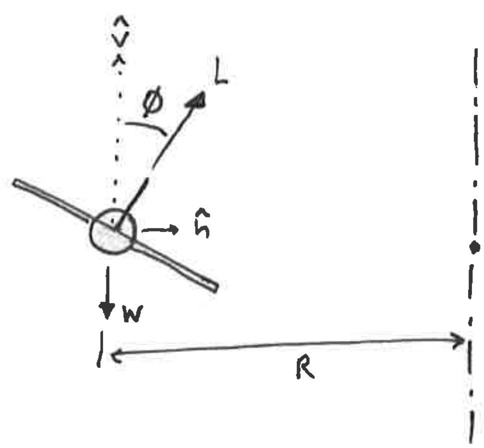
An uncoordinated "slipped" turn <u>could</u> be generated by rudder and cross controlled ailerons, but is not efficient, not elegant, not rapid, and not common. (and often not safe!).

cross controls.... see why later.

A proper turn appears as the following



top view



Rear View

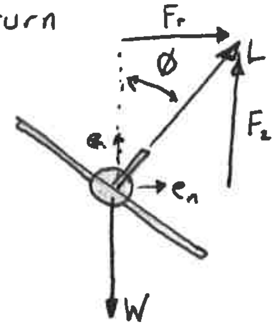
Lift in the $\hat{v}$ direction resolves in

$$F_{\hat{v}} = L\cos\phi$$

and

$$F_{\hat{n}} = L\sin\phi$$

Level turn



Lift decomposes into F_r and F_z (orthogonal) such that

$$F_z - W = 0 \qquad \text{no vertical acceleration}$$

$$F_z = L\cos\phi \quad \text{and} \quad F_r = L\sin\phi$$

$$\phi \equiv \text{bank angle}$$

Returning to the tangential and normal vectors (e_t and e_n)

e_t is in the direction of flight: $e_t \parallel V_\infty$

e_n is along the horizontal direction.

Thus, in the e_n direction

$$F_n = m\frac{V^2}{R} \quad \text{or} \quad \boxed{L\sin\phi = m\frac{V_\infty^2}{R}}$$

To sustain this level turn in the z direction

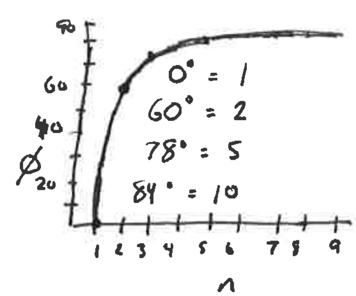
$$F_z = L\cos\phi = W \quad \text{or} \quad \boxed{L = \frac{W}{\cos\phi}}$$

lift force must increase.

$$\text{or} \quad \frac{L}{W} = \frac{1}{\cos\phi}$$

We call the ratio of L/W the load factor $n \equiv \dfrac{L}{W}$

Thus $n = \dfrac{L}{W} = \dfrac{1}{\cos\phi} \Rightarrow \phi = \text{acos}\left(\dfrac{1}{n}\right)$



$$\boxed{\text{For a level turn, } n \text{ is a function of } \phi \text{ only}}$$

Turn radius

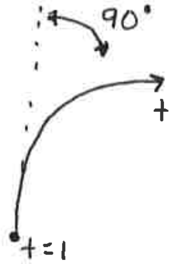
$$L\sin\phi = m\frac{V_\infty^2}{R} = \frac{W}{g}\frac{V_\infty^2}{R}$$

Solve for R: $R = \dfrac{W}{L}\dfrac{V_\infty^2}{g\sin\phi} = \dfrac{V_\infty^2}{g n \sin\phi} = \dfrac{V_\infty^2}{g n \frac{1}{n}\sqrt{n^2-1}}$

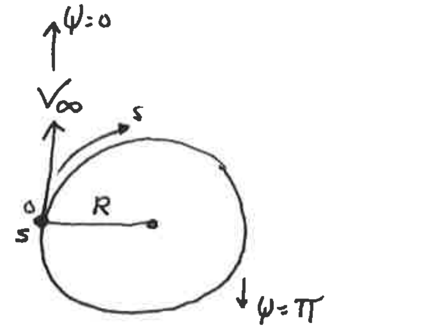
$$\boxed{R = \frac{V_\infty^2}{g\sqrt{n^2-1}}}$$

To reduce R, low V_∞ and high n.

