Lesson 13

Mission Analysis
+
Simulations

Missions:

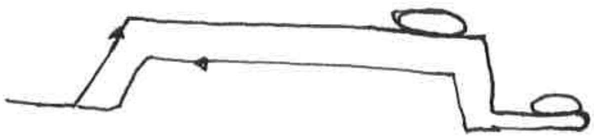
Aircraft perform missions to justify their existance

Commercial Aircraft / Cargo

Objective
Constraints

~~Cost~~ profit to carry # people from A → B

Military (fighter)



Win

Military (ISR)

Fly to a location and surveil for a given time

Acrobatic

Controllable aircraft maneuvers ~~with~~ perfection of shapes

Training

. . . ?

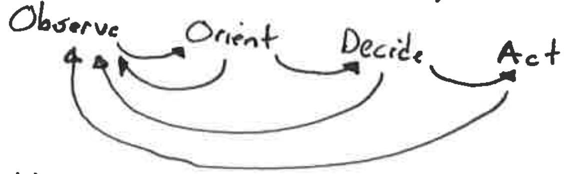
Energy Management

"When you maneuver an aircraft, you need energy... you lose energy either in gaining altitude, airspeed or both"

— Col. John Boyd

Notice how a pilot describes energy... never the less, this statement created a new concept of air combat from which the F-16 emerged.

The other main product from Boyd was the OODA loop



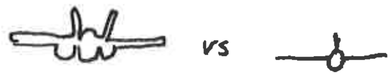
His insight was that the aircraft/pilot combination with the fastest OODA cycle ~~tends to win~~ wins.

The original program name for the F-16 was the lightweight fighter (LWF).

- The 1960s USAF trend was for heavier aircraft with high $(M>2)$ capabilities. This path eventually led to the F-15.
- The F-16 took a different strategy. Lower top speeds $(M<2)$ and much lighter.

Fact: Most air combat occurs below Mach 1.

Consider 3 representative maneuvers and a size factor

		want
• Sustained turn rate		high
• Top speed	$\longrightarrow$ vs $\rightarrow$	high ↑
• Climb rate	$\nearrow$,	high
, Size / Cost		low

• Sustained turn

$$\omega_{max} = g\sqrt{\frac{\rho}{W/S}\left(T/W\right)\frac{1}{2K} - \underbrace{\sqrt{\frac{C_{D_0}}{K}}}_{\approx 1}} \approx g\sqrt{\frac{\rho}{(W/S)}\left(\frac{T}{W}\right)\frac{\pi AR e}{2} - 1}$$

$$\text{at } V = \sqrt{\frac{2}{\rho}\frac{W}{S}}\,\sqrt[4]{\frac{K}{C_{D_0}}} \approx \sqrt{\frac{2}{\rho}\frac{W}{S}}$$

- high T/W
- low W/S
- high AR

• Top Speed

$$V_{max} = \sqrt{\frac{\frac{T}{W}\frac{W}{S} + \frac{W}{S}\sqrt{\left(\frac{T}{W}\right)^2 - 4C_{D_0}K}}{\rho C_{D_0}}} \approx \sqrt{\left(\frac{T}{W}\right)\left(\frac{W}{S}\right)\frac{2}{\rho C_{D_0}}}$$

- high T/W
- high W/S
- low C_{D_0}

• Climb rate

$$\dot{h}_{roc} = ROC = \sqrt{\left(\frac{W}{S}\right)\left(\frac{2}{3}\right)\frac{1}{\rho C_{D_0}}}\left(\frac{T}{W}\right)^{3/2}\left(1 - \frac{Z}{6} - \frac{3}{2\left(\frac{T}{W}\right)^2\left(\frac{L}{D}\right)_{max}^{2} \cdot Z}\right)$$

$$Z = 1 + \sqrt{1 + \frac{3}{(L/D)^2\left(\frac{T}{W}\right)^2}} \approx 2$$

$$= \sqrt{\left(\frac{W}{S}\right)\left(\frac{2}{3}\right)\frac{1}{\rho C_{D_0}}}\left(\frac{T}{W}\right)^{3/2}\left(1 - \tfrac{1}{3} - 0\right) \quad \text{at } V_{roc} \approx \sqrt{\left(\frac{T}{W}\right)\left(\frac{W}{S}\right)\left(\frac{2}{3}\right)\frac{1}{\rho C_{D_0}}}$$

