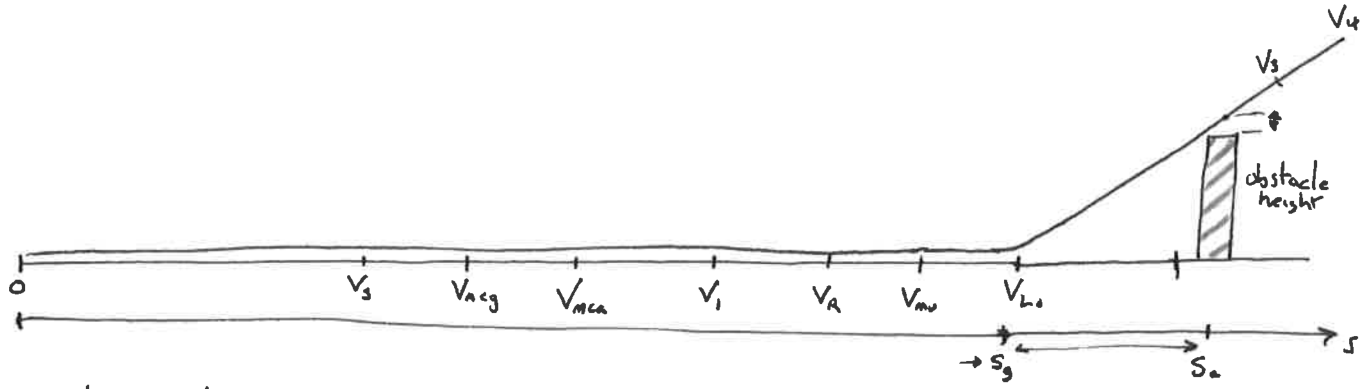
Lesson 12

Takeoff and Landing

Take off Nomenclature



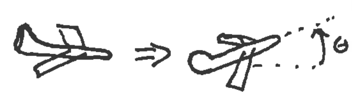
Speeds

V_s = stall speed

V_{mcg} = minimum controllable speed on ground

V_{mca} = minimum controllable speed in air

V_1 = Decision speed "minimum speed where the takeoff can continue following a failure (engine)"
Committed at this speed.

V_R = Rotation speed: pitch atitude is increased 

V_{mu} = minimum unstick speed "minimum possible T/O speed given aircraft geometry"

V_{LO} = Lift off speed

V_3 = Flap retraction Speed

V_4 = Climb out speed

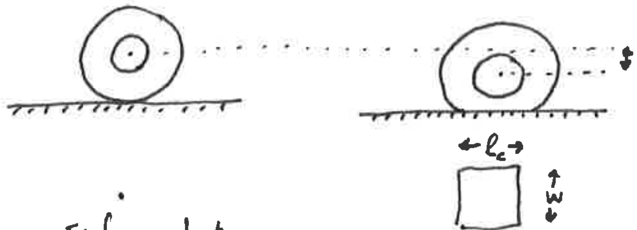
Distances

S_g = Ground roll (takeoff distance from 0 velocity to wheels off)

S_a = airborne distance (takeoff distance from wheels off to clearing an obstacle)

Ground Roll

A tire on the ground has a rolling resistance



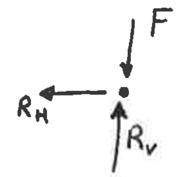
surface contact area ≈ 0

$W/A = \infty$!?!

surface contact

$A = w \cdot \ell_c$

W/A = pressure $\approx$ tire pressure



$F = R_v$

20 psi for a light plane
130 psi for civilian jet main
200 psi " " " nose
425 psi SR71 (N_2)

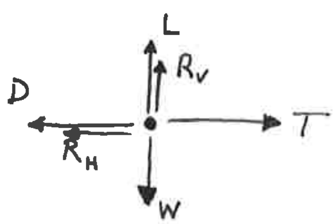
Rolling resistance depends on the applied load and the tires rolling friction coefficient (model)

$$R_H = \mu_r (W - L) = \mu_r R_v$$

μ_r depends on <u>many</u> parameters:

- tire pressure
- load
- ground condition (mud vs concrete)
  - mud: >0.1, concrete: 0.03

Free body diagram



Static:

Vertical

$$L + R_v = W \Rightarrow R_v = W - L$$

Horizontal

$$T - D - R_H = 0$$

Dynamic:

$$m\frac{dV}{dt} = T - D - R_H = T - D - \mu_r (W - L)$$

$$\boxed{m\frac{dV}{dt} = T - D - \mu_r (W - L)}$$

$$\boxed{\frac{dV}{dt} = g\left(\frac{T}{W} - \frac{D}{W} - \mu_r\left(1 - \frac{L}{W}\right)\right)} = g\frac{T}{W} - g\frac{D}{W} - g\mu_r + g\frac{L}{W}$$

T, D, and L depend on V.

$$V_1 = \int_0^t \frac{dV}{dt}\,dt \quad \text{and} \quad S_1 = \int_0^t V\,dt = \int V \frac{dt}{dV}\,dV = \int \frac{1}{2}\frac{dV^2}{(d\,V/dt)}$$

Approximate Solution

$$\frac{dV}{dt} = g\left(\frac{T}{W} - \frac{D}{W} - \mu_r + \frac{L}{W}\mu_r\right)$$

- Most aircraft (tri-gear) are at a level attitude $\Rightarrow C_L \approx 0$ or maybe 0.1
- A turbojet has $T(V)$ constant.
- Since C_L is small, C_{D_i} is small. $C_D = C_{D_0} \approx 0$ (meh!)

$$\frac{dV}{dt} = g\left(\frac{T}{W}\right) - g\mu_r = g(K_T) \qquad \text{where } K_T \equiv \frac{T}{W} - \mu_r$$

$$= g K_T$$

or

$$dV = g K_T \, dt \Rightarrow \underbrace{V = g K_T t}_{t = \frac{V}{g K_T}} \Rightarrow s = g K_T \frac{t^2}{2}$$

$$S_{TO} = \frac{1}{2} g \frac{K_T V_\infty^2}{g^2 K_T^2} = \frac{1}{2} \frac{V_\infty^2}{g K_T} \qquad \text{and} \qquad W = L_{TO} = \frac{1}{2}\rho V^2 S C_{L_{TO}}$$

$$V^2 = \frac{2W}{\rho S C_{L_{TO}}}$$

$$\boxed{S_{TO} = \frac{1}{\rho}\left(\frac{W}{S}\right)\frac{1}{C_{L_{TO}}}\frac{1}{g K_T}}$$

$$= \frac{1}{\rho}\left(\frac{W}{S}\right)\frac{1}{C_{L_{TO}}}\frac{1}{g\left(\frac{T}{W} - \mu_r\right)}$$

$$= \frac{1}{\rho}\left(\frac{W}{S}\right)\frac{1}{C_{L_{TO}}}\frac{m}{T - W\mu_r}$$

The book has a more in depth analysis method at the expense of clear terms. p 359 - 361

Square of mass!!

- ~~Improve A wing loading~~
- Inverse of $K_T C_{L_{TO}}$
- Inverse of density ($\frac{1}{\rho^2}$ if T decays with density)

Ex: Estimate the takeoff ground distance for a Cessna Mustang @ gross weight on concrete.

$W = 8645 \text{ lbf}$ $\quad T = 2 \times 1460 \text{ lbf}$ $\quad \mu_r \approx 0.03$ $\quad C_{L_{TO}} \approx 1.3$ $\quad S = 210 \text{ ft}^2$

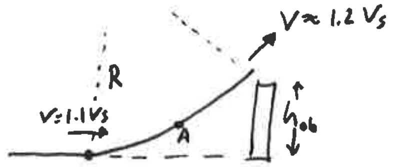
$$K_T = \frac{T}{W} - \mu_r = \frac{2 \times 1460 \text{ lbf}}{8645 \text{ lbf}} - 0.03 = 0.308 \qquad V_{TO} \approx 97 \text{ kt}$$

$$S_{TO} = \frac{8645 \text{ lbf}}{210 \text{ ft}^2} \cdot \frac{1}{1.3} \cdot \frac{s^2}{32.174 \text{ ft}} \cdot \frac{1}{0.308} \cdot \frac{\text{ft}^3}{0.00237 \text{ slug}} \cdot \frac{\text{slug ft}}{\text{lbf s}^2} = 1350 \text{ ft}$$

Does this match the reported T/O distances? No! Why.