Turn Rate



$$\frac{90^\circ}{1\ \text{sec}} = \frac{\Delta\psi}{\Delta t}$$



$S = \pi 2R = R\psi_{rad}$

$\dot{S} = V = \frac{dS}{dt} = R\frac{d}{dt}\psi$

thus $\dot{\psi} = \frac{V}{R}$

Define in terms of $\omega = \frac{d\psi}{dt} = \frac{V_\infty}{R}$ and $R = \frac{V_\infty^2}{g\sqrt{n^2-1}}$

$$\omega = \frac{V_\infty}{R} = V_\infty \cdot \frac{1}{V_\infty^2} \cdot g\sqrt{n^2-1}$$

$$\boxed{\omega = \frac{g\sqrt{n^2-1}}{V_\infty}}$$

This is in rad/s.

$$TR \equiv \text{turn rate } {}^\circ/s = \frac{180^\circ}{\pi}\frac{g\sqrt{n^2-1}}{V_\infty}$$

Ex: The new airforce trainer program (T-X) to replace the T-38 has turn requirements.

If the RFP calls for 18°/s at 6.5g, what is the maximum airspeed consistent with this requirement?

$$V_\infty \le \frac{180^\circ}{\pi}\cdot\frac{1}{TR}\cdot g\sqrt{n^2-1} = \frac{180\cancel{^\circ}}{\pi}\;\frac{1\,\cancel{sec}}{18\cancel{^\circ}}\cdot\frac{32.174\,ft}{s^{\cancel{2}}}\sqrt{6.5^2-1}$$

$$\le 658\ \frac{ft}{s} \quad \approx 390\ \text{kts}$$

Ex: An $\underbrace{\text{SR-71}}_{3000\,ft/s}$ and a $\underbrace{\text{C-150}}_{80\,ft/s}$ both pull 2g, what is the turn radius of each?

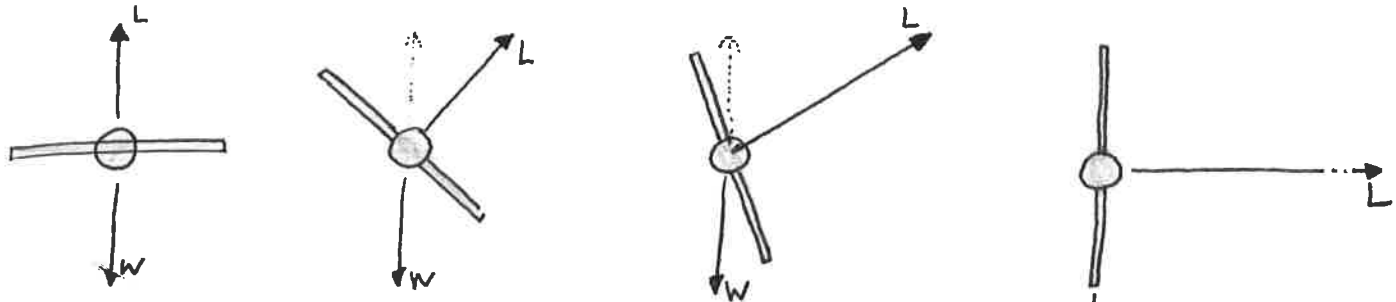
$$R = \frac{V_\infty^2}{g\sqrt{n^2-1}}$$

$$R_{SR71} = \frac{3000^2\,ft^{\cancel{2}}}{\cancel{s^2}}\;\frac{\cancel{s^2}}{32.174\,\cancel{ft}}\;\frac{1}{\sqrt{2^2-1}} = 30.6\ \text{mi}$$

$$R_{150} = \frac{80^2\,ft^2}{s^2}\;\frac{s^2}{32.174\,ft}\;\frac{1}{\sqrt{2^2-1}} = 114\ ft$$

← Need to verify $C_L \le C_{L_{max}}$

Maximum <u>sustained</u> level turn load factor



$\phi = 0°$, $n = 1$ | $\phi = 45°$, $n = \sqrt{2}$ | $\phi = 60°$, $n = 2$ | $\phi = 90°$, $n = \infty$ (Impossible)

The $\underbrace{\text{vertical component of the lift force}}_{L\cos\phi}$ $\underbrace{\text{must equal}}_{=}$ $\underbrace{W}_{W}$.

