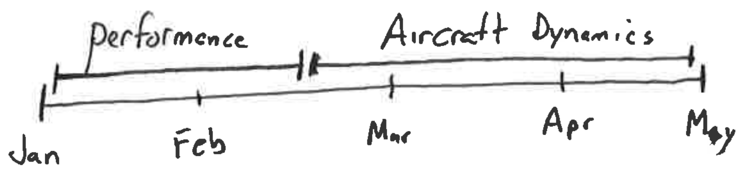
AEM 368

Stability And Control of Aircraft



Lesson 14

Static Stability and Control (Longitudinal)

Review of ordinary differential Equations (1 dim)

$$ay'' + by' + cy = 0$$

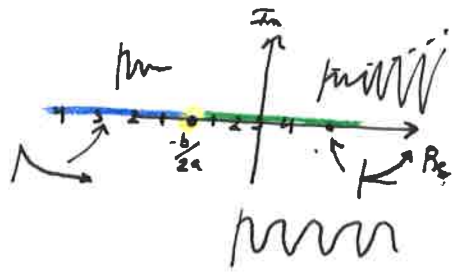
find a characteristic equation

$$ar^2 + br + c = 0$$

with solution roots $r_1, r_2 = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

- If $r_1 \neq r_2$ and both are real

$$y = Ae^{r_1 x} + Be^{r_2 x}$$



- If $r_1 = r_2$ and are real

$$y = Ae^{r_1 x} + Bxe^{r_1 x}$$

- If r_1 and r_2 are complex

$$y = Ae^{\alpha x}\cos(\beta x) + Be^{\alpha x}\sin(\beta x)$$

with

$$r_1 = \frac{-b}{2a} + \frac{1}{2a}\sqrt{b^2-4ac}$$

$$r_2 = \frac{-b}{2a} - \frac{1}{2a}\sqrt{b^2-4ac}$$

$$\Rightarrow r_1 = \bar{r}_2 = \alpha \pm i\beta$$

$$\alpha = \frac{-b}{2a} \qquad \beta = \frac{\sqrt{4ac-b^2}}{2a}$$

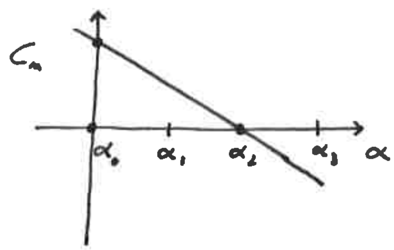
Longitudinal Static Stability

Rigid body motion is about the center of gravity (CG)

Measure moment about CG. and nondimensionalize

$$M = q S \bar{c} C_m \qquad \left[\frac{lbf}{ft^2} \cdot ft^2 \cdot ft\right] = [lbf \cdot ft] \;\checkmark$$

Now sweep through a range of α and $I\ddot{\alpha} = M$



at α_0, $C_m > 0$, $\Rightarrow$ α tends to increase $\ddot{\alpha} > 0$

at α_1, $C_m > 0$ $\Rightarrow$ α tends to increase $\ddot{\alpha} > 0$

at α_2, $C_m = 0$ $\Rightarrow$ α doesn't change $\ddot{\alpha} = 0$

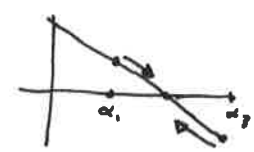
at α_3, $C_m < 0$ $\Rightarrow$ α decreases $\ddot{\alpha} < 0$

The equilibrium point is α_2 where $C_m = 0$.

Is the equilibrium point stable?

1) Reduce α to α_1
α tends to increase
2) Increase α to α_3
α tends to decrease.

this only happens when $\frac{dC_m}{d\alpha} < 0$

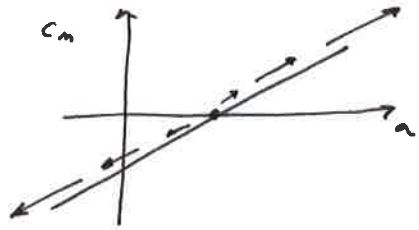


> Static Stability requires
>
> $$C_m = 0 \quad \text{and} \quad \frac{dC_m}{d\alpha} < 0$$
>
> at the given flight condition

The equilibrium point where $C_m = 0$ is often called the "trim point"

Ex: Aircraft lacking Static Stability

1)



$C_{m_\alpha} > 0$

Gov Equ:

$$I\ddot{\alpha} = M \Rightarrow \ddot{\alpha} = k C_m$$
$$= k C_{m_\alpha} \alpha$$

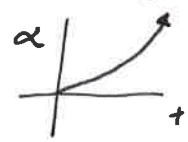
Solution to $\underset{a=1}{\ddot{\alpha}} - \underset{c=-kC_{m_\alpha}}{kC_{m_\alpha}\alpha}$ is $\quad b = 0$!

