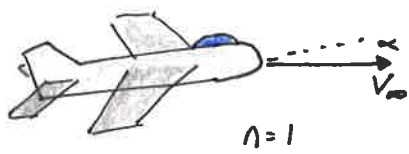


# Lesson 17

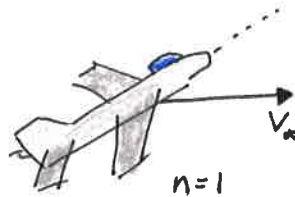
# Stall Speed and $C_{Lmax}$ from Test Flight Data

The lift coefficient is

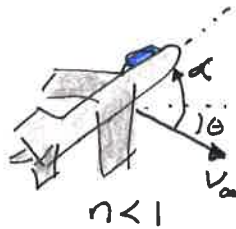
$$C_L = \frac{2nW}{\rho V^2 S} \quad \text{when } \theta \approx 0$$



slow velocity smoothly and  $\alpha$  increases



Wing no longer supports aircraft weight



$V_{cruise}$

>

$V_{s1g}$

>

$V_{min}$

How slowly should stall be approached? FAA says  $-1 \text{ kt/s}$

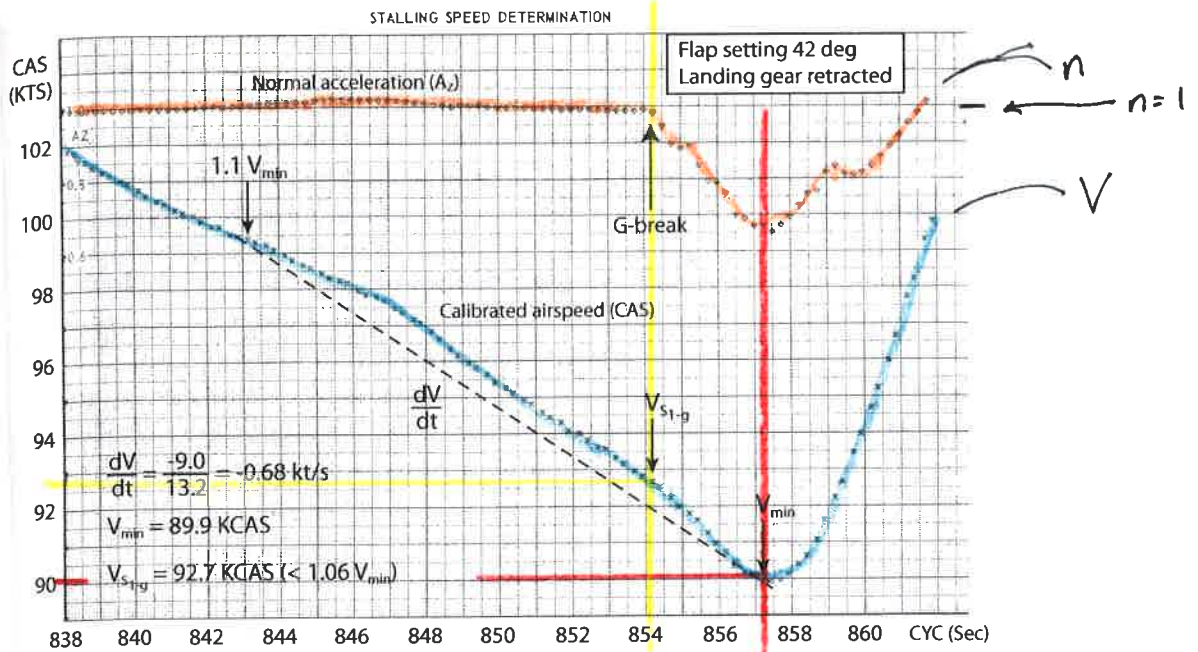


Figure 26.2 - Stall speed determination of a Fokker F-28 Mk 4000.  
Source: Fokker Report H-28.40-27.001

↑  
Velocity

Time →

Source: Aerodynamic Design of Transport Aircraft Obert

Q: For an aircraft certified after 2002, what is the stall speed?

$V_{min}$  X,  $V_{s_{1-g}}$ , something else? X

Q: Given the following data (flight test) for the Boeing 747, determine the stall speed if the aircraft were under the 2002 rules (it's not).

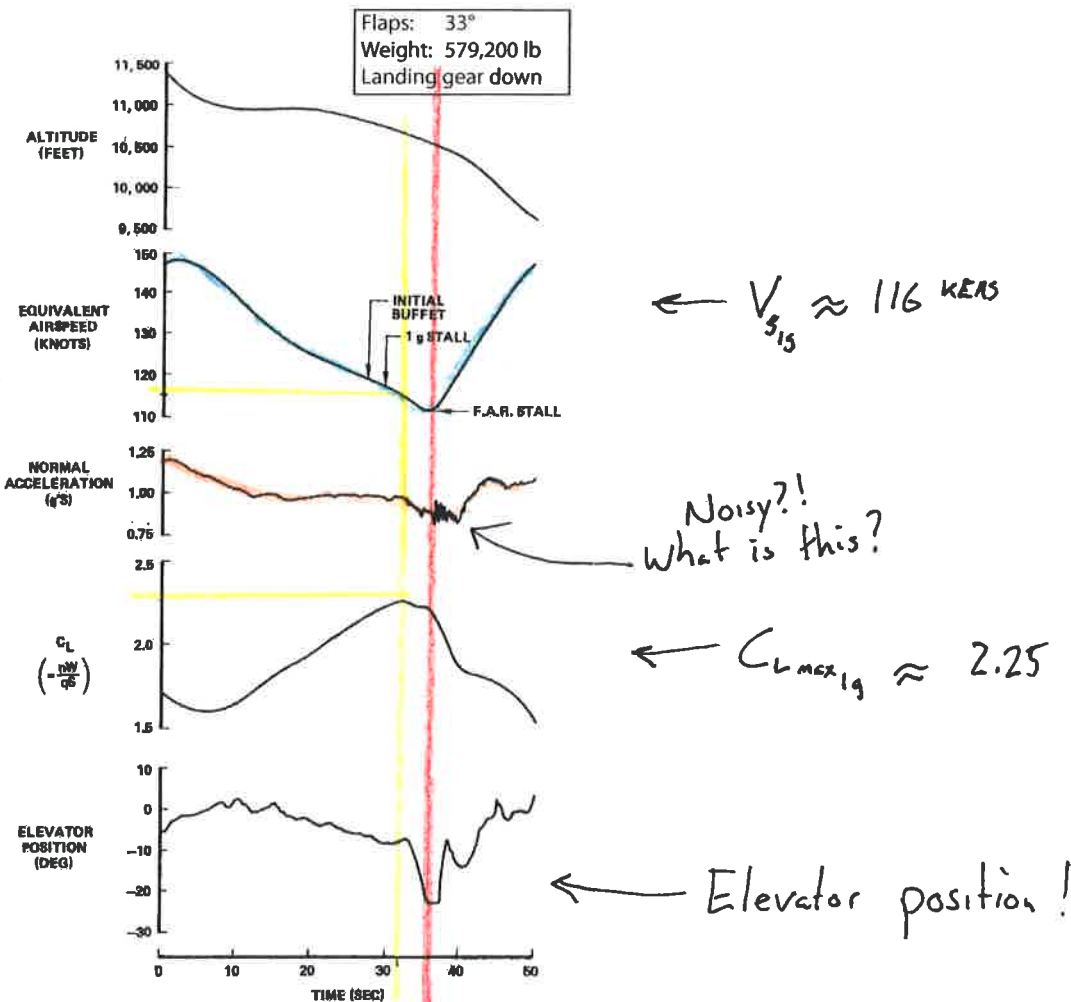


Figure 26.1 - Flight record of a stall manoeuvre of a Boeing 747  
Source: AGARD CP-102, Paper No. 21

CONTROLLABILITY AND  
MANEUVERABILITY

**§ 23.143 General.**

(a) The airplane must be safely controllable and maneuverable during all flight phases including—

- (1) Takeoff;
- (2) Climb;
- (3) Level flight;
- (4) Descent;
- (5) Go-around; and
- (6) Landing (power on and power off) with the wing flaps extended and retracted.

(b) It must be possible to make a smooth transition from one flight condition to another (including turns and slips) without danger of exceeding the limit load factor, under any probable operating condition (including, for multiengine airplanes, those conditions normally encountered in the sudden failure of any engine).