So the lift increases with bank angle ϕ. Induced drag increases with C_L^2.

$$T = D = \frac{1}{2}\rho V^2 S \left(C_{D_0} + kC_L^2\right) \quad \text{with} \quad C_L = \frac{2nW}{\rho S V^2}$$

$$T = \frac{1}{2}\rho V^2 S\left(C_{D_0} + k\frac{4n^2W^2}{\rho^2 S^2 V^4}\right)$$

Solve for n (the load factor)

$$T - \frac{1}{2}\rho V^2 S C_{D_0} = \frac{\frac{1}{2}\rho V^2 S k}{\rho^2 S^2 V^4} 4n^2W^2 = \frac{1}{2}\frac{1}{\rho}\frac{1}{S}\frac{k}{V^2} 4n^2W^2$$

$$\frac{2\rho S V^2}{k\,4W^2}\left(T - \frac{1}{2}\rho V^2 S C_{D_0}\right) = n^2$$

$$n^2 = \frac{1}{2}\frac{\rho V^2}{\cancel{4}K}\left(\frac{T}{W}\right)\left(\frac{S}{W}\right) - \frac{1}{4}\rho^2V^4\left(\frac{S^2}{W^2}\right)\frac{C_{D_0}}{K}$$

$$n = \sqrt{\frac{1}{2}\underbrace{\rho V^2}_{+}\left(\frac{T}{W}\right)\underbrace{\left(\frac{W}{S}\right)^{-1}}_{+}\frac{1}{K} - \underbrace{\frac{1}{4}\rho^2V^4\left(\frac{W}{S}\right)^{-2}\frac{C_{D_0}}{K}}_{\text{Always positive } +}}$$

the load factor is strongly determined by the thrust/weight ratio.

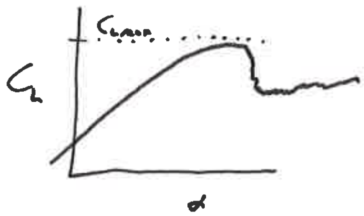
$\frac{T}{W}\uparrow \Rightarrow n\uparrow$ $\qquad$ $AR\uparrow \Rightarrow n\uparrow$

$\frac{W}{S}\uparrow \Rightarrow n\downarrow$

In the above derivation, there is an assumption about C_L

$$C_L = \frac{2nW}{\rho S V^2} \quad \text{and this } C_L \text{ is plugged into } \quad T = D = \bar{q} S \left(C_{D_0} + K C_L^2 \right)$$

<u>But</u> C_L is limited by aerodynamics to $C_{L_{max}}$



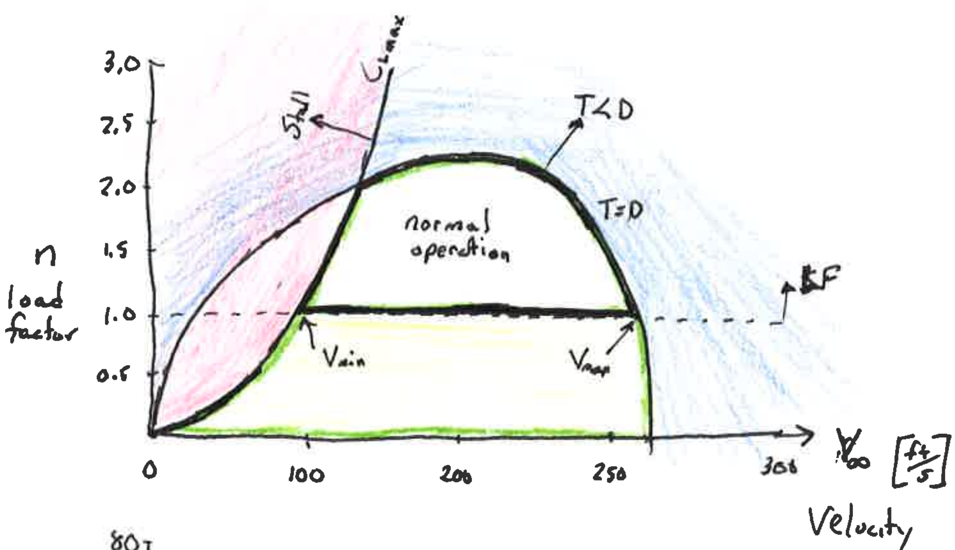
In this case, we can compute n based on a constant $C_{L_{max}}$