Balanced field Length

Take off Obstacle



Over the short distances and heights of an FAA take off obstacle, the aircraft performs a pull-up maneuver.

$$R = \frac{V^2}{g(n-1)}$$

Average speed$_A$ $\approx \frac{1.1 + 1.2}{2} = 1.15\ V_s$

Weight:

$$W = \frac{1}{2} \rho V_s^2 S C_{Lmax}$$

Assume buffer of $0.9\ C_{Lmax}$

Lift$_A$:

$$L = \frac{1}{2} \rho (1.15 V_s)^2 S\ 0.9\ C_{Lmax}$$

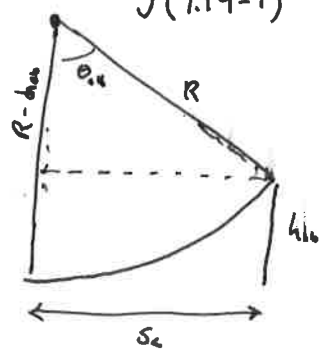
Load factor

$$n = \frac{L}{W} = \frac{\frac{1}{2} \rho V_s^2 (1.15)^2 S\ 0.9\ C_{Lmax}}{\frac{1}{2} \rho V_s S\ C_{Lmax}} = (1.15)^2\, 0.9 = 1.19$$

Radius:

$$R = \frac{(1.15\ V_s)^2}{g(1.19-1)} = \frac{6.961\ V_s^2}{g}$$

Angle



Pytha':

$$R^2 = (R - h_{ob})^2 + S_a^2$$

$$S_a^2 = R^2 - (R - h_{ob})^2$$

$$= R^2 - R^2 + 2Rh_{ob} + h_{ob}^2$$

$$S_a = \sqrt{2Rh_{ob} + h_{ob}^2}$$

Combine

$$S_a = \sqrt{\underbrace{2 \cdot \frac{6.961\ V_s^2}{g} h_{ob}} + \underbrace{h_{ob}^2}} \approx S_a \approx \sqrt{13.9 \frac{h_{ob}}{g}}\ V_s$$

$\gg$

when h_{ob} is small $\approx$ {35 ft, 50 ft, ~~100~~ or so}

$S_{a_{35ft}} \approx 3.9\ V_s$	$S_{a_{50ft}} \approx 4.65\ V_s$

V_s in ft/s

Ex: Total takeoff distance of a Mustang over a 50 ft obstacle.
SSL

$S_g = 1350^{ft}$

$S_a = 4.65 V_s$ with $V_s \approx V_{10} \approx 97^{kt} \approx 163 \frac{ft}{s}$

$= 4.65 \cdot 163 \frac{ft}{s} = 760^{ft}$

$S_{10} \approx 1350 + 760 = 2100^{ft}$

In a mustang, you are 2100 ft away from a 50 foot obstacle,
You theoretically will make the takeoff.
This is a really <u>stupid</u> thing to do Why?

tiny.cc/AEM368ScaryTakeoff

- Perfect Execution. No errors.
- Perfect Conditions.
- This is a <u>model</u> of the aircraft. Reality will be worse (Murphy's law)
- Perfect aircraft
- No failures (engine, etc)
- perfect no-slope, smooth runway.

Density Altitude

The standard atmosphere altitude equal in density to the current density at a location.

Q: What increases density altitude?

A: Hot, high, humid ← H_2O vapor lower density than N_2 (80% air)

i.e. Same pressure, higher Temp $\quad p = \rho R T \quad \Rightarrow \rho \downarrow \qquad \frac{d\rho}{dh} < 0$

Q: Estimate the takeoff distance for the Mustang at Denver CO on a std day.

Engine: $\frac{T}{T_0} \approx \frac{\rho}{\rho_0}$ $\qquad$ Atmo: $\frac{\rho}{\rho_0}$ @ 5280 ft $\approx \frac{2.02 \times 10^{-3}}{2.37 \times 10^{-3}} = 85\%$

$$K_T = \frac{T}{W} - \mu_r = \frac{T}{T_0}\frac{T_0}{W} - \mu_r = 0.85 \cdot 0.337 - 0.05 = 0.238$$

$$S_{to} \approx \frac{8645}{210 \mid 1.3 \mid 32.174 \mid 0.238 \mid 2.02 \times 10^{-3}} = 2050 \text{ ft}$$

$V_{to} \approx$ 105 kt $\qquad$ 50% further for 15% less density

Q: Now, Denver on a 95 F day (hot) and slightly humid (dew pt 70 F)

Density altitude is 9000 ft (I looked this up seperately...)

$\rho = 1.81 \times 10^{-3}$ slug/ft^3 $\qquad \frac{\rho}{\rho_0} = 0.76 \qquad K_T \approx 0.208$

$S_{to} \approx 2600$ ft

93% further

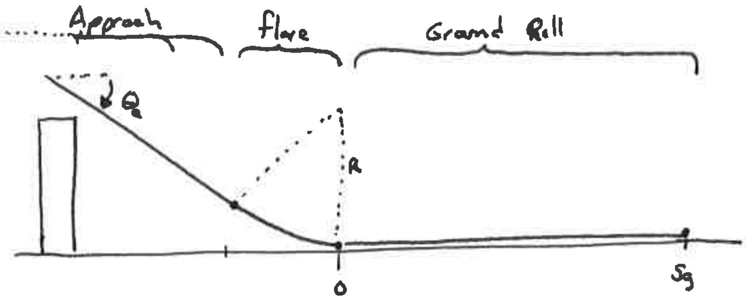
Hot, high, humid, heavy

tiny.cc/AEM368StitsonHigh

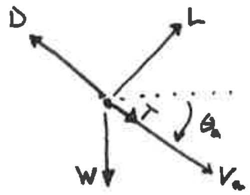
Short and Soft field takeoff

tradeoff μ_r vs $\frac{T}{W}$ and K

Landing



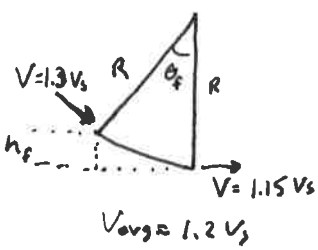
Approach:



$$L = W\cos\theta_a$$

$$D = T + W\sin\theta_a \quad \Rightarrow \quad \sin\theta_a \approx \frac{1}{(L/D)} - \left(\frac{T}{W}\right)$$

Flare:



$$R = R\cos\theta_f + h_f \quad \Rightarrow \quad h_f = R - R\cos\theta_f = R(1-\cos\theta_f)$$

$$\theta_f = \theta_a$$

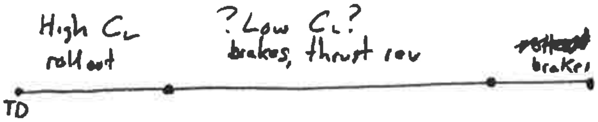
$$R = \frac{V^2}{g(n-1)} \qquad n = \left(\frac{1.3+1.15}{2}\right)^2 0.9 = \underline{1.35} \text{ to } 1.2$$

$$= \frac{(1.2)^2 V_s^2}{g\,(0.35)} \approx 4.29\,\frac{V_s^2}{g}$$

$$S_a = \frac{h_{ob} - h_f}{\tan\theta_a}$$

$$S_f = R\sin\theta_a$$

Ground Roll



Unlike the TO roll, the ground roll has distinct phases. The dynamics gov-eqn are the same

$$\frac{dV}{dt} = g\left(\frac{T}{W} - \frac{D}{W} - \mu_r + \frac{L}{W}\mu_r\right)$$

$$= \frac{g}{W}\left(\overbrace{-T_{rev}} \underbrace{- \tfrac{1}{2}\rho V^2 S C_D - \overbrace{\mu_r W} + \tfrac{1}{2}\rho V^2 S C_L \mu_r}_{\text{Aero terms}}\right)$$

$$= \underbrace{g\left(\frac{-T_{rev}}{W} - \mu_r\right)} + \underbrace{g\left(-\tfrac{1}{2}\rho V^2 \left(\frac{W}{S}\right)^{-1}\left(C_{D_0} + \Delta C_{D_0} + G k C_L^2\right) + \mu C_L\right)}$$

$$J_T \equiv \frac{T_{rev}}{W} + \mu_r \qquad J_A \equiv \frac{\rho}{2}\left(\frac{W}{S}\right)^{-1}\left(C_{D_0} + \Delta C_{D_0} + \underbrace{G k C_L^2}_{(k_1 + 6k_3)\text{ in book.}} - \mu_r C_L\right)$$

$$\frac{dV}{dt} = -g\left(J_T + J_A V^2\right)$$

Integrate.