(c) If marginal conditions exist with regard to required pilot strength, the control forces necessary must be determined by quantitative tests. In no case may the control forces under the conditions specified in paragraphs (a) and (b) of this section exceed those prescribed in the following table:

Values in pounds force applied to the relevant control	Pitch	Roll	Yaw
<b>(a) For temporary application:</b>			
Stick .....	60	30	.....
Wheel (Two hands on rim)	75	50	.....
Values in pounds force applied to the relevant control	Pitch	Roll	Yaw
Wheel (One hand on rim) ..	50	25	.....
Rudder Pedal .....	.....	.....	150
<b>(b) For prolonged application ....</b>	10	5	20

# Stick position Gradients

From earlier (Lesson 16), we found that at trim,

$$\frac{d\delta_{trim}}{dC_{L_{trim}}} = \frac{C_{m\alpha}}{C_{L\delta_e} C_{m\alpha} - C_{L\alpha} C_{m\delta_e}}$$

Using the chain rule, we can find the stick position vs ~~the~~ Velocity



$$\frac{d\delta_e}{dV} = \frac{d\delta_e}{dC_L} \frac{dC_L}{dV} \quad \text{with } C_L = \frac{2W}{\rho V^2 S} \Rightarrow \frac{dC_L}{dV} = -\frac{4W}{\rho V^3 S}$$

$$= \frac{C_{m\alpha}}{C_{L\delta_e} C_{m\alpha} - C_{L\alpha} C_{m\delta_e}} \cdot \frac{-4W}{\rho V^3 S}$$

$$\frac{d\delta_e}{dV} = \frac{4W}{\rho V^3 S} \cdot \frac{\overset{cg}{C_{m\alpha}}}{\underbrace{C_{L\alpha} C_{m\delta_e}}_+ - \underbrace{C_{L\delta_e} C_{m\alpha}}_{cg}}$$

with  $C_{m\alpha} = C_{L\alpha} \left( \frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}} \right) - \text{tail contribution}$

pilots strongly demand  $\frac{d\delta_e}{dV} > 0$

elevator <sup>"push"</sup> TED → Faster   
 "pull" TEU → Slower 

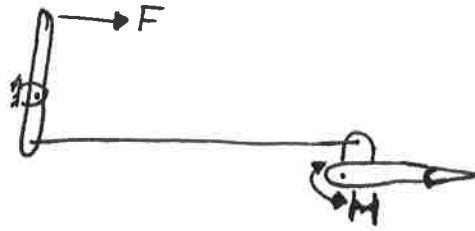
$$\frac{C_{m\alpha} < 0}{\frac{d\delta_e}{dV}} = + \frac{-}{(-) - (-)} = + \frac{-}{-} = + \quad \text{with } C_{m\alpha} < 0$$

$$\frac{C_{m\alpha} > 0}{\frac{d\delta_e}{dV}} = + \frac{+}{(+(-) - (+)(+)} = + \frac{+}{-} = -$$

A stable aircraft naturally has  $\frac{d\delta_e}{dV} > 0$ .

An unstable aircraft can be both unstable in the pitch angle and velocity.

# Stick Forces



A force on the stick generates a moment on the surface.

Look first at the hinge moment generated by the aerodynamics

$$H = q S_e \bar{c}_e C_{h_e} \approx q S_e \bar{c}_e \left( \underbrace{C_{h_0}}_{\text{Static offset 'control?'}} + \underbrace{C_{h_{\alpha}} \alpha_t}_{\text{AOA}} + \underbrace{C_{h_{\delta_e}} \delta_e}_{\text{elevator}} + \underbrace{C_{h_{\delta_t}} \delta_t}_{\text{trim tab}} \right)$$

From before, the AOA at the tail

$$\alpha_t = \alpha - \epsilon + i_h = \alpha \left( 1 - \frac{d\epsilon}{d\alpha} \right) + i_h - \epsilon_0$$

$$H = q S_e \bar{c}_e \left( C_{h_0} + C_{h_{\alpha}} \left( \alpha \left( 1 - \frac{d\epsilon}{d\alpha} \right) + i_h - \epsilon_0 \right) + C_{h_{\delta_e}} \delta_e + C_{h_{\delta_t}} \delta_t \right)$$

How does the Moment H vary with ~~the~~ Velocity?

$$\frac{dH}{dV} = \frac{dH}{d\delta_e} \frac{d\delta_e}{dV} = \left( q S_e \bar{c}_e C_{h_{\delta_e}} \right) \left( \frac{4W}{\rho V^3 S} \right) \left( \frac{C_{m_{\delta_e}}}{C_{L_{\alpha}} C_{m_{\delta_e}} - C_{L_{\delta_e}} C_{m_{\alpha}}} \right)$$

↑ calculate
↑ already derived this

$$= \eta \frac{2W}{V_{\infty}} \left( \frac{S_e}{S} \right) \bar{c}_e C_{h_{\delta_e}} \frac{C_{m_{\delta_e}}}{C_{L_{\alpha}} C_{m_{\delta_e}} - C_{L_{\delta_e}} C_{m_{\alpha}}}$$

In the small  $\delta_e$  region and a stable aircraft ( $C_{m_{\alpha}} < 0$ )

$$= \left( \frac{+}{+} \right) \left( \frac{+}{+} \right) (+) (-) \frac{(-)}{(+)(-) - (+)(-)} = +$$

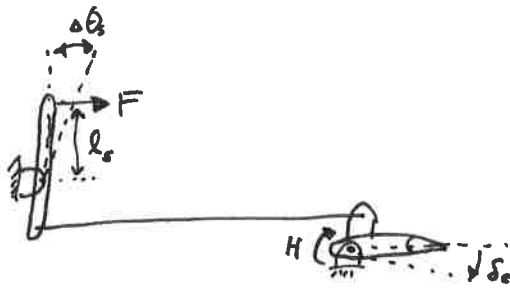
$= (-)$   
 depends only on  $C_{h_{\delta_e}}$  when  $C_{m_{\alpha}} < 0$

Increasing the velocity decreases H:  $dH = -dV$



$\frac{dH}{dV} < 0$  is a speed stable aircraft in pitch

# Stick Forces



By conservation of energy,  $F l_s \Delta \theta_s = H \Delta e$  for small  $\Delta e$  and  $\Delta \theta_s$

$$F = \underbrace{\frac{\Delta e}{\Delta \theta_s}}_{\text{Gearing ratio}} \frac{1}{l_s} H = G_e H \quad \left( \begin{array}{l} \text{The stick force} \\ \text{counteracts a} \\ \text{hinge moment } H \end{array} \right)$$

$$F = G_e g \eta S_e \bar{c}_e C_{n_e} = G_e g \eta S_e \bar{c}_e \left( C_{n_0} + C_{n_{\alpha}} \left( \alpha \left( 1 - \frac{dG}{d\alpha} \right) + i_n - \epsilon_0 \right) + C_{n_{\delta_e}} \delta_e + C_{n_{\delta_e}} \delta_e \right)$$

But write  $\alpha$  as:  $\alpha = \frac{d\alpha}{dC_L} C_L + \alpha_0 = \frac{d\alpha}{dC_L} \frac{W}{gS} + \alpha_0$  where  $\alpha_0$  is  $C_L = 0$

$\delta_e = \frac{d\delta_e}{dC_L} C_L + \delta_{e_0} = \frac{d\delta_e}{dC_L} \frac{W}{gS} + \delta_{e_0}$   $\delta_{e_0}$  is for  $C_L = 0$

$$F = G_e g \eta S_e \bar{c}_e \left( C_{n_0} + C_{n_{\alpha}} \left( 1 - \frac{dG}{d\alpha} \right) \alpha_0 + C_{n_{\alpha}} (i_n - \epsilon_0) + C_{n_{\delta_e}} \delta_{e_0} \right) + G_e \eta S_e \bar{c}_e \left( C_{n_{\alpha}} \left( 1 - \frac{dG}{d\alpha} \right) \frac{d\alpha}{dC_L} \frac{W}{gS} + C_{n_{\delta_e}} \frac{d\delta_e}{dC_L} \frac{W}{gS} \right)$$

Notice! One term has the static offsets and dynamic pressure.  
The other term has wing loading and NO dynamic pressure!!