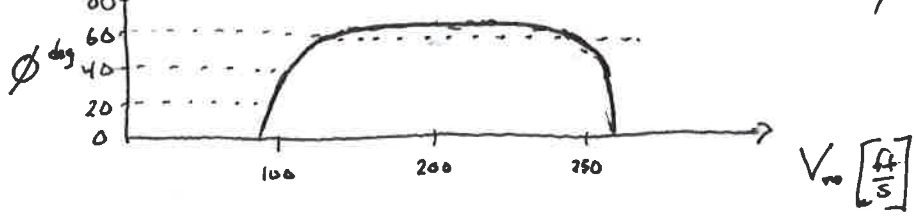
$$n = \frac{L}{W} = \frac{\frac{1}{2}\rho V^2 S C_L}{W} \quad \Rightarrow \quad n_{max_{C_L}} = \frac{1}{2}\rho V^2 \left(\frac{S}{W}\right) C_{L_{max}}$$

So for an aircraft,

$$n = \min\left(\underbrace{n_{max_{C_L}}}_{\text{above}} , \underbrace{n_{max_{T=D}}}_{\text{prev page}} \right)$$

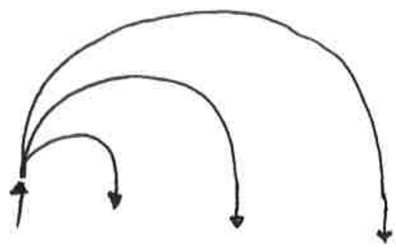
Ex: Our Bonanza Example





Aside for the pilots:
Now you know why your CFI made you practice 60° banked turns!

Minimum Turn Radius



Why might this be useful?

- Flying in a canyon
- Minimize detection or threat (military)

$$R = \frac{V_\infty^2}{g\sqrt{n^2-1}} \qquad \text{minimize } R \Rightarrow \frac{dR}{dV_\infty} = 0$$

However, $n(V_\infty)$ so this is messy (see ADP p 330).

Velocity:

$$(V)_{R_{min}} = \sqrt{\frac{4k\,(W/S)}{\rho\,(T/W)}} = 2\sqrt{\frac{k}{\rho}\left(\frac{W}{S}\right)\left(\frac{T}{W}\right)^{-1}}$$

Load Factor:

$$(n)_{R_{min}} = \sqrt{2 - \frac{4kC_{D_0}}{(T/W)^2}} = 2\sqrt{\frac{1}{2} - \frac{kC_{D_0}}{1}\left(\frac{T}{W}\right)^{-2}}$$

Radius:

$$R_{min} = \frac{4k}{g\rho}\left(\frac{W}{S}\right)\left(\frac{T}{W}\right)^{-1}\frac{1}{\sqrt{1-4\underbrace{kC_{D_0}}\left(\frac{T}{W}\right)^{-2}}}$$

$kC_{D_0} \ll 1$ unless the a/c is really inefficient

Approximate

$$R_{min} \approx \underbrace{\frac{4k}{g\rho}\left(\frac{W}{S}\right)\left(\frac{T}{W}\right)^{-1}}$$

High wing loading increases radius.

High thrust to weight decreases radius.