Two identical roots (real)

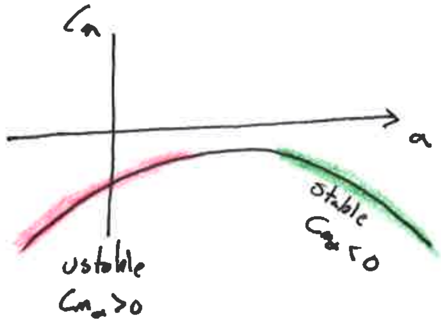
$$r = \sqrt{kC_{m_\alpha}} = \frac{-0}{2A} \pm \frac{\sqrt{0 - 4 \cdot 1 \cdot -kC_{m_\alpha}}}{2 \cdot 1}$$

Solution

$$\alpha = \underbrace{Ae^{\sqrt{kC_{m_\alpha}}t} + Bte^{\sqrt{kC_{m_\alpha}}t}}_{\text{diverge away with time}}$$

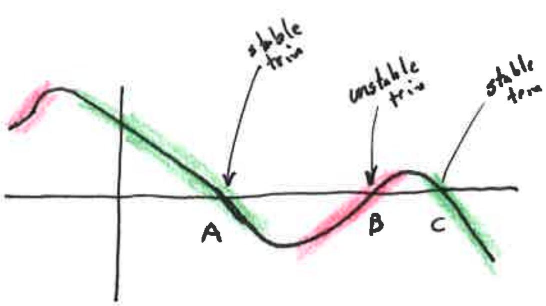


2)



$C_m \neq 0$

3)

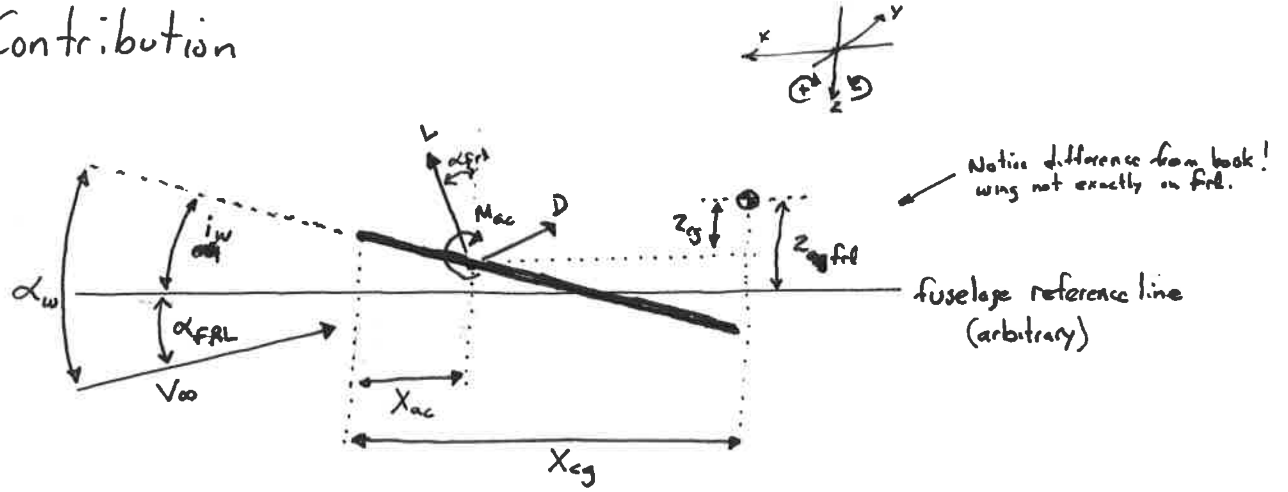


Multiple trim points

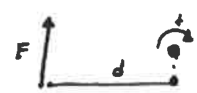
A and C are stable

B is unstable

Wing Contribution



Moment about the CG:



$$M_{cg_{wing}} = M_{ac_w} + \underbrace{L_w \cos(\alpha_w - i_w)}_{\text{Lift component normal to frl}} \cdot \underbrace{(X_{cg} - X_{ac})}_{\text{distance from L to cg}} + \underbrace{D_w \sin(\alpha_w - i_w)}_{\text{drag} \perp \text{frl}} \cdot \underbrace{(X_{cg} - X_{ac})}_{\text{distance from D to frl}}$$

$$+ \underbrace{L_w \sin(\alpha_w - i_w)}_{\text{Lift} \parallel \text{frl}} \cdot \underbrace{(z_{cg})}_{\text{distance (height)}} - \underbrace{D_w \cos(\alpha_w - i_w)}_{\text{Drag} \parallel \text{frl}} \cdot \underbrace{(Z_{cg})}_{\text{distance (height)}}$$

non-Dimensionalize (divide by $q S \bar{c}$)

$$C_{m_{cg_w}} = C_{m_{ac_w}} + C_{L_w} \cos(\alpha_w - i_w) \cdot \left(\frac{X_{cg}}{\bar{c}} - \frac{X_{ac}}{\bar{c}} \right)$$

$$+ C_{D_w} \sin(\alpha_w - i_w) \cdot \left(\frac{X_{cg}}{\bar{c}} - \frac{X_{ac}}{\bar{c}} \right)$$

$$+ C_{L_w} \sin(\alpha_w - i_w) \cdot \left(\frac{Z_{cg}}{\bar{c}} \right)$$

$$+ -C_{D_w} \cos(\alpha_w - i_w) \cdot \left(\frac{Z_{cg}}{\bar{c}} \right)$$

When the angles are small;

$$\cos(\alpha_w - i_w) \approx 1$$

$$\sin(\alpha_w - i_w) \approx \alpha_w - i_w$$

$$C_L > C_D$$

and the wing is mounted near the cg (in the vertical direction)