The stick force is approximately

$$F = \eta G S_e \bar{c}_e \left( gA + \frac{W}{gS} \frac{C_{n_{\delta_e}}}{C_{n_{\alpha}}} SM_{free} \right)$$

$$A = C_{n_0} + C_{n_{\alpha}} \left( 1 - \frac{dG}{d\alpha} \right) \alpha_0 + C_{n_{\alpha}} (i_n - \epsilon_0) + C_{n_{\delta_e}} \delta_{e_0}$$

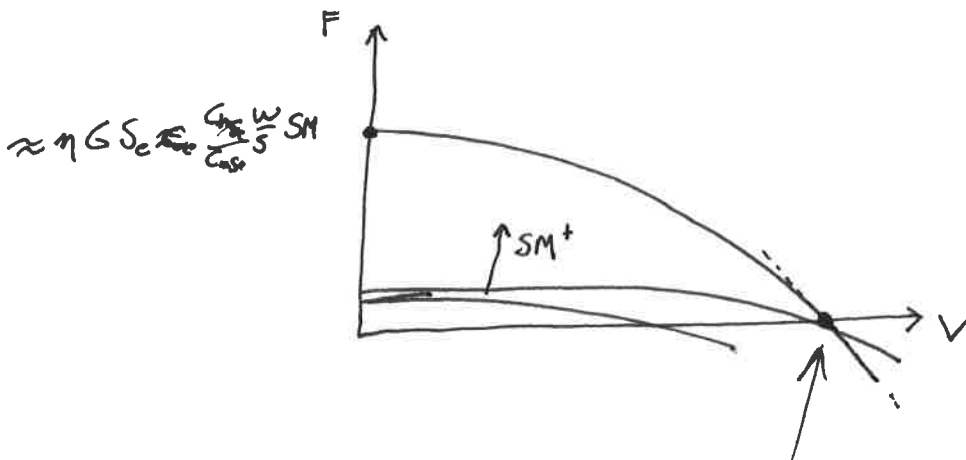
and  $SM_{free}$  is the static margin (elevator free)

# Stick Force Gradients

How does  $F$  change with velocity?

$$\frac{dF}{dV} \approx \eta G_c S_c \bar{c}_c \frac{1}{2} \rho V \cdot 2 \cdot A = C \cdot V$$

Plot  $F$  vs  $V$



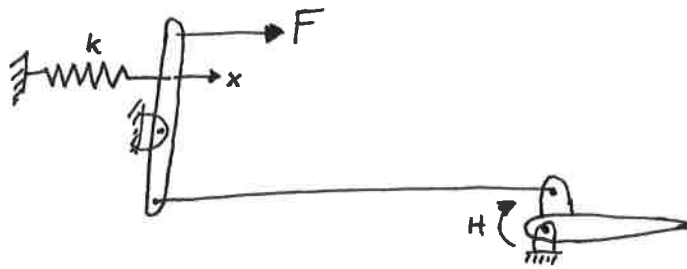
At this point,  $gA + \frac{W}{S} \frac{C_{hs}}{C_{hs}} SM_{tra} = 0$

Thus,  $A < 0$  and thus  $\frac{dF}{dV} < 0$

Notice that given a particular trim speed,  
 a larger Static Margin gives a  
more speed stable aircraft (i.e. the slope of  $F$  wrt  $V$  is  
 more negative)



# Adding a spring



The spring adds

$$F_{spring} = kx = \bar{k} \delta_e \quad \text{where } \bar{k} \text{ is the geometric gear ratio}$$

The total stick force is  $F = F_{aero} + F_{spring}$

$$F = \eta G S_e \bar{c}_e \left( gA + \frac{W}{S} \frac{C_{L\delta_e}}{C_{N\delta_e}} SM_{Free} \right) + \bar{k} \delta_e$$

$$\uparrow$$

$$\frac{d\delta_e}{dC_L} C_L + \delta_e = \frac{d\delta_e}{dC_L} \frac{W}{gS} + \delta_e$$

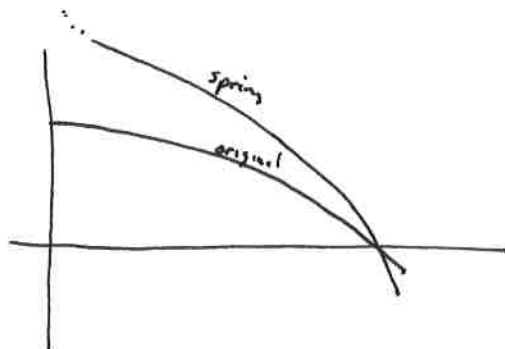
$$= \eta G S_e \bar{c}_e \left( gA + \frac{W}{S} \frac{C_{N\delta_e}}{C_{M\delta_e}} SM_{Free} \right) + \bar{k} \frac{1}{C_{L\delta_e}} \frac{W}{gS} + \bar{k} \delta_e$$

Now

$$\frac{dF}{dV} \approx \underbrace{C_1 V}_{\text{from previous}} + \bar{k} C_2 \frac{1}{V^3}$$

Thus at low speeds, adding a spring keeps  $\frac{dF}{dV}$  negative (and stable)

This is a common fix to overly sensitive control systems



## Stick Force per "g"

How many pounds does the pilot need to pull to generate a load factor  $n$ :

$$\frac{\partial F}{\partial n} \approx \eta \bar{g} S_e \bar{c}_e G_c \left( C_{L_{trim}} \frac{C_{hs_e}}{C_{hs_c}} S M_{trim} + \left( \frac{g(x_{ac} - x_{cg})}{U_{\infty}^2} \right) \left( C_{hs_e} - \frac{1.1 C_{hs_e}}{\tau_e} \right) \right)$$

A fighter aircraft is different than a bomber.

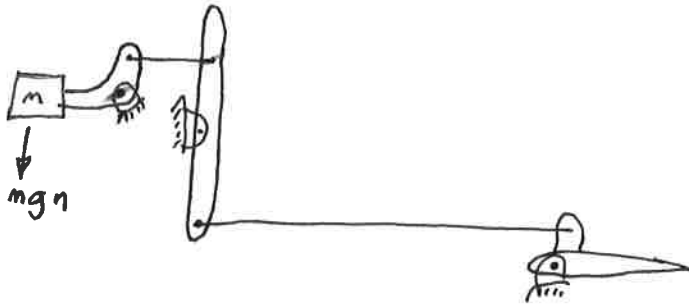
- too small and the pilot can "pull the wings off"
- too high and the pilot struggles to perform maneuvers

FAR 23 and 25 give requirements based on weight

$$F_{n-max} > 20 \text{ lbs stick}$$
$$> 15 \text{ lbs yoke}$$

"There must be no excessive decrease in the gradient of the curve of stick force versus maneuvering load factor"

Bob Weight (fix a low  $\frac{dF}{dn}$ )



As the load factor increases, the weight of the bob increases.  
This pulls the stick forward, which increases  $\frac{dF}{dn}$ .

See:

[tiny.cc/AEM368Bob](http://tiny.cc/AEM368Bob)

# 30 *Tail surface design*

Tail surfaces perform three functions:

1. They provide static and dynamic stability.
2. They enable aircraft control.
3. They provide a state of equilibrium in each flight condition.

The first and second items are covered in detail in textbooks and courses on Stability and Control.

The ability to maintain a state-of-equilibrium is often taken for granted, yet to cover extreme flight conditions it often sets design requirements for horizontal and vertical tail surfaces and their control surfaces.

Examples are minimum control speed with a failed engine ( $V_{MC}$ ), extreme out-of-trim conditions or maximum cross-wind capability.

In general the following design requirements can be formulated for tail surfaces:

1. They shall provide a sufficiently large contribution to static and dynamic longitudinal, directional (and sometimes lateral) stability. This determines primarily their lift gradients

$$\frac{dC_{l_h}}{d\alpha_h} S_h \quad \text{and} \quad \frac{dC_{l_v}}{d\alpha_v} S_v .$$

This requires a maximum aspect ratio and for high aspect ratios minimum sweep.

2. They shall provide sufficient control capability, which again determines their lift slope. This also requires a maximum aspect ratio and for high aspect ratios minimum sweep.