Radius increases with altitude

Radius increases with induced drag (i.e. $\frac{1}{AR}$)

$C_{L_{max}}$

Check to ensure $C_L < C_{L_{max}}$

Maximum Turn Rate

Maximize ω

Summary:

$$V_{\omega_{max}} = \sqrt{\frac{2}{\rho}\left(\frac{W}{S}\right)}\left(\frac{K}{C_{D_0}}\right)^{1/4}$$

$$n_{\omega_{max}} = \sqrt{\left(\frac{T}{W}\right)\frac{1}{\sqrt{K C_{D_0}}} - 1}$$

$$\omega_{max} = g\sqrt{\frac{\rho}{1}\left(\frac{W}{S}\right)^{1}\cdot\left(\frac{T}{W}\frac{1}{2K} - \sqrt{\frac{C_{D_0}}{K}}\right)}$$

* fix the error in your book "g" not "g"

Ex:

Which has a higher turn rate, an F-16 at max thrust (afterburner) or an F-16 at military power (dry)?

$W/S = \frac{26500\ lb}{300\ ft^2}$ $\quad T/W_{AB} = \frac{28600}{26500}$ $\quad T/W_{dry} = \frac{17155}{26500}$

F-16 $\quad C_{D_0}$ depends on Mach #. Subsonic $C_{D_0} \approx 175$ counts (clean!)

From flight test data, I estimated $K \approx 0.16$

From lifting line $K \approx \frac{1}{\pi A R e} \approx \frac{1}{\pi \cdot 3.5 \cdot 0.9} \approx 0.1$

} difference? strakes

Δ

Use WT data!

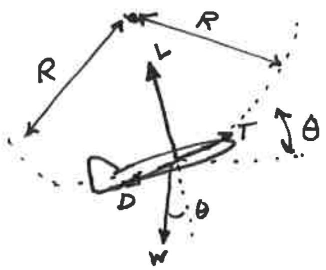
$$V_{\omega_{max}} = \sqrt{\frac{2\ \cancel{ft^4}\,\cancel{ft^3}}{0.002375\ slugs}\,\frac{26500\ lbf}{300\ ft^2}\,\frac{slug\ ft}{\cancel{lbf}\ s^2}}\ \sqrt[4]{\frac{0.16}{0.0175}} = 475\ ft/s \approx 280\ kt$$

$$n_{\omega_{max}} = \sqrt{\frac{28600}{26500}\sqrt{\frac{1}{0.16\cdot 0.0175}} - 1} = 4.4 \qquad n_{\omega_{max_{dry}}} \approx 3.35$$

$$\boxed{\omega_{max_{AB}} = 17\ °/s} \qquad \omega_{max_{dry}} = 12.5\ °/s$$

In a dog fight, you need the $\frac{T}{W}$ that the afterburner provides.

Instantaneous Turn Performance. ("pitch" rate)



In the e_n direction,

$$m \frac{V_\infty^2}{R} = L - W\cos\theta$$

Solve for R:

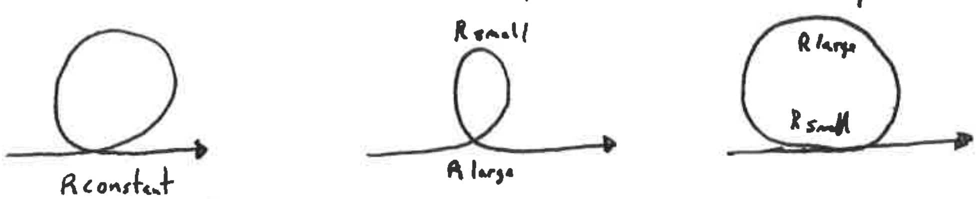
$$R = \frac{m V_\infty^2}{L - W\cos\theta} = \frac{W}{g}\frac{V_\infty^2}{L - W\cos\theta} = \frac{V_\infty^2}{g}\frac{1}{\left(\frac{L}{W}\right) - \cos\theta}$$

$$\boxed{R_{inst} = \frac{V_\infty^2}{g(n - \cos\theta)}}$$

Turn rate

$$\omega = \frac{V_\infty}{R} = \frac{V_\infty \,\big|\, g(n-\cos\theta)}{\big|\, V_\infty^2} = \boxed{\frac{g(n-\cos\theta)}{V_\infty} = \omega}$$