- ~~high~~ moderate W/S
- high (T/W)

• Size

Size $\approx S$

$\approx T$ since $T = \dot{m}\,\Delta V = \rho V A\,\Delta V$

- low S
- low T

$\underbrace{\text{Match sustained turn Velocity}}$ with $\underbrace{\text{Climb rate Velocity}}$

$$V_{\dot\omega} = \sqrt{\frac{2}{\rho}\frac{W}{S}}\sqrt[4]{\frac{K}{C_{D_0}}} \approx \sqrt{\frac{2}{\rho}\frac{W}{S}} \qquad V_{\dot h} \approx \sqrt{\left(\frac{T}{W}\right)\left(\frac{W}{S}\right)\left(\frac{2}{3}\right)\frac{1}{\rho C_{D_0}}}$$

$$\frac{V_{\dot h}}{V_{\dot\omega}} = \sqrt{\left(\frac{T}{W}\right)\left(\frac{W}{S}\right)\left(\frac{2}{3}\right)\frac{1}{\rho C_{D_0}} \cdot \frac{\rho S}{2W}}$$

$$= \sqrt{\frac{T}{W}\frac{1}{3}\frac{1}{C_{D_0}}} \approx \sqrt{\frac{1}{3}\cdot\frac{1}{0.02}} = \sqrt{\frac{1}{0.06}} \approx 4!$$

If $T/W \approx 1$, then a relatively high C_{D_0} is ok.

If top speed is not as critical, C_{D_0} can be larger and W/S can be lower.

Conclusion:

A lightweight fighter has turn rate advantages.

has T/W advantages ($W \downarrow$ and simple engine inlet $T \uparrow$ for $M<1$)

has less visible/radar area

has a slower top speed

is much less expensive

This is the F-16's description

• 78 SIGNIFICANT VARIATIONS • M = .2 – 2.2 • $\alpha = 28^{\circ}$ • $\beta = 12^{\circ}$

	Configurations Tested	Wings Λ_{LE}	Wings Fixed Camber	Wings Variable Camber	Wings Airfoil(s)	Inlets Side	Inlets Bottom	Inlets Bottom	Vertical Tails Twin	Vertical Tails Single	Vortex Lift (Forebody Strakes)	Wind Tunnel Test Hours
Conventional Forebody	785	40°	✓		[illegible]		✓			✓	✓	48
		35°	✓		[illegible]		✓			✓		20
	786	40°		✓	[illegible]		✓		✓		✓	48
Wing/Forebody Shaping	401F-0	35°		✓	[illegible]		✓		✓		✓ STRAKES	107
		40°	✓		64A204		✓		✓			
	401F-2	40°		✓	64A204		✓	✓	✓	✓	✓	91
		35°		✓	[illegible]				✓			
	401F-3	40°		✓	64A204			✓	✓			20
	401F-4	40°		✓	64A204			✓	✓	✓	✓ STRAKES	39
	401F-8	40°		✓	64A204	✓			✓			29
	401F-5	40°		✓	64A204			✓	HORIZ TAIL POSITIONS	✓	✓	130
	401F-5A	40°		✓	64A204			✓		✓	✓	30
	401F-10	40°		✓	64A204		✓			✓	✓	30
	401F-10A	40°		✓	64A204		✓			✓	✓	32
	401F-16	40°		✓	64A204			✓		✓	✓	442
		45°	✓	✓	[illegible]							
	401F-16E	40°		✓	64A204			✓		✓	✓	126
		Wing Moved Forward 14 inches										