AERODYNAMIC DESIGN OF TRANSPORT AIRCRAFT

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3. Control shall be possible with acceptable control forces. This requires a maximum aspect ratio because control force

$$F = C_h \frac{1}{2} \rho V^2 S_c \bar{c}_c$$

where

$C_h$  = hinge moment coefficient

$\frac{1}{2} \rho V^2$  = dynamic pressure

$S_c$  = control surface area

$\bar{c}_c$  = control surface mean aerodynamic chord

4. The tail surfaces shall be able to cope with high tailplane angles-of-attack, both for the horizontal tail (in particular at higher speeds with flaps deflected) and for the vertical tail surface (high cross-winds). In this case a low aspect ratio is required and sweep is beneficial. The requirement to be able to cope with high tailplane and fin angles-of-attack is aggravated when flight in icing conditions is possible.
5. The tail surfaces shall be able to provide a maximum force sufficiently large to balance the total tail-off forces and moments so that static equilibrium is achieved in all flight conditions. This leads to specific requirements on tail surface areas and on the maximum lift coefficient for the tail surfaces with varying degree of control surface deflection, including the effect of ice roughness.
6. For high-speed aircraft the Mach-number at which serious flow separation occurs shall preferably lie above the design dive Mach-number  $M_D$ . Serious flow separation on the stabilizer will aggravate the effect of the changes in tail-off pitching moment due to changes in the wing flow (See pages 29.1-29.5). This applies in particular to aircraft with reversible control systems.

Therefore on stabilizers for high-speed aircraft the sweep angle is often about 5 deg higher than on the wing. Furthermore the section is often 1 or 2% (relative to the chord) thinner than on the outboard wing.

Note that flow break-down on the tail surfaces should preferably not occur below  $M_D$  also with deflected control surfaces required for small side slip corrections or pull-up manoeuvres with  $n = 1.5$ .

390

---

Furthermore the following shall be kept in mind:

1. A high aspect ratio has an adverse (although relatively small) effect on weight. Also, in particular for T-tails the flutter analysis requires extra care. A few degrees anhedral (negative dihedral) has a very beneficial effect.
2. Excessive taper ratio may lead to premature tip stall. This risk is higher when sweep is applied although the stall is then more gradual with less loss in lift. On the other hand tapering leads to lower weight.

Source:

Aerodynamic Design  
of Transport Aircraft

Obert

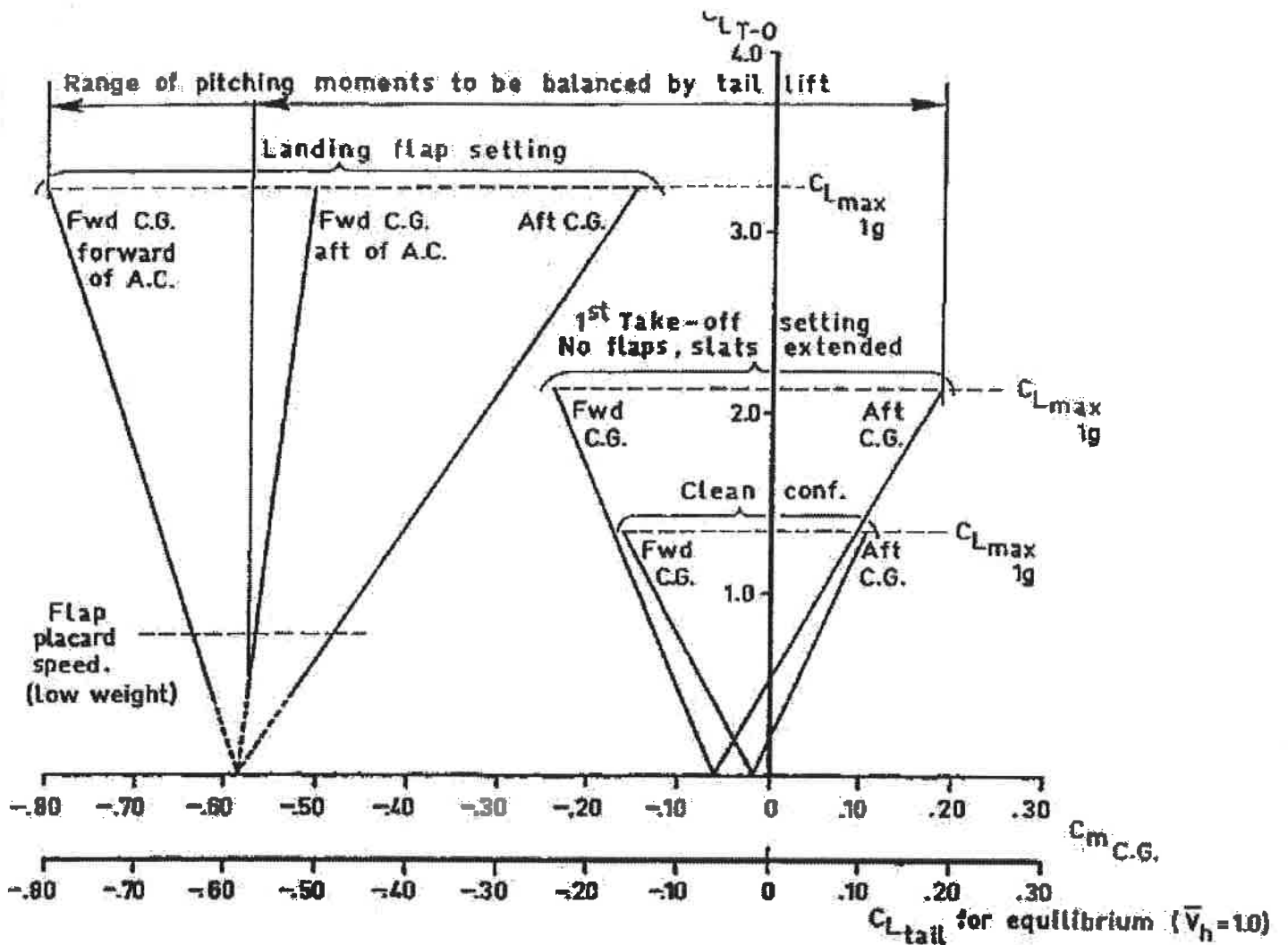


Figure 31.11 - Pitching moment, aircraft less tail. Source: AGARD CP-160, Paper No. 10  
 Flap placard speed = maximum speed at which aircraft is allowed to fly with flaps deployed