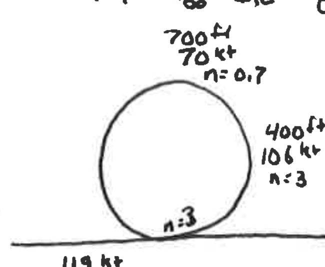
Ex: Given a constant load factor, describe a loop with $\frac{T}{W} < 1$ and $n \approx 2$



A: We know that V will ~~dec~~ decrease as h increases

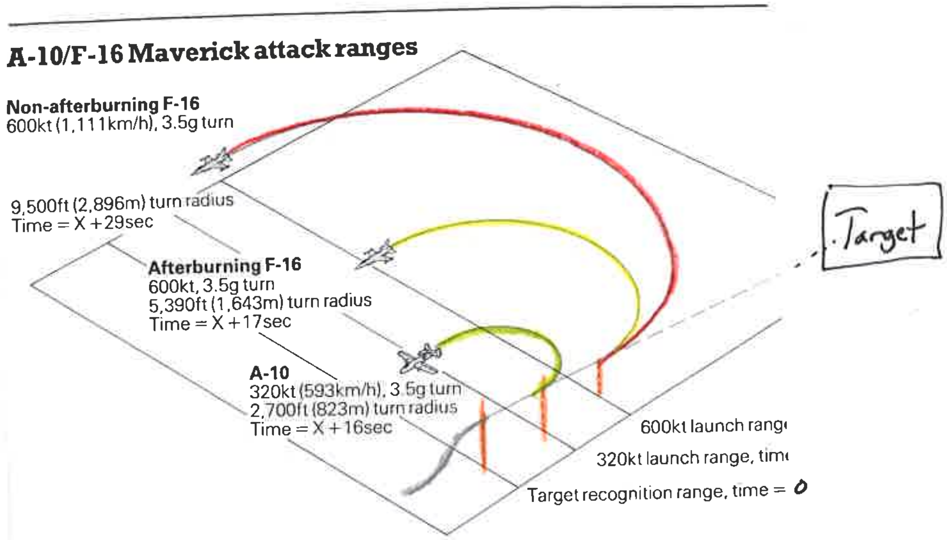
Aerobatic pilots vary n with V_∞ and θ to create a perfect loop.

C150
Aerobat.