$$\frac{Z_{cg}}{\bar{c}} \approx 0$$

The moment about the CG becomes

$$C_{m_{cg_w}} = C_{m_{ac_w}} + C_{L_w} \cos\cancelto{1}{(\alpha_w - i_w)} \cdot \left(\frac{X_{cg}}{\bar{c}} - \frac{X_{ac}}{\bar{c}} \right)$$
$$+ \cancelto{0}{C_{D_w}} \sin\cancelto{\alpha_w - i_w}{(\alpha_w - i_w)} \cdot \left(\frac{X_{cg}}{\bar{c}} - \frac{X_{ac}}{\bar{c}} \right)$$
$$+ C_{L_w} \sin(\alpha_w - i_w) \cdot \cancelto{0}{\frac{Z_{cg}}{\bar{c}}} + - \cancelto{0}{C_{D_w}} \cos(\alpha_w - i_w) \cancelto{0}{\frac{Z_{cg}}{\bar{c}}}$$

$$\boxed{C_{m_{cg_w}} = C_{m_{ac_w}} + C_{L_w} \left(\frac{X_{cg}}{\bar{c}} - \frac{X_{ac}}{\bar{c}} \right)}$$

$$\underbrace{C_{m_{cg_w}}}_{\text{the moment about the CG}} = \underbrace{C_{m_{ac_w}}}_{\text{the moment about the wing}} + \underbrace{C_{L_w}}_{\text{lift coefficient}} \underbrace{\left(\frac{X_{cg}}{\bar{c}} - \frac{X_{ac}}{\bar{c}} \right)}_{\text{distance from a.c. to cg}}$$

Expand C_{L_w} (the wing's lift coefficient)

book typo "−"

$$C_{m_{cg_w}} = C_{m_{ac_w}} + \left(C_{L_{0_w}} + C_{L_\alpha} \alpha \right) \left(\frac{X_{cg}}{\bar{c}} - \frac{X_{ac}}{\bar{c}} \right)$$

Expand $C_{m_{cg_w}} = C_{m_{0_w}} + C_{m_{\alpha_w}} \alpha$

$$C_{m_{0_w}} + C_{m_{\alpha_w}} \alpha = C_{m_{ac_w}} + C_{L_{0_w}} \left(\frac{X_{cg}}{\bar{c}} - \frac{X_{ac}}{\bar{c}} \right) + C_{L_\alpha} \left(\frac{X_{cg}}{\bar{c}} - \frac{X_{ac}}{\bar{c}} \right) \alpha$$

Equate the constant terms

$$C_{m_{0_w}} = C_{m_{ac_w}} + C_{L_{0_w}}\left(\frac{X_{cg}}{\bar{c}} - \frac{X_{ac}}{\bar{c}}\right)$$

I personally prefer not to use this particular equation.

And the α terms

$$C_{m_{\alpha_w}} = C_{L_{\alpha_w}}\left(\frac{X_{cg}}{\bar{c}} - \frac{X_{ac}}{\bar{c}}\right)$$

Recall that Static Stability required two conditions

$$C_m = 0 \quad \text{and} \quad \frac{dC_m}{d\alpha} < 0$$

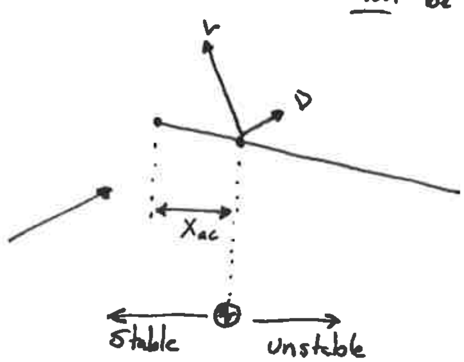
1) $C_m = 0$

For the moment to be zero, $C_{m_{ac_w}} + C_{L_{0_w}}\left(\frac{X_{cg}}{\bar{c}} - \frac{X_{ac}}{\bar{c}}\right) = 0$

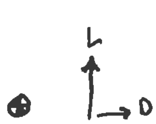
not much to say yet....

2) $\frac{dC_m}{d\alpha} \equiv C_{m_\alpha} < 0$

For stability, $C_{m_\alpha} \leq 0 \Rightarrow \underbrace{C_{L_\alpha}}_{\text{"+"}} \underbrace{\left(\frac{X_{cg}}{\bar{c}} - \frac{X_{ac}}{\bar{c}}\right)}_{\text{thus this term must be negative}} \leq 0 \Rightarrow \underline{X_{ac} > X_{cg}}$



with a forward CG, { an increase in α tends to create a nose down C_m moment
a decrease in α tends to create a nose up moment



Return to 1)

$$C_{m_{cg_w}} = 0 \Rightarrow 0 = C_{m_{ac_w}} + \underbrace{C_{L_w}}_{(C_{L_0} + C_{L_\alpha}\alpha)} \left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}} \right)$$

Must be negative for stability

$$0 = C_{m_{ac_w}} + \left(C_{L_0} + C_{L_\alpha}\alpha\right)\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right)^{-}$$

Flat plate

From thin airfoil theory, $C_\ell = 2\pi\alpha$

$C_{m_{c/4}} = 0$ (symmetrical)

$$0 = \cancelto{0}{C_{m_{ac_w}}} + \cancelto{0}{C_{L_0}} + \cancelto{2\pi}{C_{L_\alpha}}\alpha\,(-)$$

which is only zero when $\alpha = 0$

Not useful! $C_{L_\alpha} \cdot \cancelto{0}{\alpha} = 0$

No lift!