TOTAL WIND TUNNEL HOURS 1272

Simulation

$$F = ma$$

Excess Power $= P_s = \frac{dh}{dt} + \frac{V_\infty}{g}\frac{dV_\infty}{dt}$

$m\frac{dV}{dt} = F_t$

$m\frac{V^2}{R} = F_n$

Dynamics

$C_L(\alpha)$ $\quad$ $C_D(C_L)$

Flight configuration: flaps, spoilers, etc

Aero

c $\quad$ c_t $\quad$ T

η $\quad$ N

Propulsion

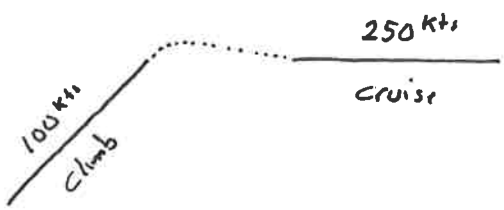
Integrate forward in time given the desired flight profile

$$h = \int_{t_0}^{t_1} \dot{h}\, dt \approx \dot{h}\,\Delta t + h_0$$

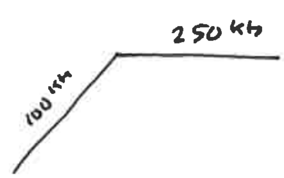
$$W_{\substack{fuel\\burn}} = \int T c_t \, dt$$

$$V = \int \frac{dV}{dt} dt \approx \frac{dV}{dt}\Delta t + V_0$$

Transitions are necessary (sometimes)

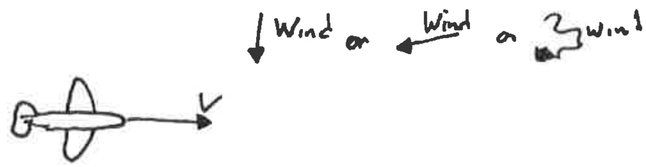


$\approx$

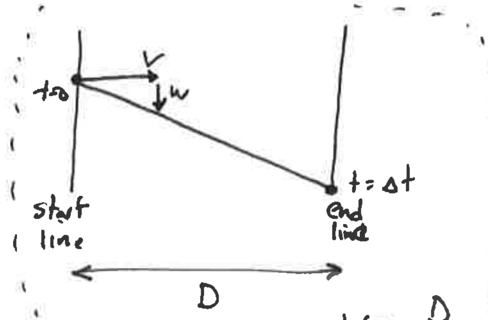


Flight Testing for Performance

Airspeed calibration: (with a production aircraft)

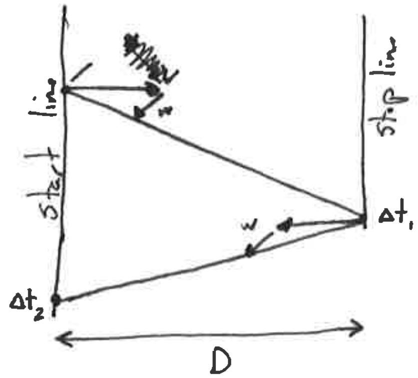


Orthogonal wind (rare)



$$V = \frac{D}{\Delta t}$$

Arbitrary Wind



- Fly orthogonal to a start line and stop line (easy to do out west!)
- Measure time
- Reverse heading and fly back to start
- Measure time

$$V = \frac{2D}{\Delta t_1 + \Delta t_2}$$

Ex: In 2001, your professor was required to take a flight test class. A C-172 RG had a 2 mile course with the following data

Dir	time	IAS	Press Alt.	OAT
West	64.5 s	100 kt	1340 ft	36 °F
East	56.7 s	104 kt	1360 ft	37 °F

Compute the airspeed calibration at 100 kt. (i.e. IAS → CAS)

Low speed, so $V_{True} \approx \frac{V_{CAS}}{\sqrt{\sigma}}$ with $\sigma = \frac{\rho}{\rho_{sl}}$

Ideal Gas law $p = \rho R T \Rightarrow \frac{p}{p_{ssl}} = \frac{\rho}{\rho_{ssl}} \frac{R}{R_{ssl}} \frac{T}{T_{ssl}} = \delta = \sigma \theta$

CAS = IAS + C

Find std day $\underbrace{P \text{ and } T}_{\text{Actually find } \delta \text{ and } \theta}$ at 1340^{ft}.

$$\delta_{1340} = \left(1 - 6.8755 \times 10^{-6} \cdot h\right)^{5.255}$$ good only below 36^{kft} !