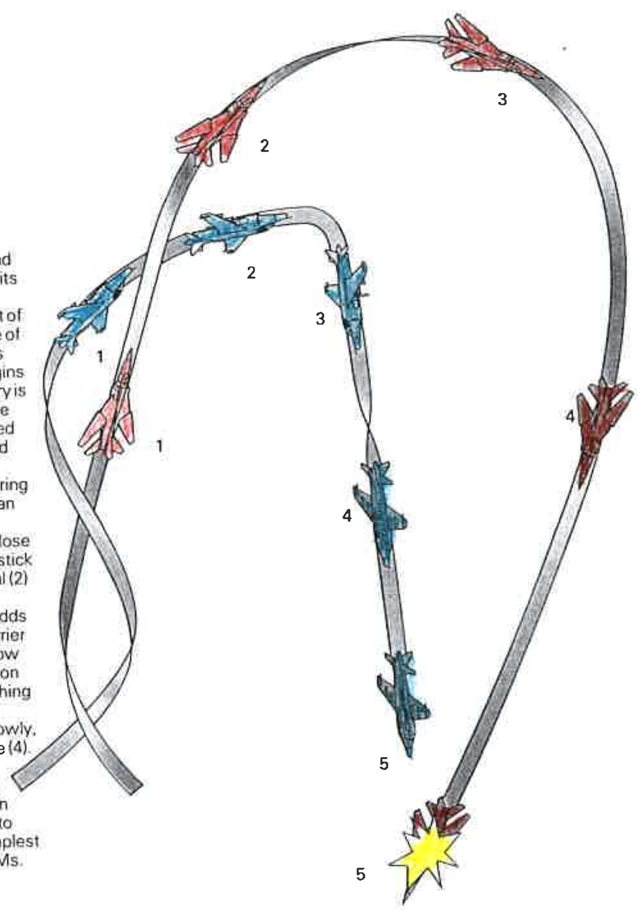


source: Kershner, Basic Aerobatic Flight Manual

Turn Rate Applications



Source: Modern Fighting Aircraft A-10, Sweetman



Right: In this so-called "climb and flip" the Harrier performs one of its numerous "impossible" manoeuvres, which are now part of the routine air-combat repertoire of all experienced US Marine Corps Harrier pilots. The sequence begins with the Harrier (whose trajectory is indicated by a blue line in all these illustrations) and its adversary (red line) climbing in a steep spiral and losing speed, the enemy close behind and eager to get within firing parameters before the Harrier can pull one of its tricks. From this position (1), with the enemy in close trail, the Harrier pilot using light stick forces pulls well past the vertical (2) and, as the speed bleeds away through the 200-knot level, he adds a small nozzle angle (3). The Harrier very quickly flips to a 90° nose-low attitude. The enemy has no option but to follow a semi-ballistic arching curve to end up going steeply downhill. Still travelling quite slowly, the Harrier goes into full reverse (4). There is no way the enemy can avoid going on down past what seems to be a Harrier stopped in mid-air. When the enemy gets to position (5) he presents the simplest possible target, for guns or AAMs.

Source: MFA: Harrier, Gunston

Energy Height

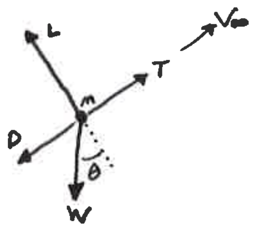
From basic physics, the total energy of a system is kinetic + potential

$$E = mgh + \frac{1}{2}mV^2$$

Specific energy is energy per weight

$$\underbrace{H_e}_{\text{energy height}} = \frac{E}{W} = \frac{mgh + \frac{1}{2}mV^2}{mg} = \boxed{\underbrace{h}_{\text{altitude}} + \underbrace{\frac{V_\infty^2}{2g}}_{\text{velocity}} = H_e}$$

Excess Power



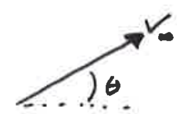
In e_t direction, $T - D - W\sin\theta = m\frac{dV}{dt}$

and $W = mg$

$$T - D = W\sin\theta + \frac{W}{g}\frac{dV}{dt} = W\left(\sin\theta + \frac{1}{g}\frac{dV}{dt}\right)$$

Mult by V_∞ to get power and divide by W

$$V_\infty\frac{(T-D)}{W} = V_\infty\left(\sin\theta + \frac{1}{g}\frac{dV}{dt}\right)$$



$\Rightarrow \dot{h} = V_\infty \sin\theta$

$$= \underbrace{\dot{h} + \frac{V_\infty}{g}\frac{dV}{dt}}$$

Now, take derivative of H_e

$$\frac{dH_e}{dt} = \frac{d}{dt}\left(h + \frac{V_\infty^2}{2g}\right) = \frac{dh}{dt} + \underbrace{\frac{2V_\infty}{2g}\frac{dV_\infty}{dt}}$$

Notice

$$\frac{dH_e}{dt} = V_\infty\left(\frac{T-D}{W}\right) \equiv \frac{\text{Excess Power}}{W} = P_s \equiv \text{Specific Excess Power}$$

$$\boxed{\text{Specific Excess Power contributes to either altitude or velocity}}$$

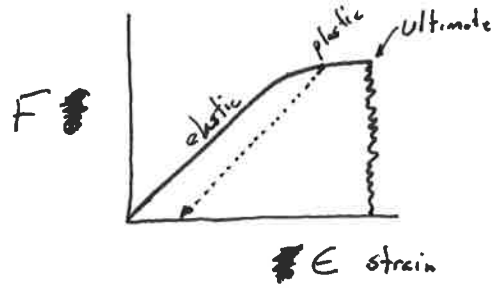
V-n diagram "Flight Envelope"

Where aerodynamics and structures interact.