X

Circular Arc Airfoil

$$C_\ell = 2\pi\left(\alpha + \frac{2f}{c}\right)$$

$$C_{m_{c/4}} = -\pi \frac{f}{c}$$

plug into $C_m = 0$ eqn

$$0 = \underbrace{\cancelto{-\pi f/c}{C_{m_{ac_w}}}}_{-} + \underbrace{\left(\underbrace{\cancelto{4\pi f/c}{C_{L_0}}}_{+} + \underbrace{\cancelto{2\pi}{C_{L_\alpha}}\alpha}_{+}\right)}_{+}\underbrace{(-1)}_{-}$$

For $C_L > 0$, $\alpha > -\frac{2f}{c}$

and thus $0 = \text{"}-\text{"} + \text{"}-\text{"}$

Not possible!

Not trimmable!

X

Inverted Circular Arc

$$C_\ell = 2\pi\left(\alpha - \frac{2f}{c}\right)$$

$$C_{m_{c/4}} = \pi f/c$$

$C_L > 0$ when $\alpha > 2f/c$

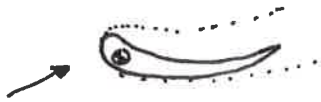
$$0 = \underbrace{\cancelto{\pi f/c}{C_{m_{ac_w}}}}_{+} + \underbrace{\underbrace{\left(\cancelto{-4\pi f/c}{C_{L_0}} + \cancelto{2\pi}{C_{L_\alpha}}\alpha\right)}_{+}\underbrace{(-)}_{-}}_{-}$$

Trimmable! Stable!

✓

The inverted camber airfoil is not an efficient choice from aerodynamics ...

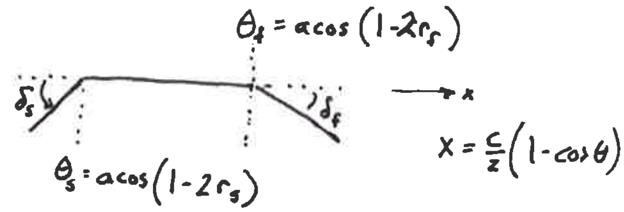
Why?



Separation and Boundary layer growth

We need an airfoil with a "nicer" leading edge, $C_{L_0} > 0$, and trimmable

Recall an airfoil with a slat and flap

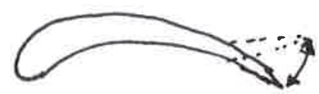


From Aerodynamics

$$C_\ell \approx 2\pi A_0 + \pi A_1$$

$$C_{m_{1/4c}} = -\frac{\pi}{4}(A_1 - A_2)$$

models



$$A_0 = \alpha - \frac{1}{\pi}\left(\delta_s\theta_s + \delta_f\theta_f\right) + \delta_f$$

$$A_n = \frac{2}{n\pi}\left(\delta_s \sin n\theta_s + \delta_f \sin n\theta_f\right)$$

$$A_1 = \frac{2}{\pi}\left(\delta_s\sqrt{4r_s + 4r_s^2} + \delta_f\sqrt{4r_f + 4r_f^2}\right) = \frac{2}{\pi}\delta_s \sin\theta_s + \frac{2}{\pi}\delta_f \sin \theta_f$$

$$C_\ell = 2\pi\alpha + 2\pi\delta_f\left(1 - \frac{\theta_f}{\pi} + \frac{\sin\theta_f}{\pi}\right) + 2\pi\delta_s\left(-\frac{\theta_s}{\pi} + \frac{\sin\theta_s}{\pi}\right)$$

$$A_2 = \frac{1}{\pi}\delta_s \sin 2\theta_s + \frac{1}{\pi}\delta_f \sin(2\theta_f)$$

$$C_{m_{1/4}} = -\frac{\pi}{4}\left(\frac{2}{\pi}\delta_s\sin\theta_s + \frac{2}{\pi}\delta_f\sin\theta_f - \frac{\delta_s}{\pi}\sin 2\theta_s - \frac{\delta_f}{\pi}\sin 2\theta_f\right)$$

After trials, you can find a stable and trimmed flying wing

$r_s = 0.2$ $\theta_s = 5°$

$r_f = 0.8$

$\Rightarrow$

reflexed trailing edge

$\frac{X_{cg}}{c} - \frac{X_{ac}}{c} = -0.1$

δ_f	C_L	α
-1	0	0°
-2	0.07	2°
-5	0.4	7°
-8	0.75	12° stall!

Very poor choice for high C_L since C_L decreases with the trailing edge deflection nessesary to trim to a high C_L.