# Loss of Control of Light Aircraft: A cost effective approach to Flight Test

With the \$  
price tag



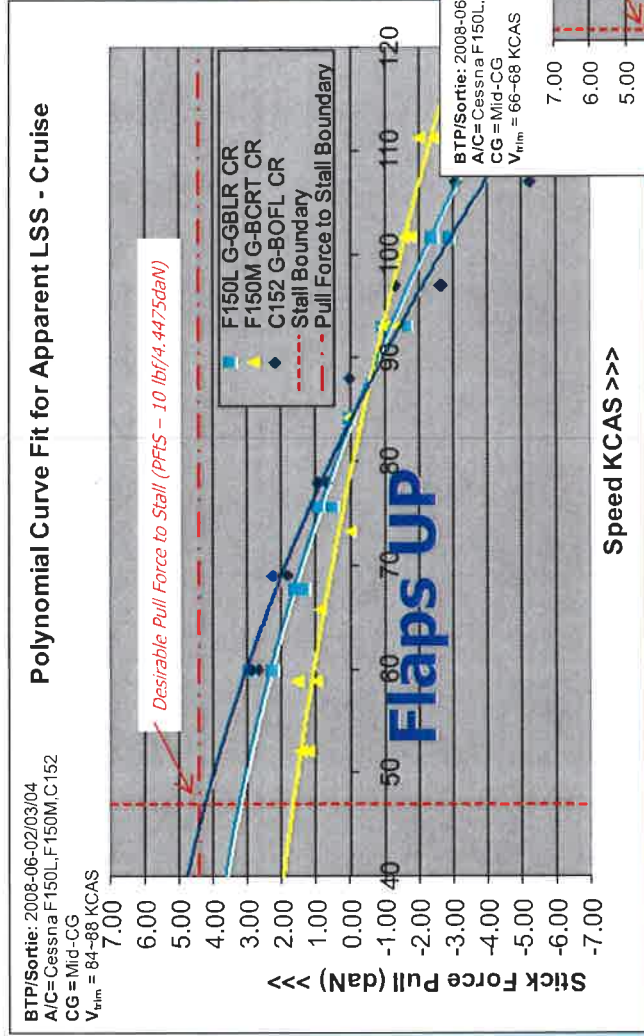
**Mike Bromfield**  
**Guy Gratton**

To the 41st SFTE International Symposium

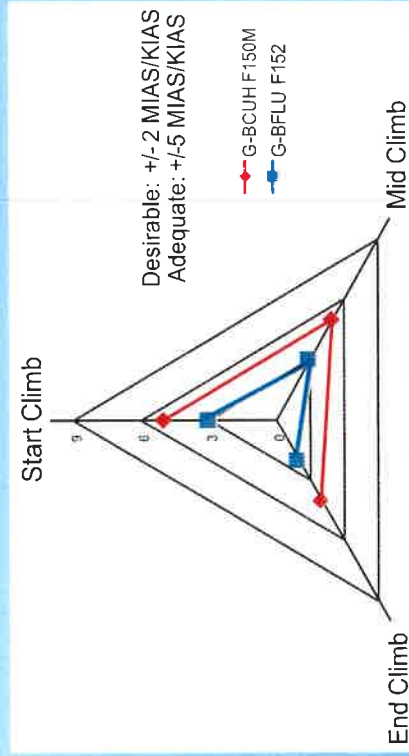
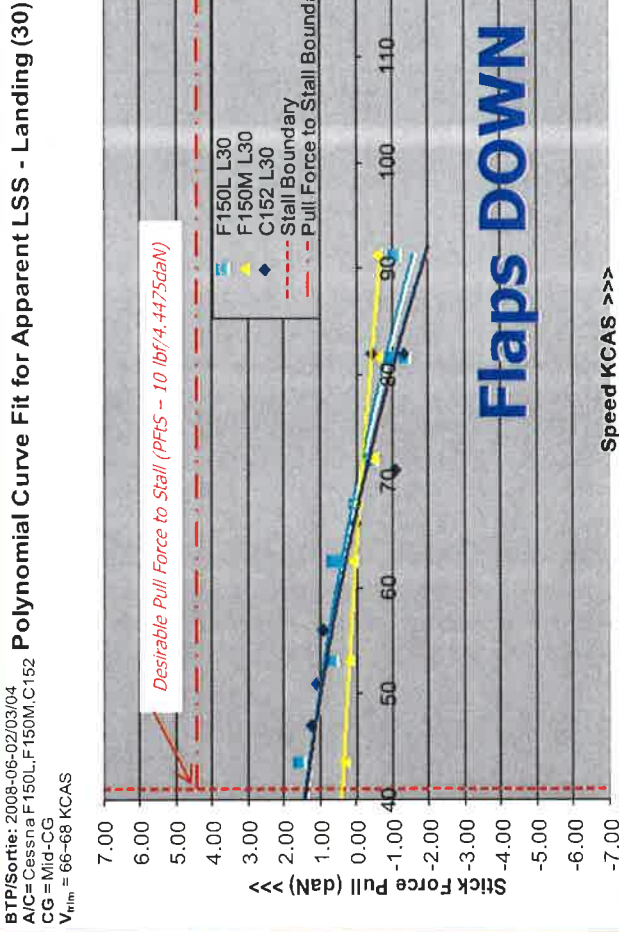




# Stick Force to Change Airspeed Cessna 150L,M & 152 with Flaps a) UP b) DOWN (L30)



BTP/Sortie: 2008-06-02/03/04  
 A/C=Cessna F150L,F150M,C152  
 CG = Mid-CG  
 $V_{min} = 66-68$  KCAS

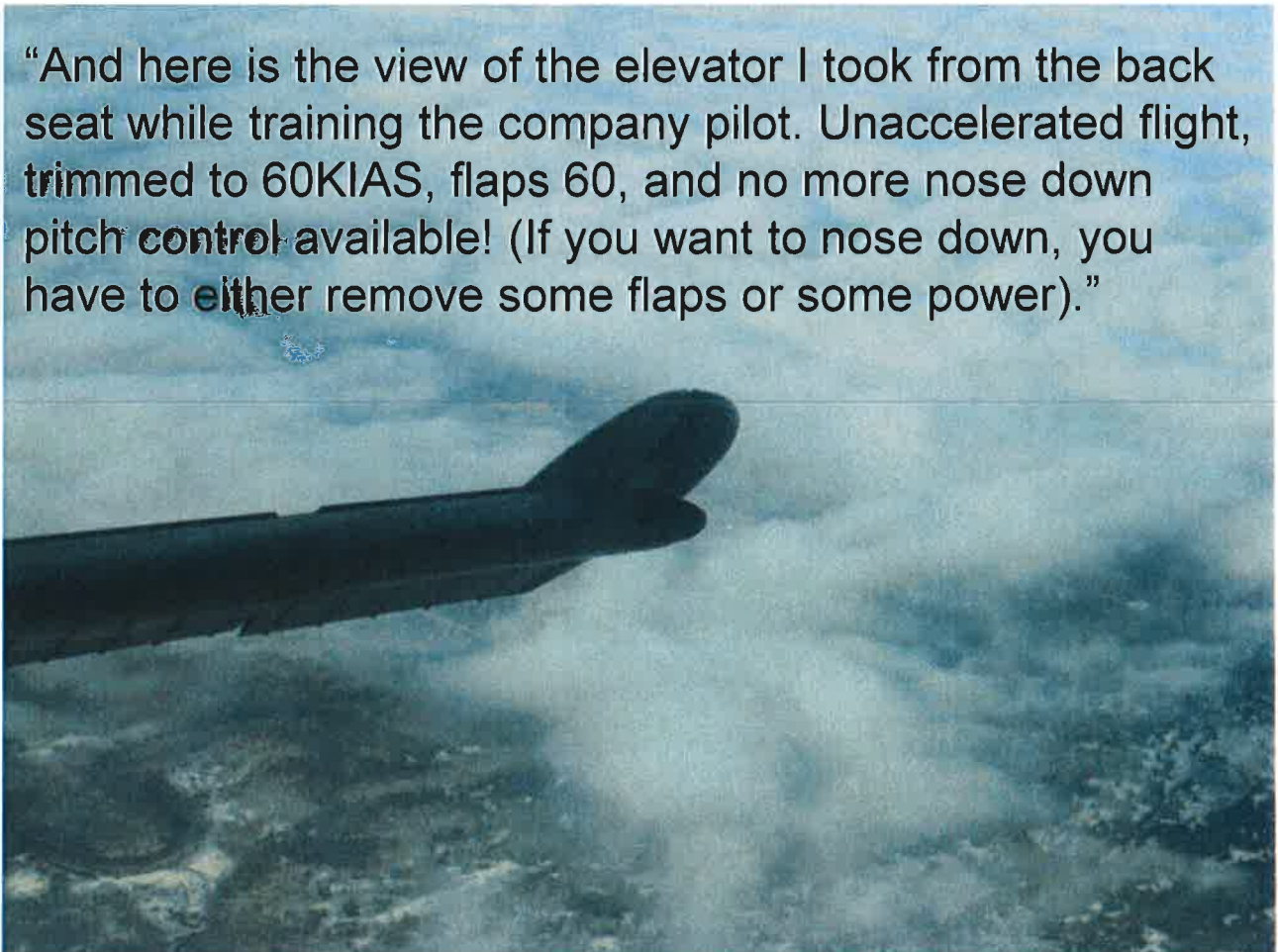


<http://www.pprune.org/flight-testing/484274-minimum-stick-force-gradient.html>



Siai Marchetti 1019

“And here is the view of the elevator I took from the back seat while training the company pilot. Unaccelerated flight, trimmed to 60KIAS, flaps 60, and no more nose down pitch control available! (If you want to nose down, you have to either remove some flaps or some power).”