From structures, you design an aircraft to meet a particular load factor

e.g. $n^+ = 5 \quad n^- = 3$ at a particular weight

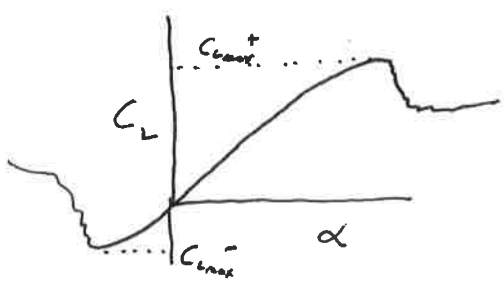
you design a buffer region between the load limit (plastic) and the ultimate load (breaks)



From aerodynamics:

$$L = \frac{1}{2}\rho V^2 C_L S \quad \Rightarrow \quad n = \frac{L}{W} = \frac{1}{2}\rho V^2 C_L \left(\frac{W}{S}\right)^{-1}$$

And a maximum C_L is determined by aero



$$\Rightarrow \quad n_{max} = \frac{1}{2}\rho V^2 C_{L_{max}} \left(\frac{W}{S}\right)^{-1}$$

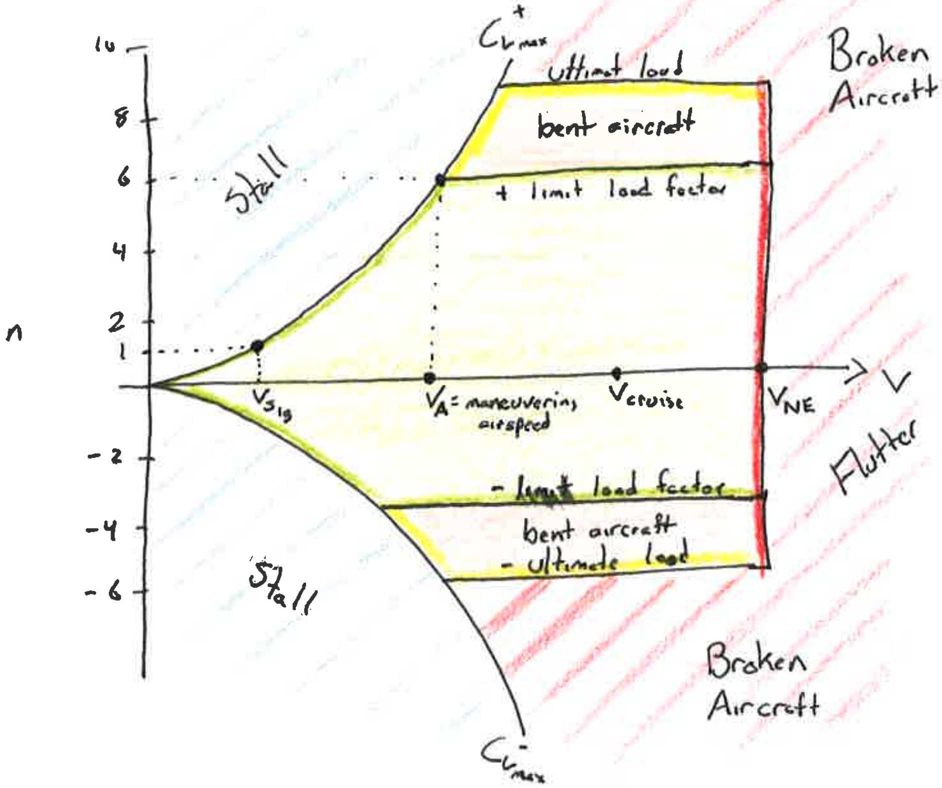
From aerodynamics + performance + aero-structural dynamics

- The aircraft has a maximum "red-line" airspeed. V_{NE}

  Above this airspeed, parts may not remain on the aircraft!

- ~~Certain~~ All aircraft will exhibit "flutter" above a certain dynamic pressure at certain flight conditions

V-n



$V_{S_{1g}}$ is the stall speed at $n=1$

V_A = maneuvering speed, the intersection of aero and structural limits.

At V_A, the aircraft can not be broken/bent by "normal" acceleration (e_n).

At V_A, the aircraft **can** be broken/bent by control deflections and non e_n accelerations.

V_C = Cruise speed (FAA certification)

V_{NE} = Never exceed.

The aircraft can not sustain operation outside of the stall $C_{L_{max}}$ curves.