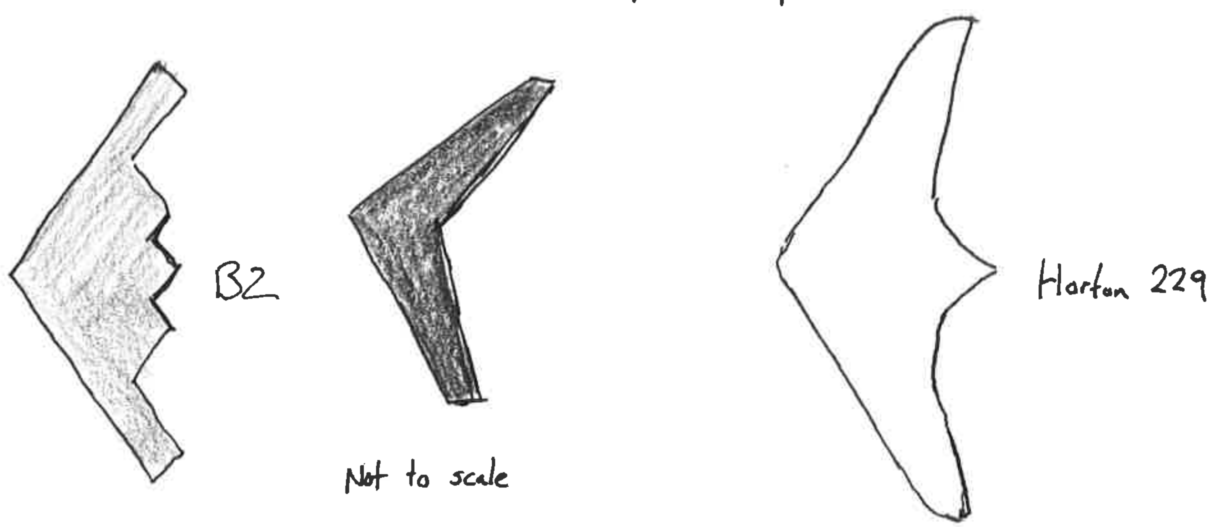
C_{L_0} is still rather low compared to non-reflexed airfoil

Flying Wings

Unswept flying wings are not common.

- poor $C_{L_{max}}$
- poor pitch damping (see later in class)
- Often sensitive to δ_f without very forward cg, in which case, th δ_f must be large
- Aero is still poor wrt BL growth and stall.
  purposeful decambering?!