$$S_g - S_{fr} = \int_0^{V_{TD}} \frac{d\left(V_\infty^2\right)}{2g\left(J_T + J_A V_\infty^2\right)}$$

when J_T and J_A are constant,

$$S_g = N V_{TD} + \frac{1}{2g J_A} \ln\left(1 + \frac{J_A}{J_T} V_{TD}^2\right)$$

Estimate the ground roll of a Mustang (2 seconds at V_{TO}

$$J_T = \cancel{\frac{T_{rev}}{W}}^{\,0} + \mu_r = +0.5 \ \text{(brakes)}$$

$$J_A = \frac{\rho}{2}\left(\frac{W}{S}\right)^{-1}\left(C_{D_0} + \cancel{\frac{G}{\pi ARe}C_L^2}^{\,0} - \cancel{\mu_r C_L}^{\,0}\right) \qquad \text{Assume } C_L \approx 0$$

$$= \frac{0.00237\ \text{slugs}}{ft^3} \cdot \frac{210\ ft^2}{8645\ lbf} \cdot 0.0250 \cdot \frac{lbf\, s^2}{slug\, ft} = 1.44\times10^{-6}\ \frac{s^2}{ft^2}$$

$$S_g = \underbrace{2^s \cdot 163\frac{ft}{s}}_{326\ ft} + \underbrace{\frac{1}{2}\,\frac{1}{32.174}\frac{s^2}{ft}\,\frac{ft^2}{1.44\times10^{-6}\ s^2}\ln\left(1+\underbrace{\frac{1.44\times10^{-6}\frac{s^2}{ft^2}}{+0.5}163^2\frac{ft^2}{s^2}}_{\text{small}}\right)}_{795\ ft}$$

$$= 1121\ ft$$

Approach

$$\theta_a \approx 5^\circ \qquad h_{ob} = 50^{ft} \implies h_f = 4.29\frac{V_s^2}{g}\left(1-\cos\theta_a\right) = 14^{ft}$$

$$S_a = \frac{h_{ob} - h_f}{\tan\theta_a} = \frac{50-14}{\tan 5^\circ} = 416^{ft}$$

$$S_f = R\sin\theta_a = 4.29\frac{V_s^2}{g}\sin\theta_a = 311^{ft}$$

Total landing

$$1121 + 416 + 311 = 1848^{ft}$$

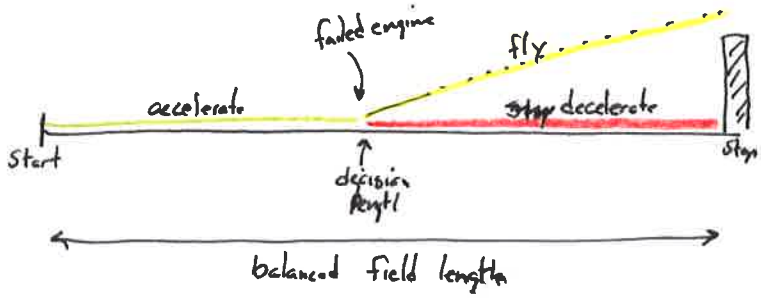
Balanced Field Length

Given an aircraft and an obstacle, the balanced field length is where

- The length necessary to accelerate and clear the obstacle (one engine failed)

equals

- The length necessary to accelerate to the decision speed and decelerate to a stop



The concept of BFL gives the pilot survivable options.

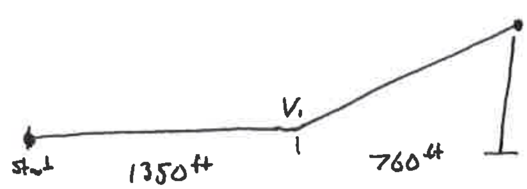
The legal requirements vary depending the type and operation of the aircraft.

ie. ρ
N_e
T
W
⋮

Ex:

Estimate the BFL of a Mustang with a 50 foot obstacle with no failed engine

Fly



$1350 + 760 = 2100$ ft

Decelerate at V_1

$1350 + 1121 = 2471$ ft

with both engines operational, BFL = 2471

What other cases need to be evaluated?

Lesson 12 part 2

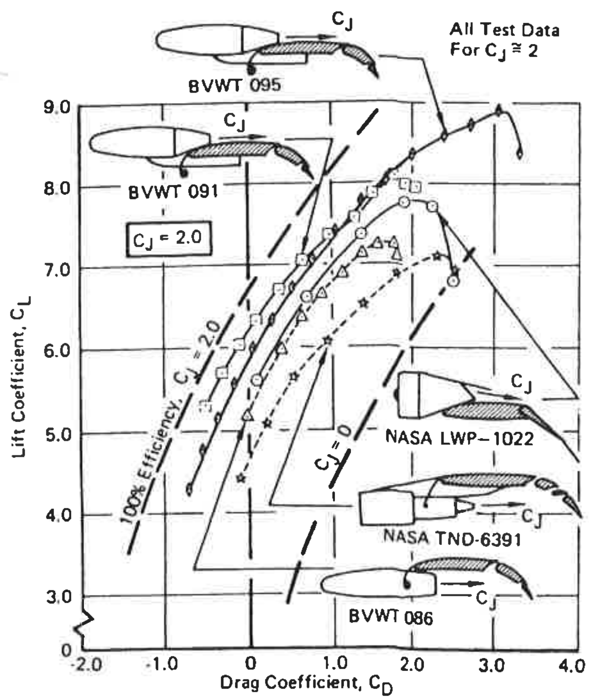
Applications of STOL/VTOL

YC14 STOL TO/Landing

$$S_{to} \approx \frac{1}{\rho}\left(\frac{W}{S}\right)\underbrace{\frac{1}{C_{L_{to}}}}\,\frac{1}{g}\,\frac{1}{K_T}$$

Upper surface blowing:

$C_{L_{max}} \approx 9$



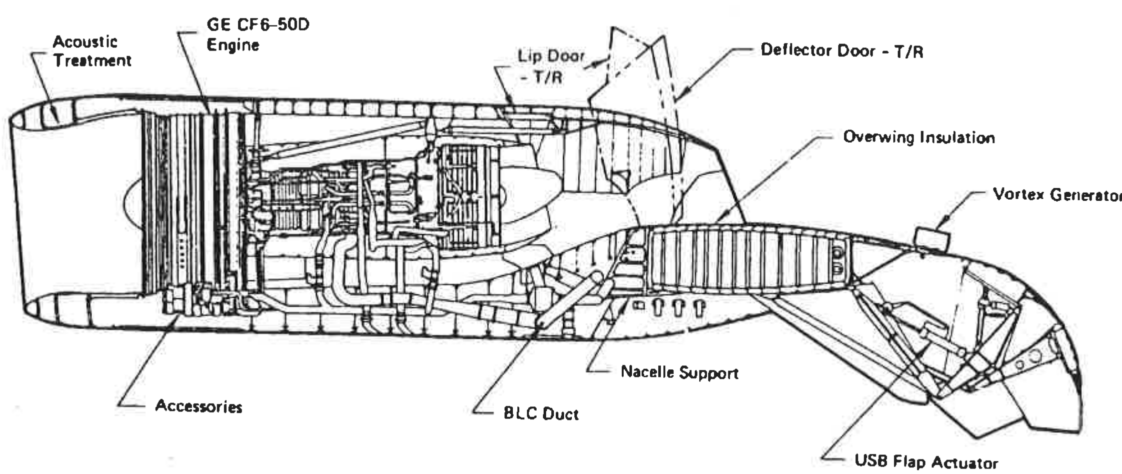


Fig. 21 Engine and nacelle cross section. Reprinted with permission from AIAA Preprint AIAA-74-972 © 1974.