$$= 0.9525$$

$$\delta_{1360} = 0.9518$$

$$\theta_{1340_{std}} = \left(1 - 6.8755 \times 10^{-6} \cdot h\right) = 0.99078$$ on a std day

but we are actually at $36^\circ F$

$$\theta_{1340} = \frac{36^{\circ F} + 459.67}{59^{F} + 459.67} = 0.9556$$

$$\theta_{1360} = \frac{37 + 459}{59 + 459} = 0.9575$$

Thus,

$$\sigma = \frac{\delta}{\theta} \Rightarrow \qquad \sigma_{1340} = \frac{0.9525}{0.9556} \qquad \sigma_{1360} = \frac{0.9518}{0.9575}$$

- What do we know?

$$\boxed{V_{grnd_W}}\,\boxed{\Delta t_W} + \boxed{V_{grnd_E}}\,\boxed{\Delta t_E} = 2 \cdot \text{Distance}$$

$$V_{grnd_W} = V_{true_W} + \text{Wind} \quad , \quad V_{grnd_E} = V_{true_E} + \text{Wind}$$

$$V_{true_W} = \frac{V_{cas_W}}{\sqrt{\sigma}}$$

$$V_{true_E} = \frac{V_{cas_E}}{\sqrt{\sigma}}$$

$$V_{grnd_W} = \frac{Dist}{\Delta t_W} \qquad V_{grnd_E} = \frac{Dist}{\Delta t_E}$$

$$\left(V_{true_W} - W\right)\Delta t_W = Dist \quad \left(V_{true_E} + W\right)\Delta t_E = Dist \; = \cancel{Dist}$$

$$V_{true_W} = \frac{V_{CAS_W}}{\sqrt{\sigma_W}} = \frac{V_{IAS_W} + C}{\sqrt{\sigma_W}}$$

$$V_{true_E} = \frac{V_{CAS_E}}{\sqrt{\sigma_E}} \qquad \frac{V_{IAS_E} + C}{\sqrt{\sigma_E}}$$

$$\left(\frac{V_{IAS_W} + C}{\sqrt{\sigma_W}} - W\right)\Delta t_W = Dist$$

$$\left(\frac{V_{IAS_E} + C}{\sqrt{\sigma_E}} + W\right)\Delta t_E = Dist$$

$$\cancel{C = \left(\frac{Dist}{\Delta t_W} + W\right)\sqrt{\sigma_W} - V_{IAS_W}}$$

$$\frac{C}{\sqrt{\sigma_W}} - W = \frac{Dist}{\Delta t_W} - \frac{V_{IAS_W}}{\sqrt{\sigma_W}}$$

$$\frac{C}{\sqrt{\sigma_E}} + W = \frac{Dist}{\Delta t_E} - \frac{V_{IAS_E}}{\sqrt{\sigma_E}}$$

$$\begin{bmatrix} \frac{1}{\sqrt{\sigma_W}} & -1 \\ \frac{1}{\sqrt{\sigma_E}} & +1 \end{bmatrix} \begin{pmatrix} C \\ W \end{pmatrix} = \begin{pmatrix} \cdots \\ \cdots \end{pmatrix}$$

$$\begin{pmatrix} C \\ W \end{pmatrix} = \frac{\begin{pmatrix} 1 & 1 \\ -\frac{1}{\sqrt{\sigma_E}} & \frac{1}{\sqrt{\sigma_W}} \end{pmatrix}}{\frac{1}{\sqrt{\sigma_W}} + \frac{1}{\sqrt{\sigma_E}}} \begin{pmatrix} \frac{Dist}{\Delta t_W} - \frac{V_{IAS_W}}{\sqrt{\sigma_W}} \\ \frac{Dist}{\Delta t_E} - \frac{V_{IAS_E}}{\sqrt{\sigma_E}} \end{pmatrix}$$

$$C = \frac{\overbrace{\frac{Dist}{\Delta t_W}}^{V_{gnd}} - \frac{V_{IAS_W}}{\sqrt{\sigma_W}} + \frac{Dist}{\Delta t_E} - \frac{V_{IAS_E}}{\sqrt{\sigma_E}}}{\frac{1}{\sqrt{\sigma_W}} + \frac{1}{\sqrt{\sigma_E}}}$$

$$\boxed{C = 1.501 \text{ kt}}$$