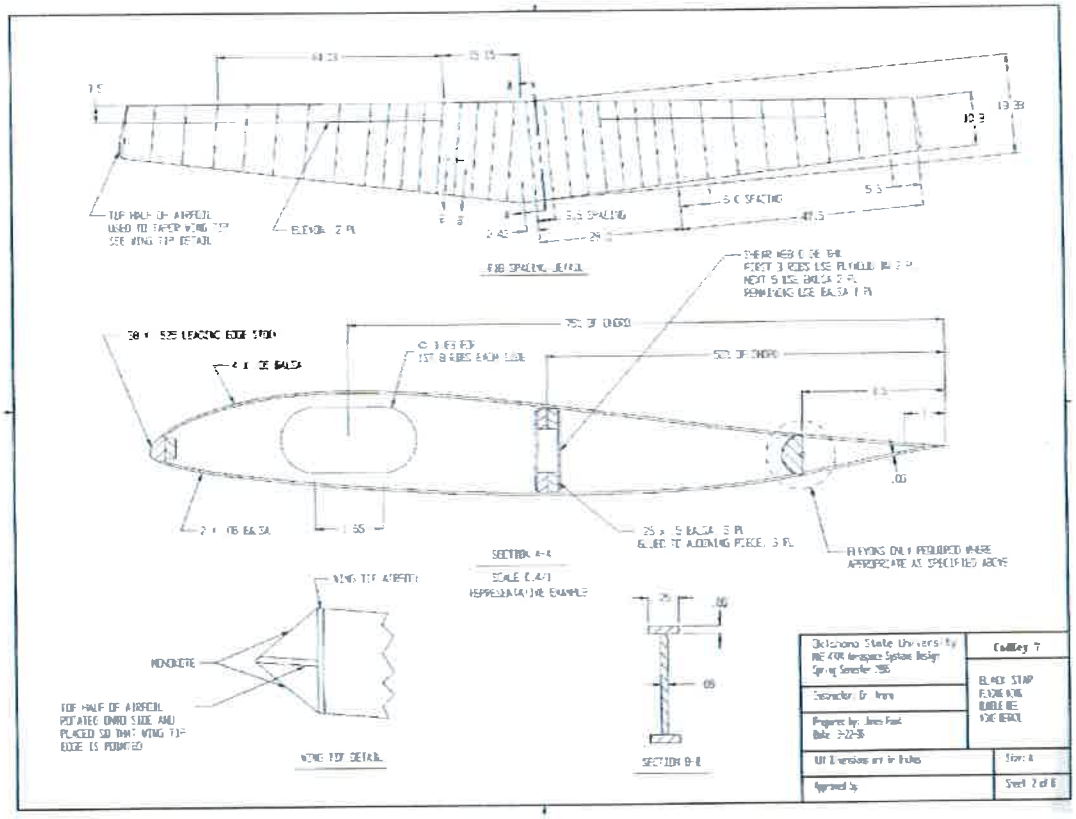
Most flying wings are the swept concept

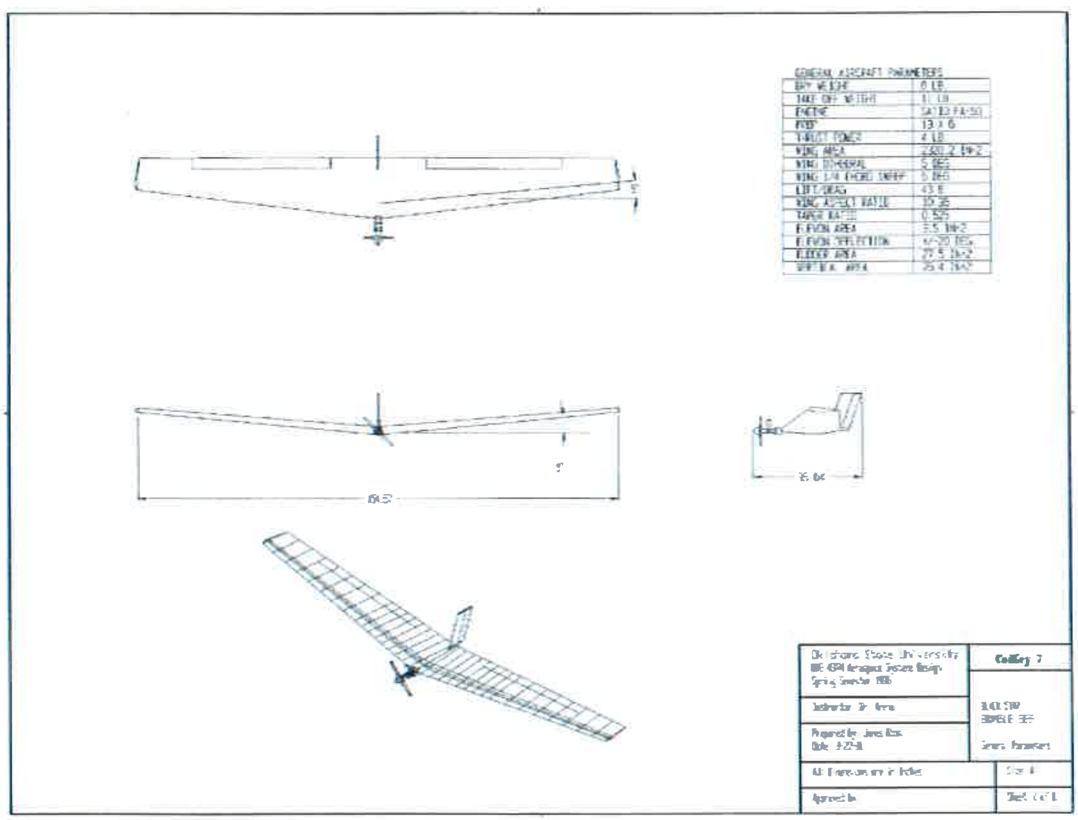


These achieve stability and trim through wing twist. This twist profile also allows the vertical stab to be removed (i.e. natural yaw stability ... like a bird)

There are some persistent myths of aerodynamic efficiency associated with flying wings. For the most part, these are wishful thinking.

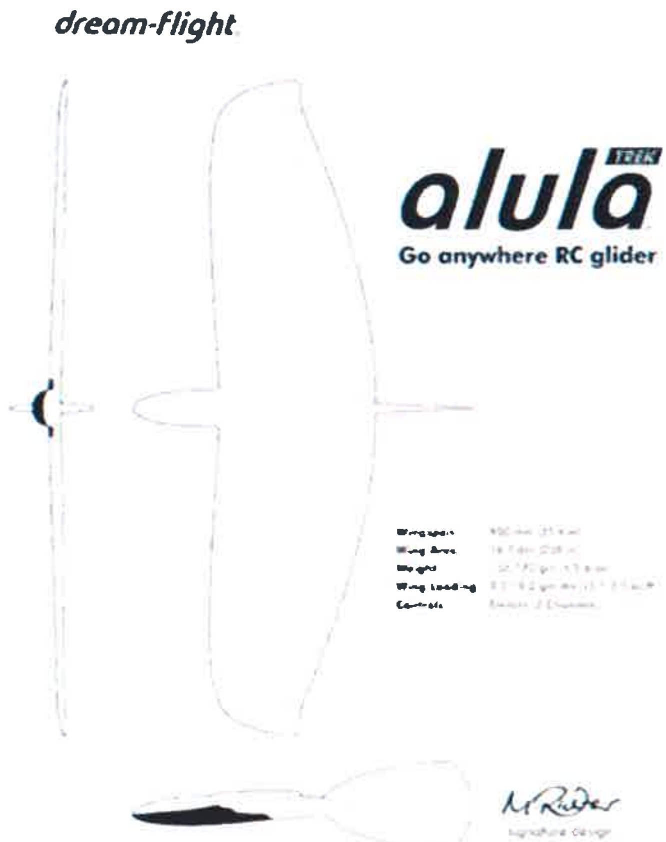
Stealth is not a myth, but is also not completely "invisible".





Source: aerodesign.okstate.edu

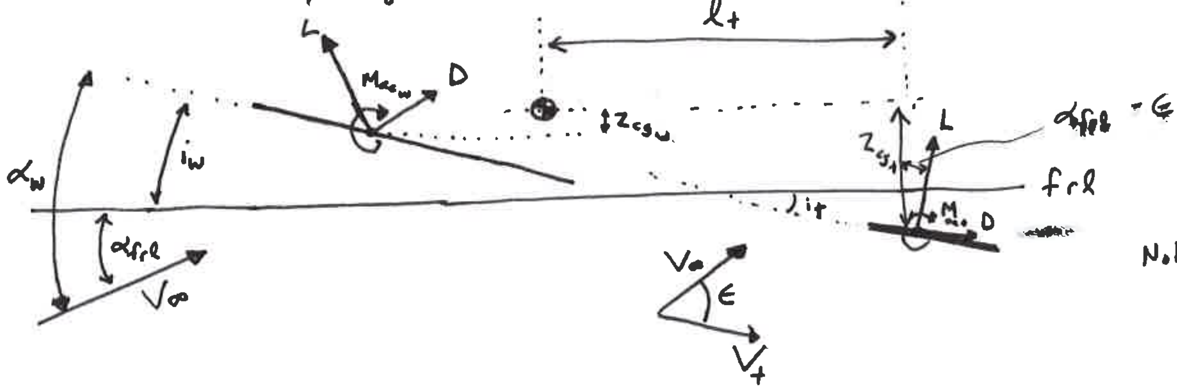
YB-49 (Source: DFRC)



Tail Contribution to Static Stability (Aft tail)

An aft tail operates in the downwash and wake of the wing, this complicates the aerodynamics associated with static stability.