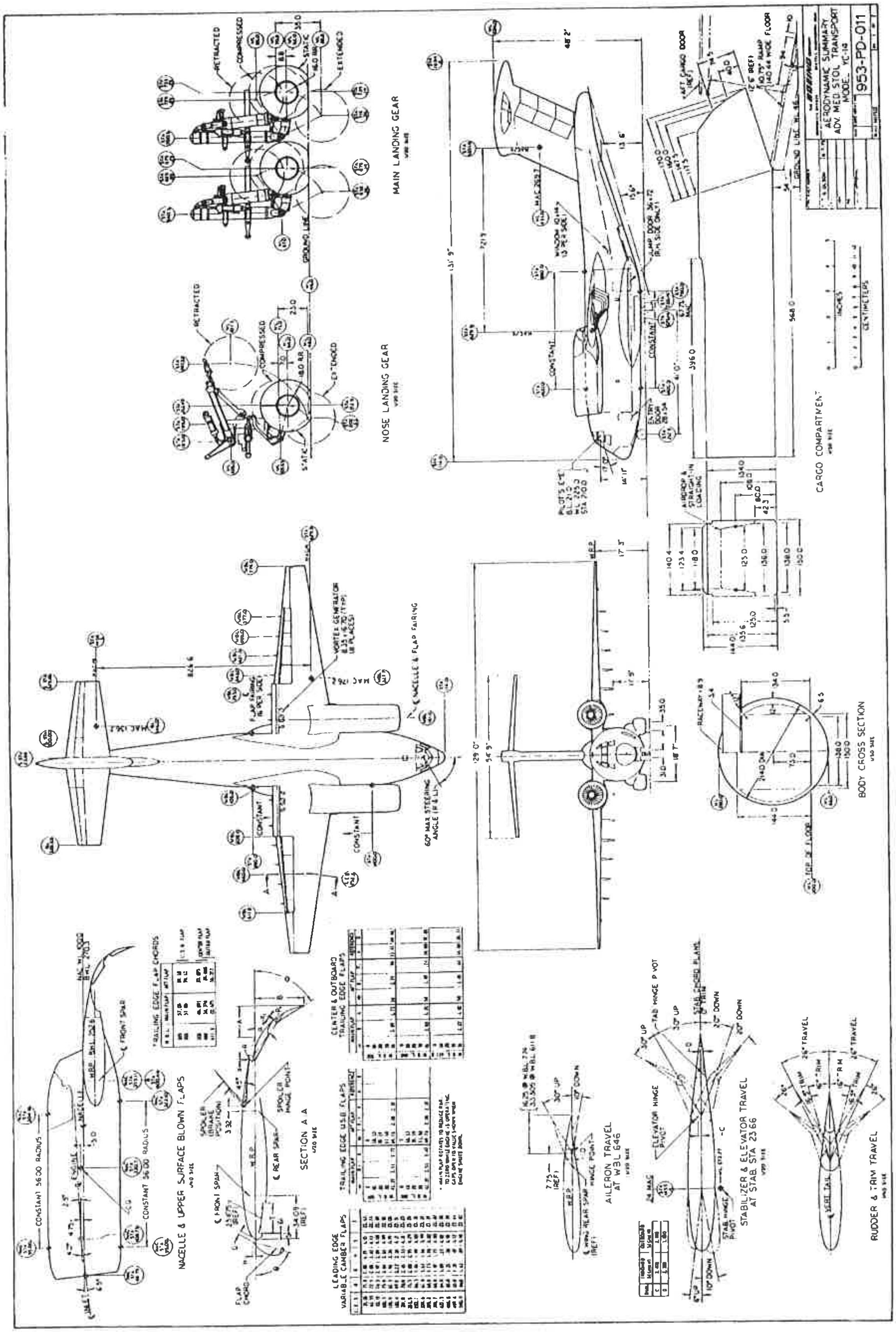
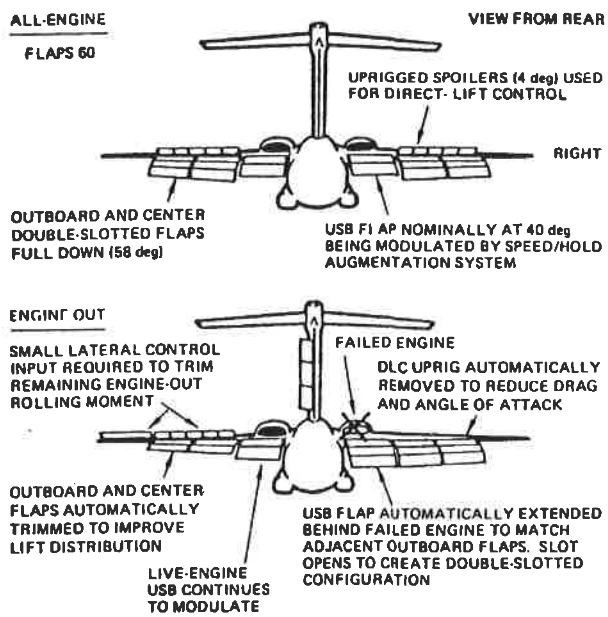


Fig. 25 YC-14 configuration details.

YC-14 Flight Control System.

What happens if an engine fails?



Approach at 80 kts "Like a big Cessna 182"

tiny.cc/aem368YC14

VJ101

In the 1960s, the German Airforce had a problem. The front line fighters required dedicated airfields with long runways. (Ex. F104 liftoff at ≈190^{kts})

Just a few nuclear blasts could knock out their entire airforce. Even a few carefully placed conventional bombs could prevent T/O. (See: Iraq 91).

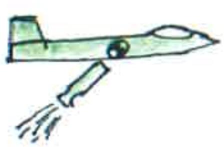
Solutions:

- Dispersé and Decentralize
- VTOL

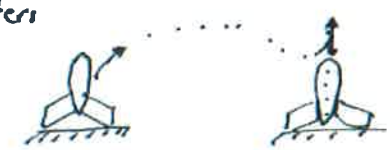
Q: How do you design a front line air superiority fighter with F104 performance in a VTOL configuration?

A: F-35 is still having deployment issues using modern materials. (present day)

A: F104 "Zell" ZLL (zero length launch) .... Landing?



A: Tailsitters

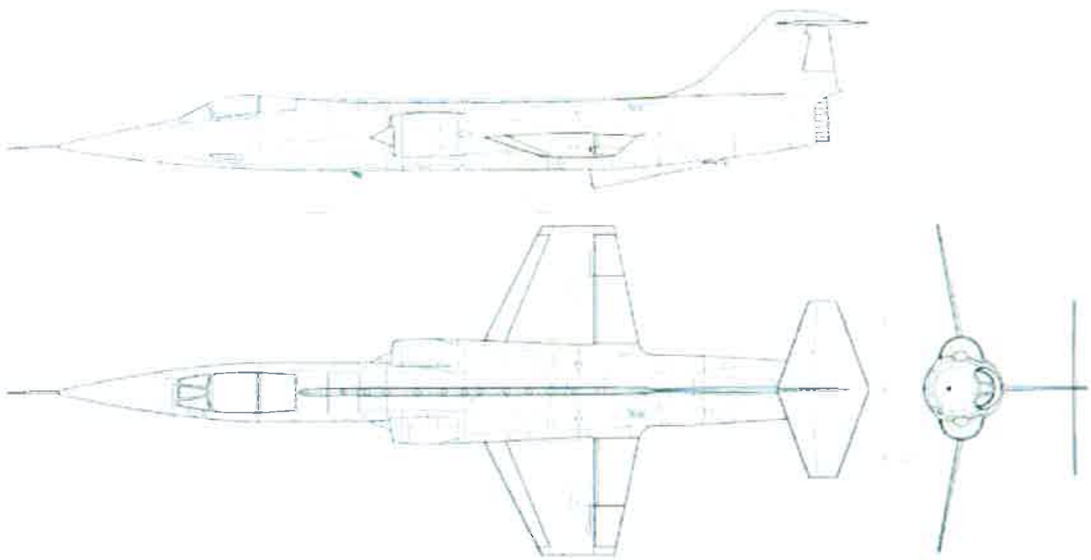


problem:
- Sears-Haack says "long and slender" for high Mach aircraft.
- Landing stability of tailsitters says "wide and short".