# Load - CG diagram

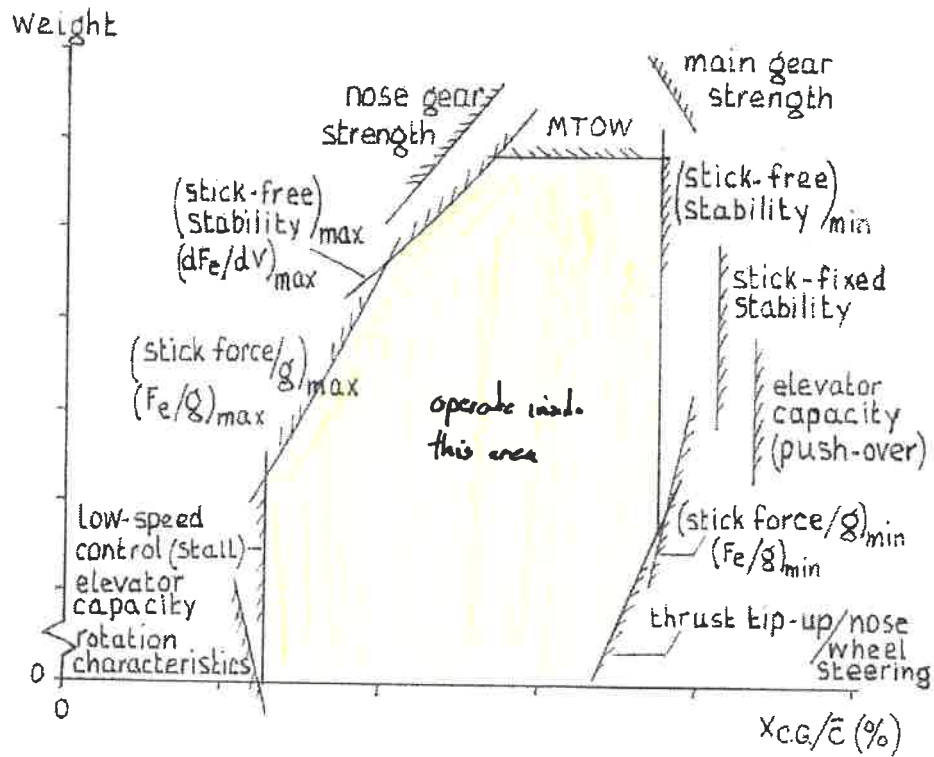
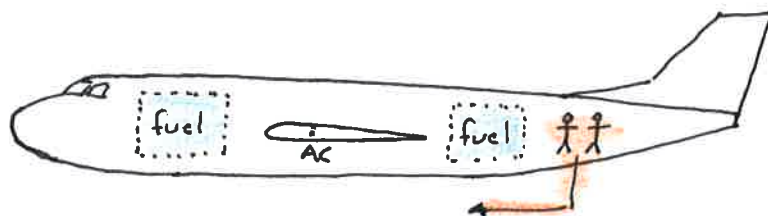
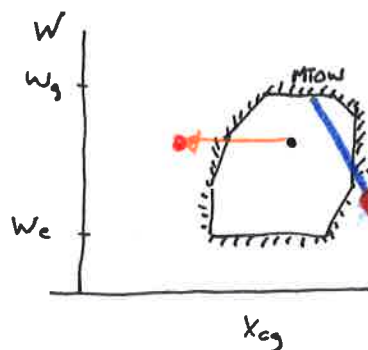


Figure 31.12 - Loading diagrams of some jet transport aircraft: Limits of the loading diagram.

A common failure is to feed from the wrong fuel tank during flight.



Distribution of passengers + baggage



burning fuel from a forward tank moves the cg aft slowly until there is a crisis!



Full-scale wind tunnel (NASA Langley 30'x60')

- Electric motor (266 hp) driving propeller ( $T_c = T/qS$ )

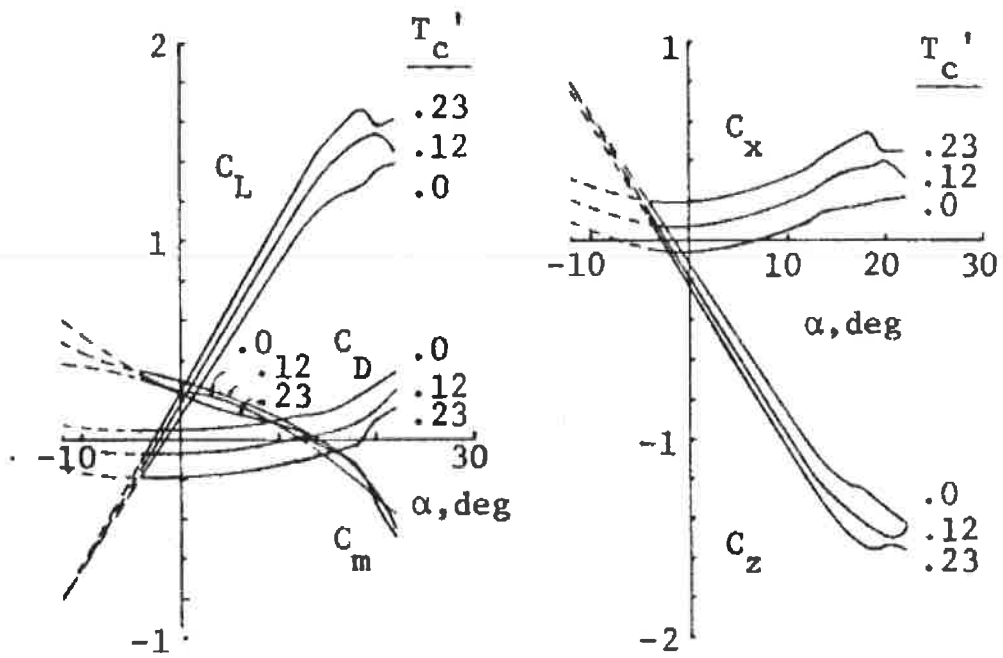
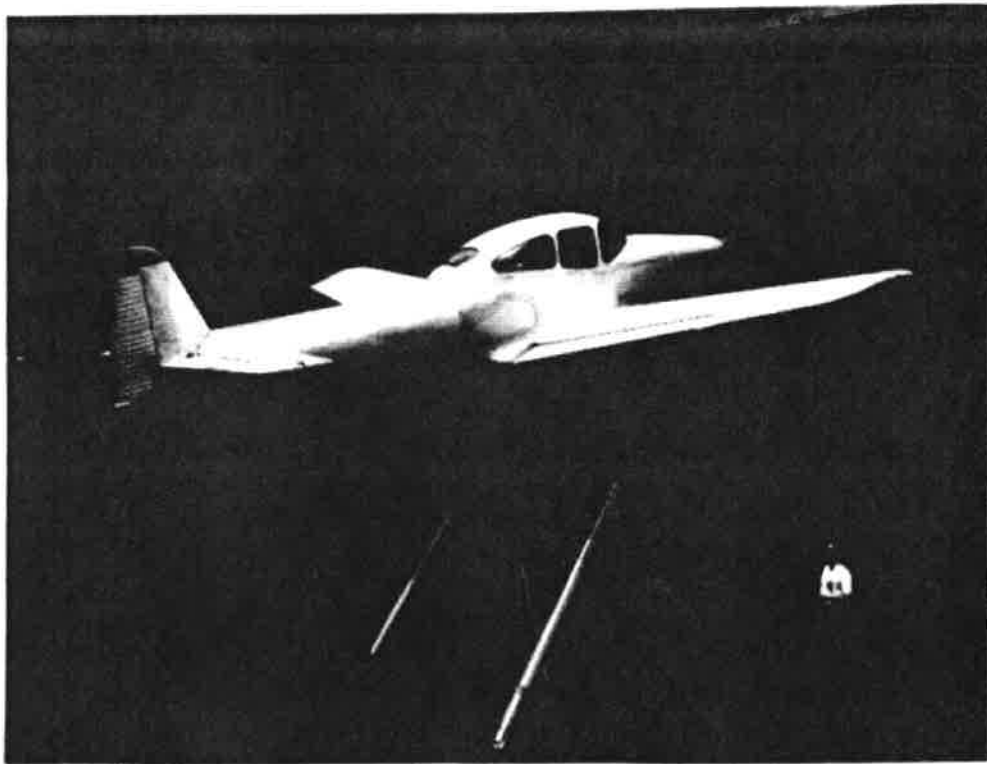


FIGURE 2. Static Longitudinal Coefficients from Full-Scale Wind Tunnel Tests.