The Freebody diagram is



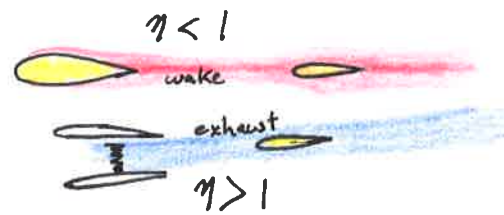
Notice in this figure $\alpha_{frl} - \epsilon < 0$!!

At the tail

$$\alpha_t = \underbrace{\alpha_w - i_w}_{\alpha_{frl}} \underbrace{- \epsilon}_{\text{downwash}} \underbrace{+ i_t}_{\substack{\text{tail}\\\text{incidence}\\\text{angle}}}$$

The dynamic pressure ratio at the tail

$$\eta = \frac{q_t}{q_w} = \frac{\frac{1}{2}\rho V_t^2}{\frac{1}{2}\rho V_w^2}$$



Assuming small angles again gives

$$L_{aircraft} = L_w + L_t$$

nondim' by qS_w

$$\Rightarrow \underbrace{\frac{L}{qS_w}}_{C_L} = \underbrace{\frac{L_t}{qS_w}}_{C_{L_w}} + \underbrace{\frac{L_t}{qS_w}}$$

$$\frac{L_t}{qS_w} \cdot \frac{S_t}{S_t} \cdot \frac{q_t}{q_t} = C_{L_t} \frac{S_t}{S_w} \eta$$

$$C_L = C_{L_w} + C_{L_t} \frac{S_t}{S_w} \eta$$

Sum of moments about CG (from tail)

$$M_t = -l_t L_t \cos(\alpha_{frl} - \epsilon) - l_t D_t \sin(\alpha_{frl} - \epsilon)$$
$$- z_{cg_t} D_t \cos(\alpha_{frl} - \epsilon) + z_{cg_t} L_t \sin(\alpha_{frl} - \epsilon) + M_{ac_t}$$

Be careful with sign of sine terms. $\alpha_{frl} - \epsilon < 0$!

Reduce terms

$$M_t = -l_t L_t \cancelto{1}{\cos(\dots)} - l_t \cancelto{\ll L_t}{D_t} \cancelto{\approx 0}{\sin(\dots)} - z_{cg} \cancelto{\ll L_t}{D_t} \cancelto{1}{\cos(\dots)} + z_{cg_t} L_t \cancelto{\approx 0}{\sin(\dots)}$$

$$= -l_t L_t$$

Non-dim by <u>wing</u> $q_w S_w \bar{c}_w$

$$\overbrace{\cancel{\frac{M_t}{q_w S_w \bar{c}_w}}}^{C_{m_t}} = \frac{-l_t L_t}{q_w S_w \bar{c}_w} = \frac{-l_t}{q_w S_w \bar{c}_w} \cdot q_t S_t C_{L_t} = \underbrace{\frac{-l_t S_t}{\bar{c} S_w}}_{\boxed{\text{Tail volume ratio} \equiv V_H}} \eta C_{L_t}$$

$$\boxed{C_{m_t} = -V_H \eta C_{L_t} = -\frac{l_t}{\bar{c}} \frac{S_t}{S_w} \eta C_{L_t}}$$

Angle of attack at tail

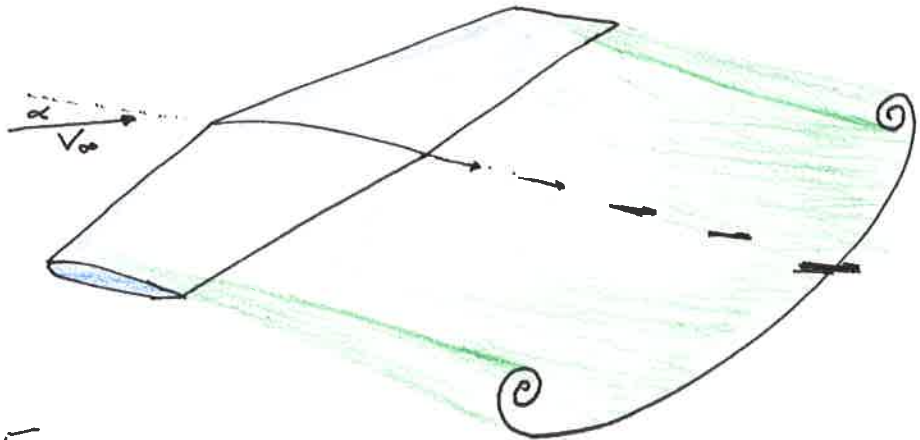
$$\alpha_t = \overbrace{\cancel{\alpha_{FRL}}}^{\alpha_w - i_w} - \underbrace{\epsilon}_{\text{downwash}} + \overbrace{i_t}^{\text{tail incidence angle}} = \alpha_w - i_w - \epsilon + i_t$$

Tail lift coeff.