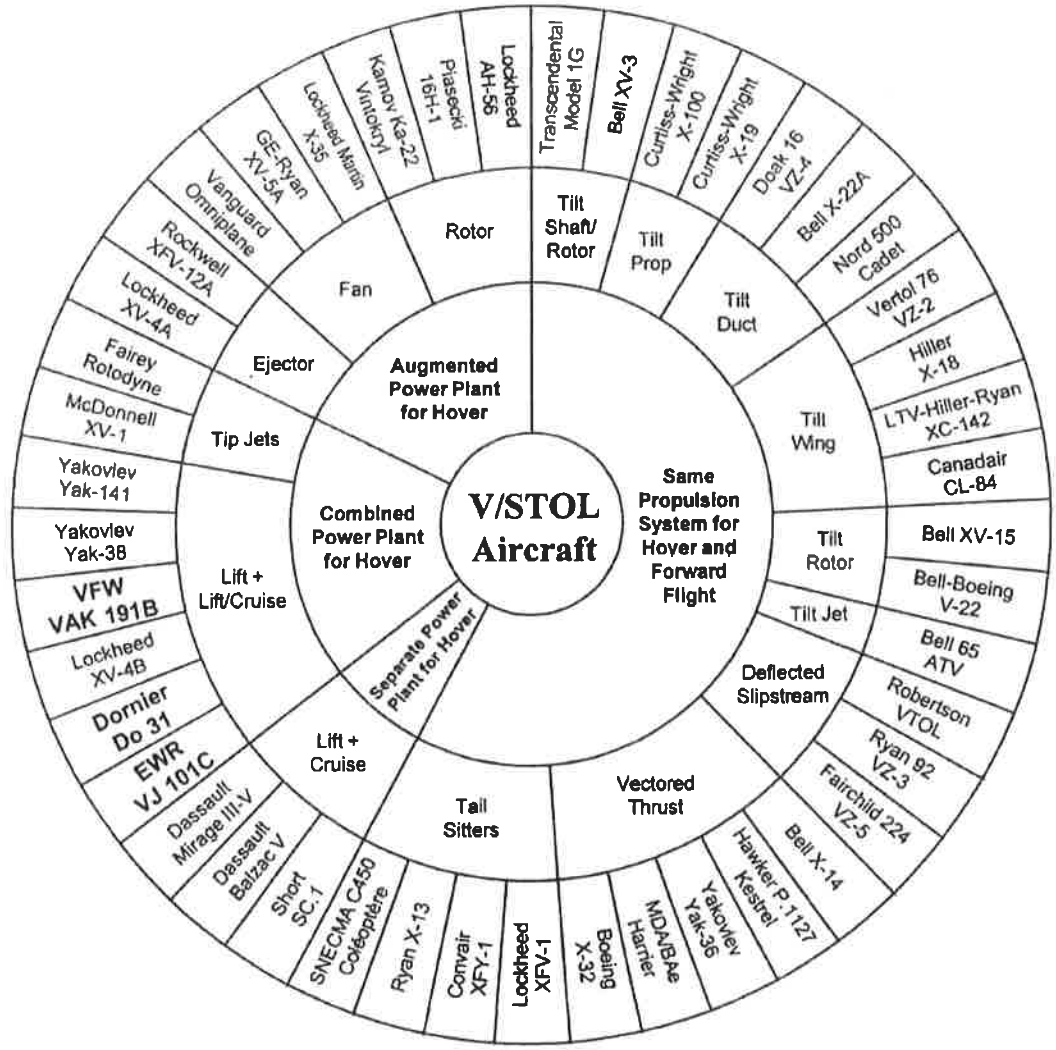
A: Vectored Thrust

VJ 101: tilt the cruise engines

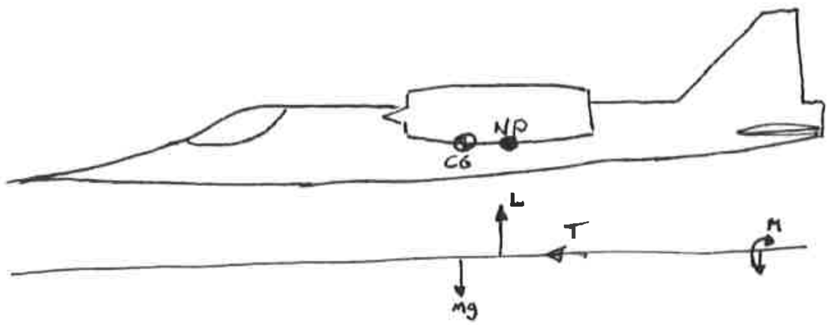
VAK 191B: Nozzle vectored thrust (eg. Harrier)



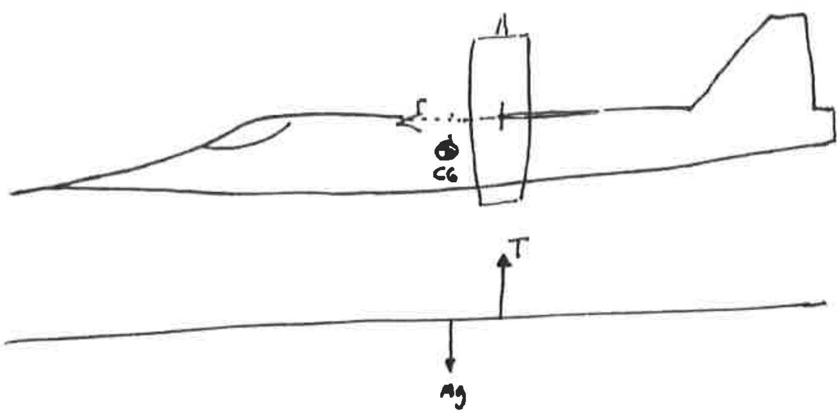
Figures from: German V/STOL Fighter Program
by Albert C. Piccirillo. AIAA, 1997



Forces and Moments (Conceptual VTOL system)

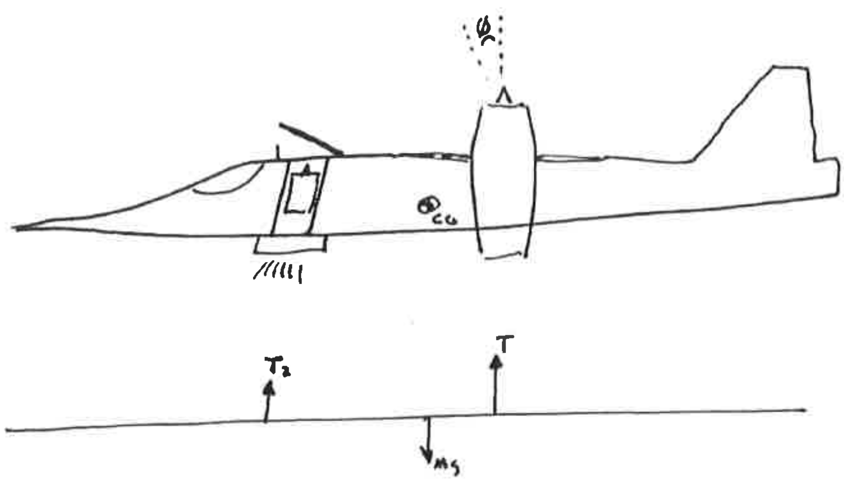


✓ Stable



X not balanced

- We clearly need an additional moment (Nose Up). Ahead of the CG would be most efficient. Add lift engines ahead of CG.



✓

- pitch control with lift engines
- roll control with wingtip (cruise) engines
- yaw control with differential tilt angle of wingtip cruise engines

Engine Systems

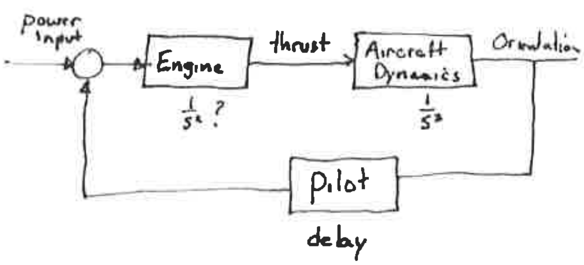
RB 145 axial flow turbojet — Thrust: 2750 lb dry — 3650 lb Afterburner

The VJ 101 used thrust modulation for pitch and roll control and yaw. Turbojet engines have a rotating core, which gives a spool up time of seconds.