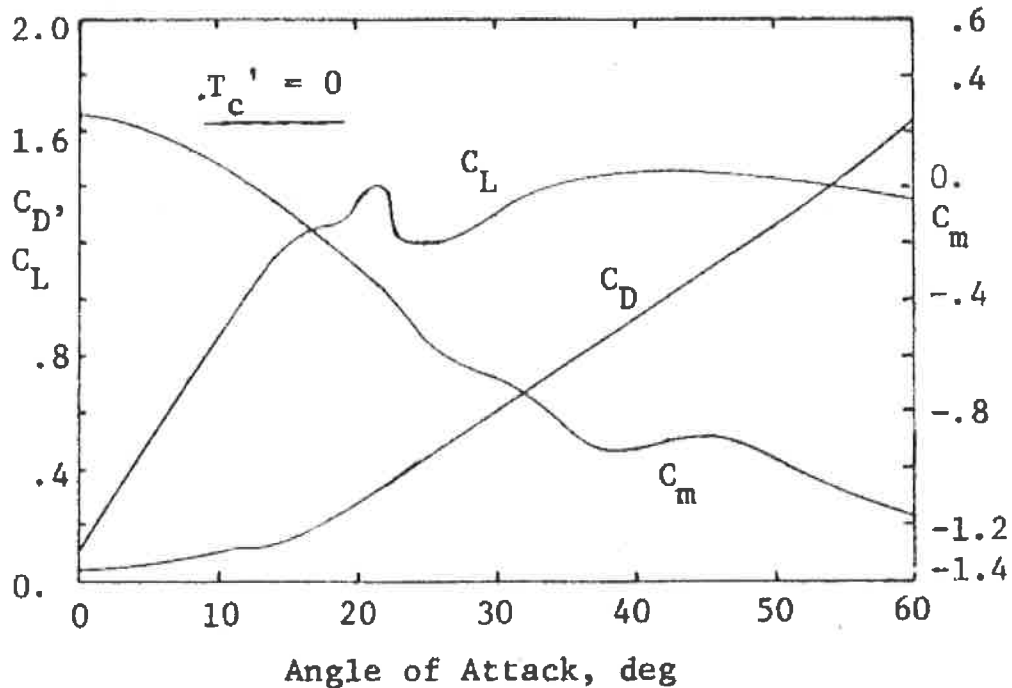


FIGURE 3. Combined Static Longitudinal Model (zero thrust).

The combined zero-thrust static longitudinal coefficients, illustrated in Fig. 3, have three characteristics of note. The wing stalling effects are confined principally to  $C_L$ , which shows a pre-stall flattening at  $\alpha = 18^\circ$  and a stall break at  $\alpha = 22^\circ$ .  $C_D$  is quadratic in  $\alpha$  for low angles, but it is linear in the  $20^\circ$ -to- $50^\circ$  range. The pitching moment slope changes sign for  $\alpha = 38^\circ$  to  $45^\circ$ , implying a region of local instability and the possibility of a superstall equilibrium beyond  $\alpha = 45^\circ$ ; however, neither problem can occur unless the aircraft can trim in the region. With the rotational center at 25% m.a.c. (as shown), the maximum  $\Delta C_m$  that can be produced by the elevator is 0.54, far short of the 0.9 required for trim at high  $\alpha$ . An aft center-of-gravity (c.g.) shift of more than 20% would be required to experience pitch instability or superstall with full negative  $\delta E$ ; this would place the c.g. well behind its certified aft limit.

Stall strongly depends on power setting:

- Case A: full power & up elevator
- Case B: cut power & up elevator

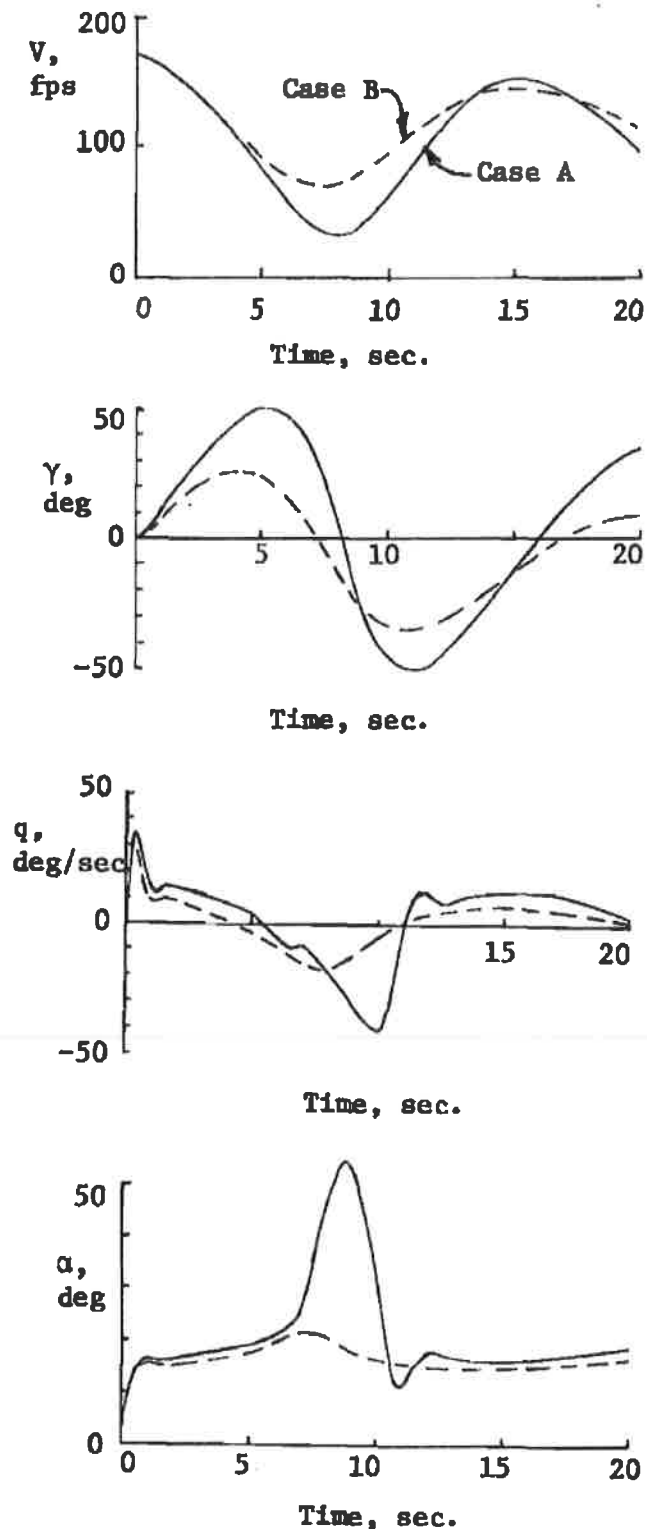
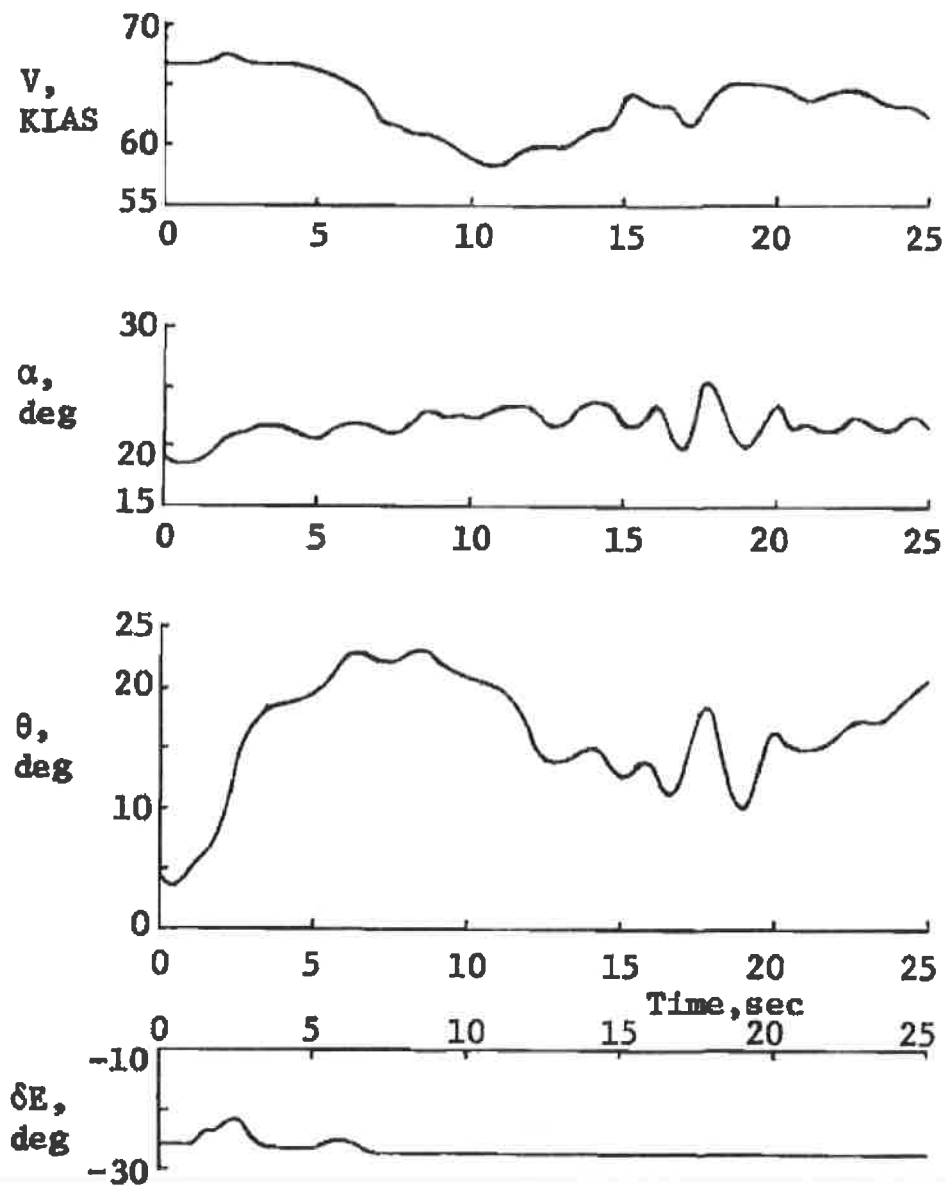