Horizontal α_{ZL} is lumped into here.

$$C_{L_t} = C_{L_{\alpha_t}} \alpha_t = C_{L_{\alpha_t}} \left(\underbrace{\alpha_w}_{\text{flight condition}} - \underbrace{i_w}_{\substack{\text{geometry} \\ \text{(easy)}}} - \underbrace{\epsilon}_{\substack{\text{aero} \\ \text{(hard)}}} + \underbrace{i_t}_{\substack{\text{geometry} \\ \text{(easy)}}} \right)$$

Downwash



Taylor Series ...

$$\epsilon = \epsilon_0 + \frac{d\epsilon}{d\alpha}\alpha_w$$

book typo Eq 2.20

For an elliptical ~~drag~~ lift distribution

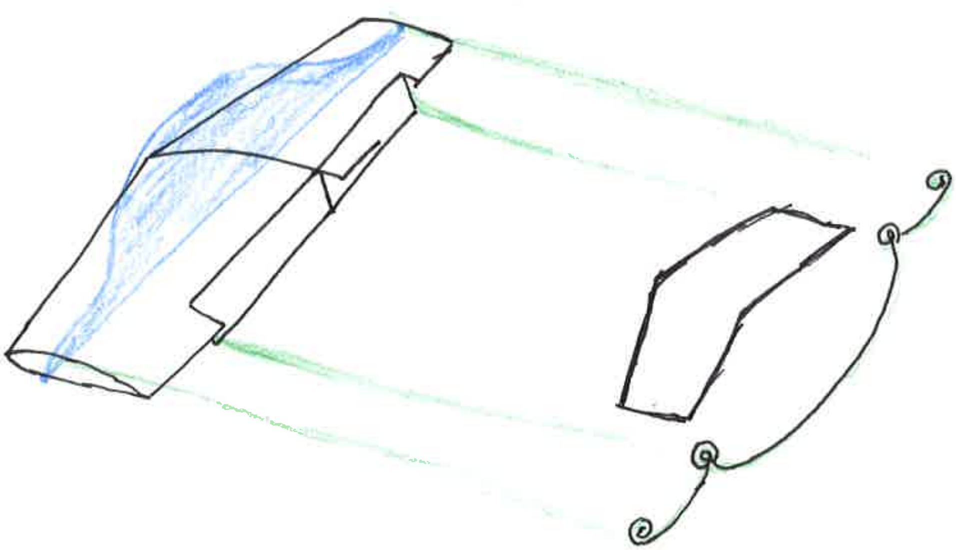
$$\epsilon = \frac{2C_{L_w}}{\pi AR_w} = \epsilon_0 + \frac{d\epsilon}{d\alpha}\alpha_w$$

$$= \frac{2C_{L_{w_{\alpha=0}}}}{\pi AR_w} + \frac{2C_{L_{\alpha_w}}}{\pi AR_w}\alpha_w$$

Non-elliptical

downwash varies along span

Average value is a reasonable estimate



Moment Coeff about CG (tail)

$$C_{m_t} = -V_H \eta C_{L_t} = -V_H \eta C_{L_{\alpha_t}}\left(\alpha_w - i_w - \underbrace{\epsilon}_{\epsilon_0 + \frac{d\epsilon}{d\alpha}\alpha} + i_t\right)$$

$$= \underbrace{\eta V_H C_{L_{\alpha_t}}\left(\epsilon_0 + i_w - i_t\right)}_{\text{geometry}} - \eta V_H C_{L_{\alpha_t}}\underbrace{\left(\alpha_w - \frac{d\epsilon}{d\alpha}\alpha_w\right)}_{\left(1-\frac{d\epsilon}{d\alpha}\right)\alpha}$$

Aero

this is now the wing α <u>not</u> α_t

$$= C_{m_{0_t}} + C_{m_\alpha}\alpha$$

with
$$C_{m_{0_t}} = \eta V_H C_{L_{\alpha_t}}\left(\epsilon_0 + i_w - i_t\right)$$
$$C_{m_{\alpha_t}} = -\eta V_H C_{L_{\alpha_t}}\left(1 - \frac{d\epsilon}{d\alpha}\right)$$

<u>Conviential Aircraft</u>

$$C_{m_0} = C_{m_{0_w}} + C_{m_{0_t}} = C_{m_{ac_w}} + C_{L_{0_w}}\left(\frac{X_{cg}}{\bar{c}} - \frac{X_{ac}}{\bar{c}}\right) + \eta V_H C_{L_{\alpha_t}}\left(\epsilon_0 + i_w - i_t\right)$$

easy to trim a/c

and

$$C_{m_\alpha} = C_{m_{\alpha_w}} + C_{m_{\alpha_t}} = C_{L_{\alpha_w}}\left(\frac{X_{cg}}{\bar{c}} - \frac{X_{ac}}{\bar{c}}\right) \underbrace{- \eta V_H C_{L_{\alpha_t}}\left(1 - \frac{d\epsilon}{d\alpha}\right)}_{\text{Horizontal provides a strong stabilizing effect.}}$$

- Unlike the flying wing, the CG <u>can</u> be aft of the quarter chord provided a large enough tail.

- Tail incidence i_t is an effective trim mechanism.