Modern civilian engines are required under 14 CFR $\underbrace{33.73}_{\text{"engines"}}$ to provide 95% takeoff power within 5 seconds from idle.

Early jets were <u>not</u> known for fast power response.

How to design a control system?

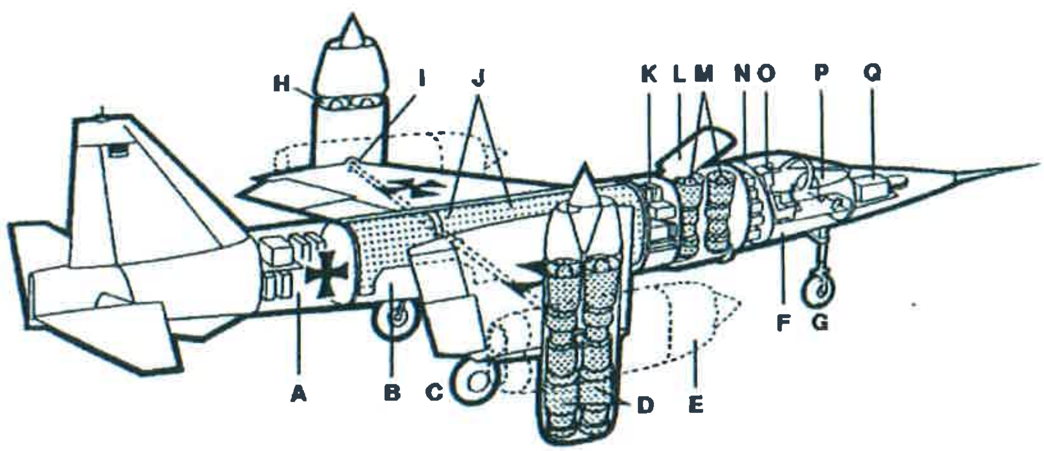


$$I\ddot{\Theta} = M$$

$$H(s) = \frac{A(s)\,E(s)}{1 + P(s)\,A(s)\,E(s)}$$

Succesfull operation in VTOL mode requires slow steady maneuvers.

In practice, transition took 90 seconds from/to VTOL.



A. Equipment bay
B. Main landing gear bay
C. Rearward retracting main wheels
D. Afterburning RB.145 turbojets in swivelling nacelles
E. Nacelle in forward position
F. Nosewheel bay
G. Rearward retracting nosewheel
H. Nose section of nacelle raised for V/STOL flight
I. Hollow shaft on which nacelles swivel
J. Two-cell fuselage tank
K. Avionics bay
L. Retractable air intake door for lift engines
M. RB.145 lift engines
N. Avionics bay
O. Pilot ejection seat
P. Instrument panel
Q. Nose radar installation (planned)

Fig. 10 Cutaway drawing of the afterburner-equipped VJ 101 X2.

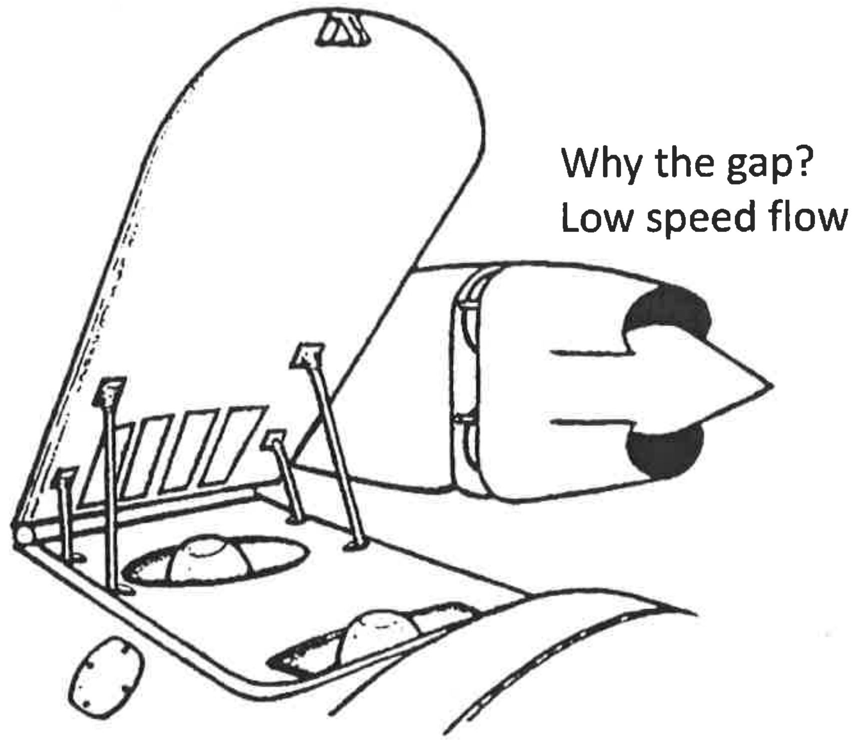


Fig. 12 VJ 101C fuselage and nacelle inlets in extended (low-speed) positions.

How do you evaluate and design the FCS?

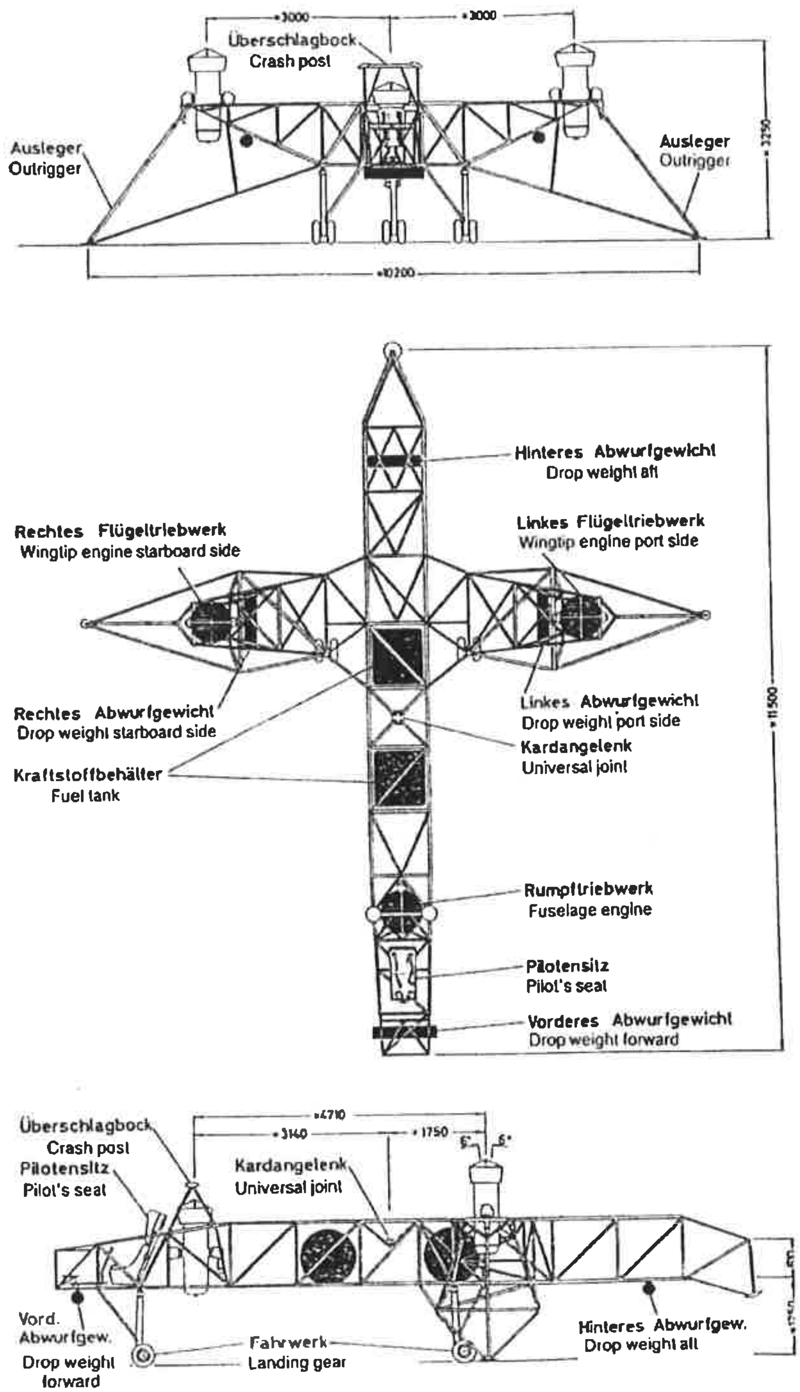
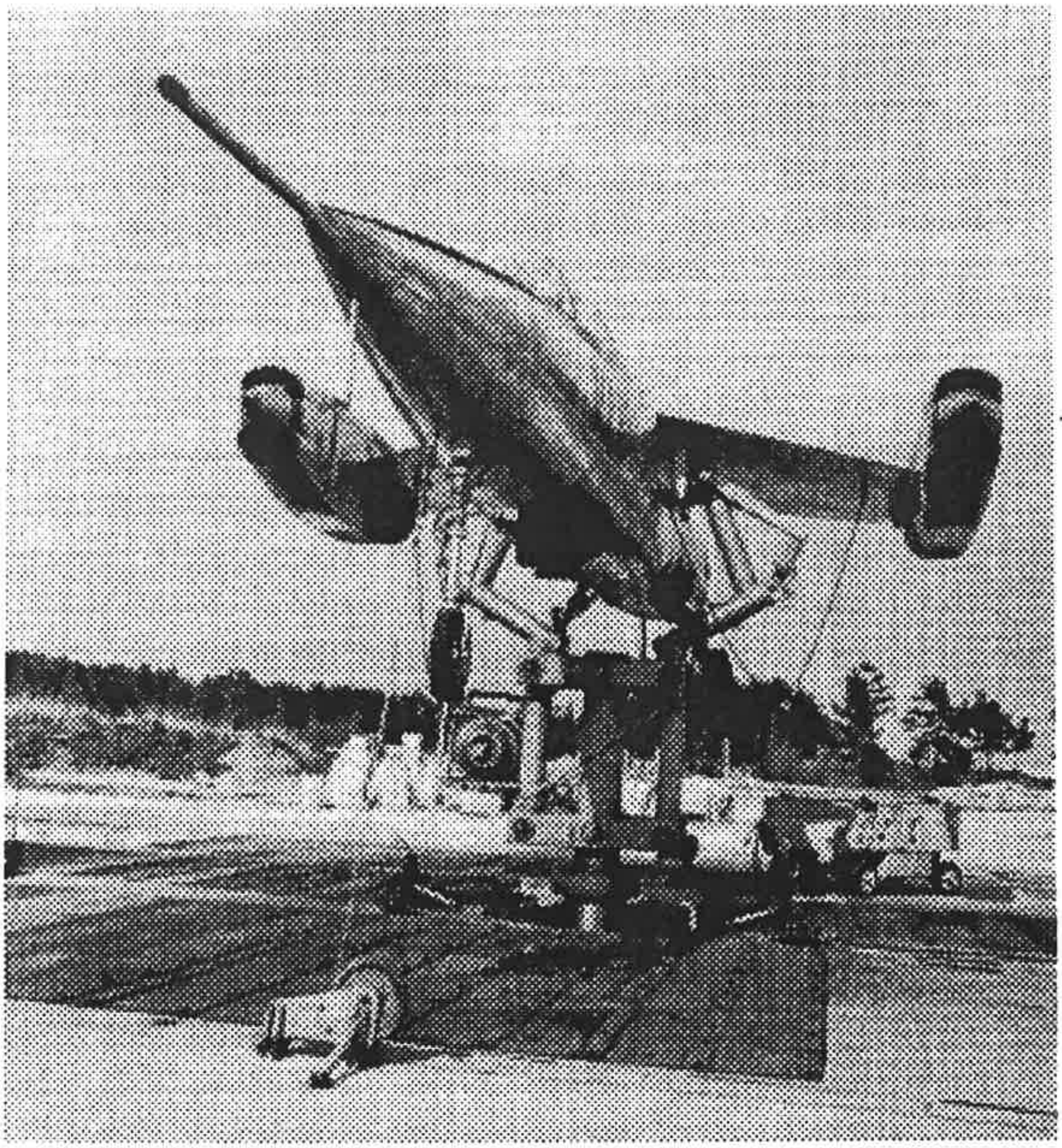


Fig. 7 The hover test rig used to evaluate VJ 101C flight control system options.



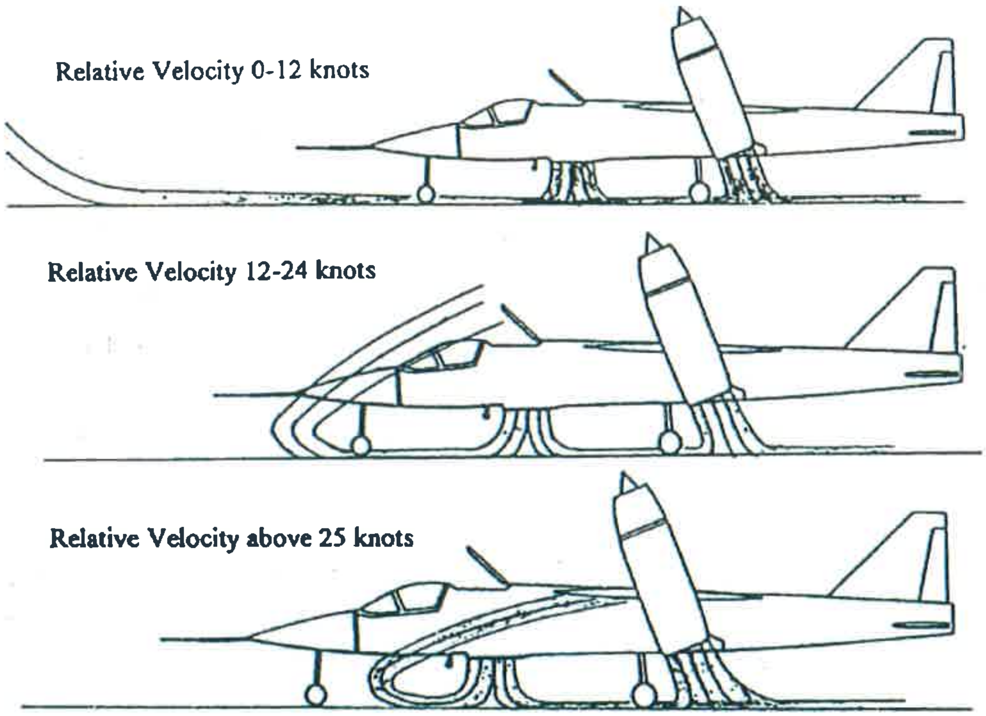
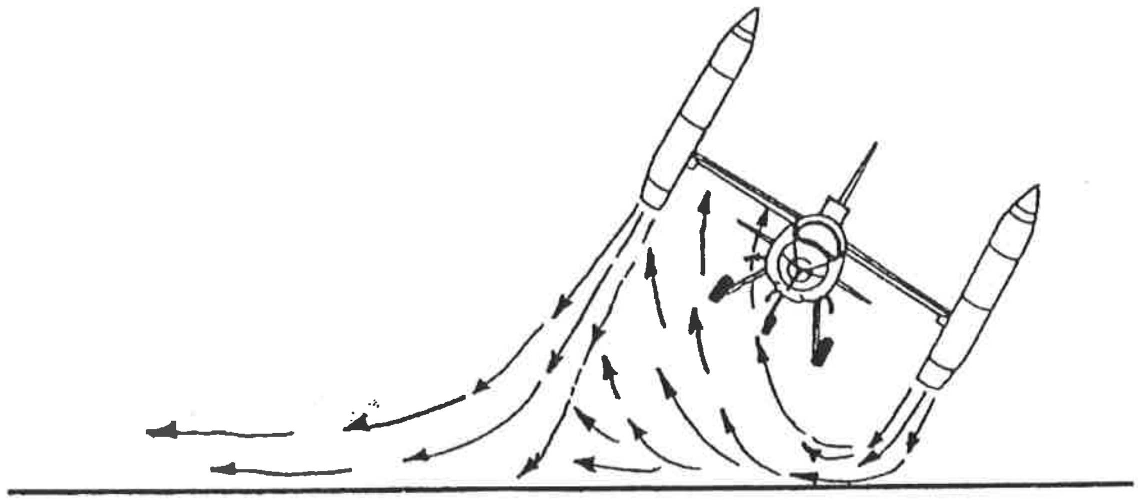


Fig. 26 VJ 101 critical forward velocity (center) for hot gas ingestion during RVTOs.

F35 System

Joint Strike Fighter
F-35 Lightning II Propulsion
F135 Short Take-Off Vertical Landing

Cold flow is a HUGE advantage



tiny.cc /AEM368VJ101

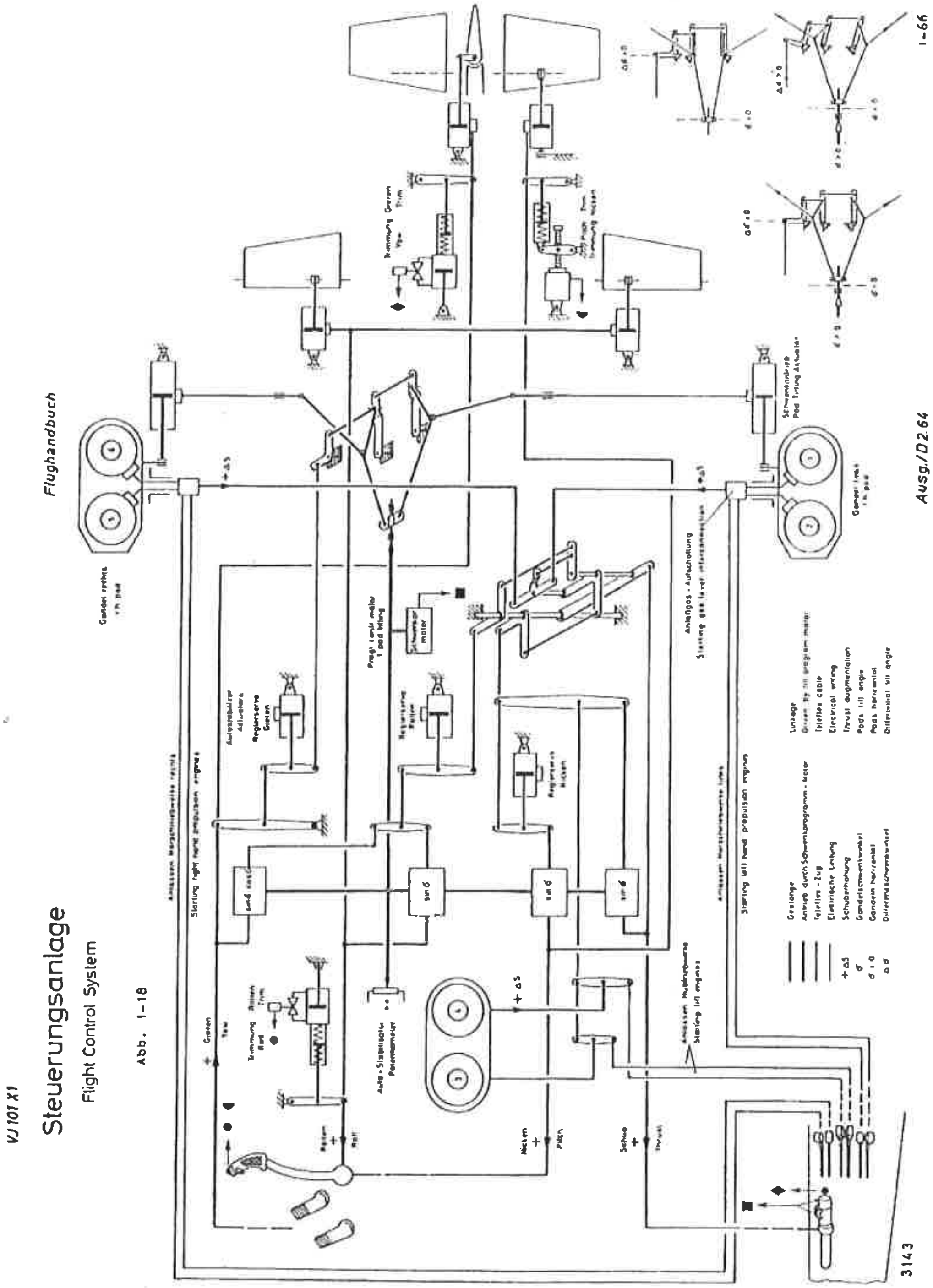


Fig. 17 VJ 101C flight control system.

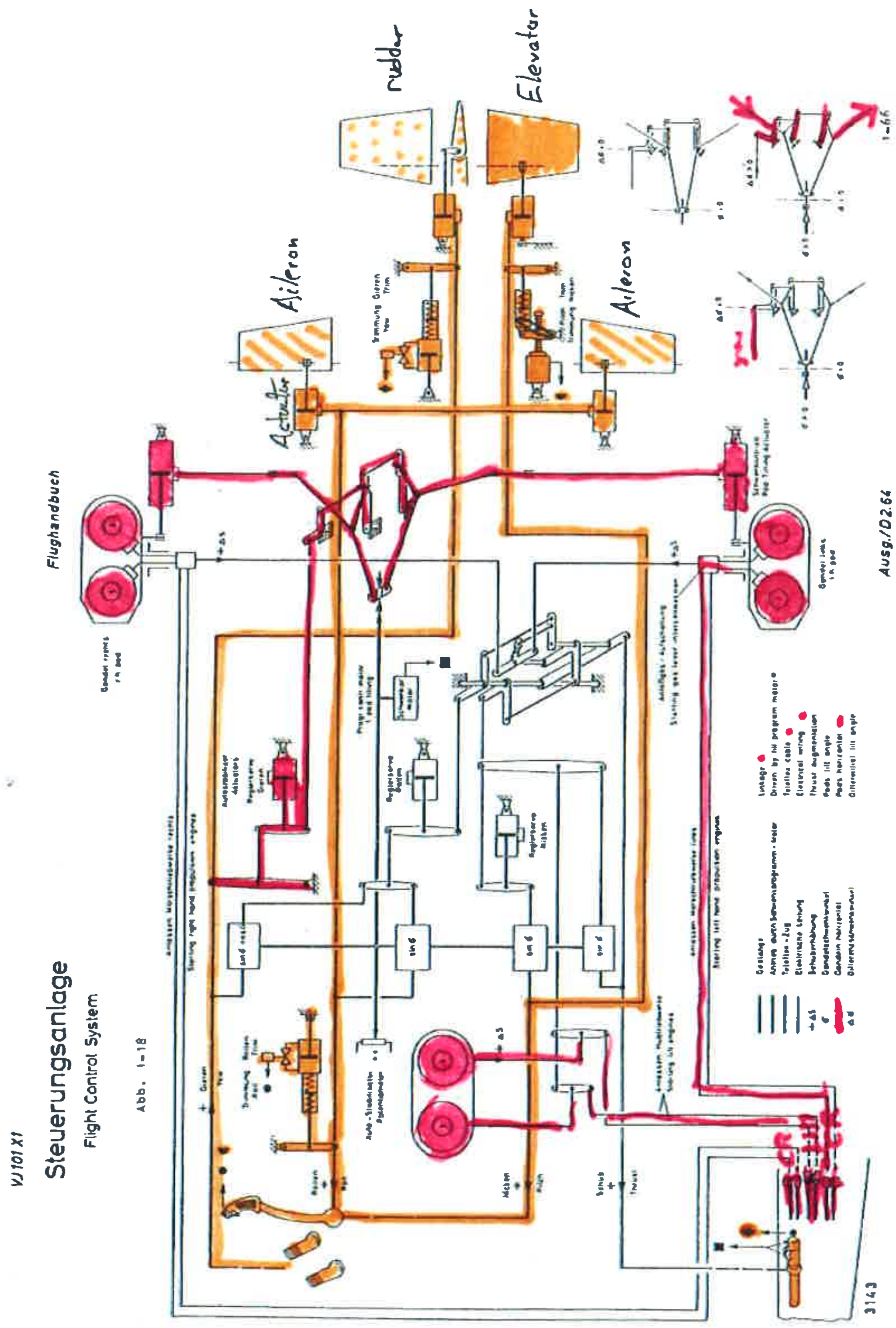


Fig. 17 VJ 101C flight control system.