FIGURE 15. Effect of Power on the Abrupt Stall.  
Altitude = 2286 m(7500 ft).



**FIGURE 21. Effect of Adding Power During the Stall.**

INVESTIGATION OF THE STALLING CHARACTERISTICS  
OF A GENERAL AVIATION AIRCRAFT

ICAS-80-22.2

Robert F. Stengel\* and W. Barry Nixon\*\*  
Princeton University  
Flight Research Laboratory  
Department of Mechanical and Aerospace Engineering  
Princeton, New Jersey, U.S.A.



# Boeing 747

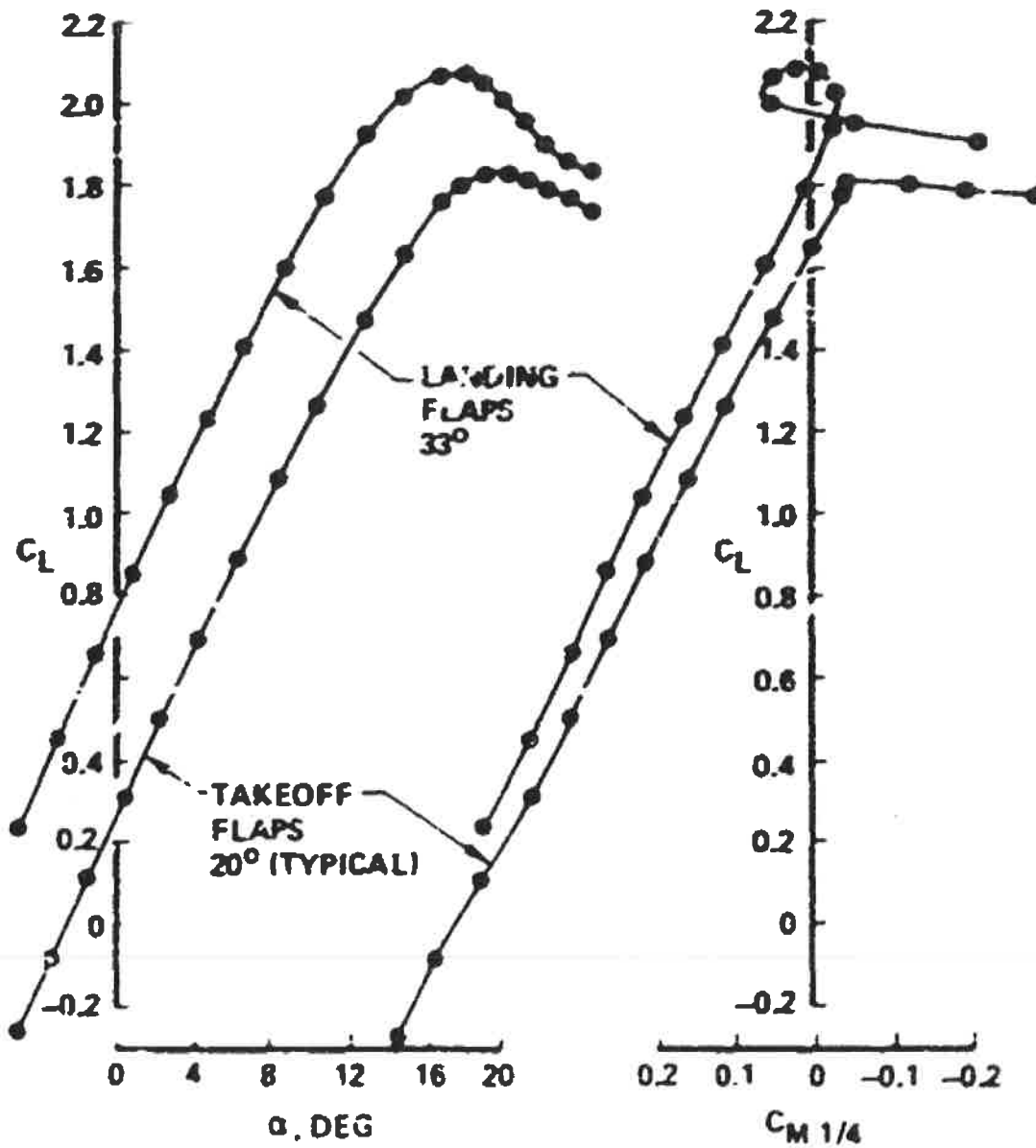


FIGURE 3 HIGH LIFT WIND TUNNEL DATA (LOW REYNOLDS NUMBER)

# Fokker F-28

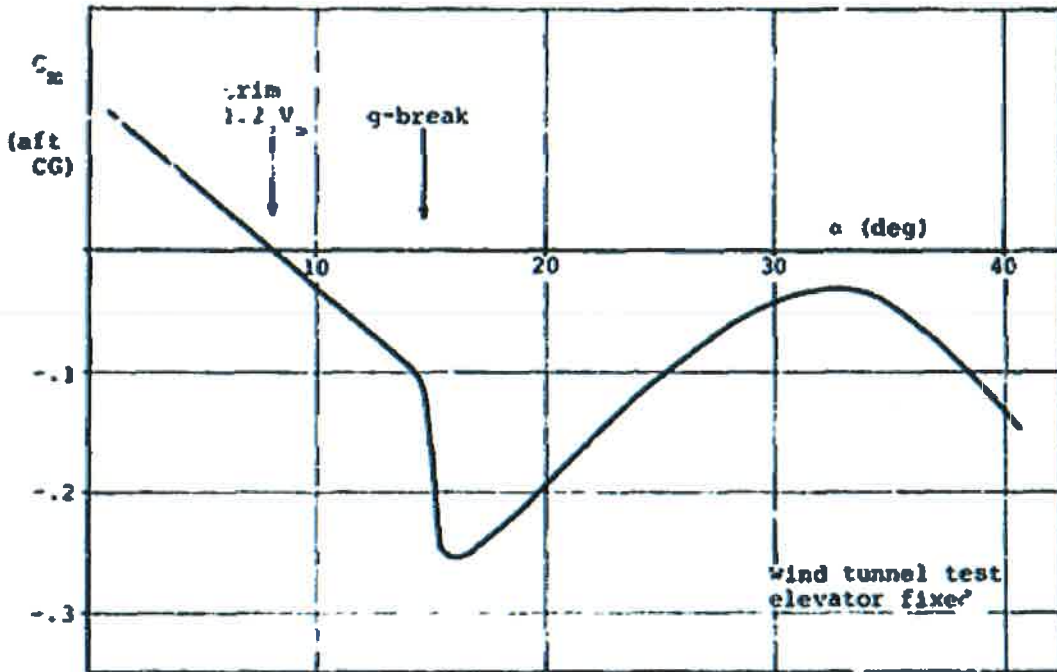


Fig. 3 F28 pitching moment characteristics

If the tailplane is immersed in the wake of the separated flow of the main wing, the aircraft will lose its longitudinal stability. It will remain unstable and pitch-up until a new equilibrium is found at a very high angle-of-attack. This may result in a "locked-in stall" or "deep stall", from which recovery may be extremely difficult. The principle of deep stall will be explained with the aid of figure 26.35.

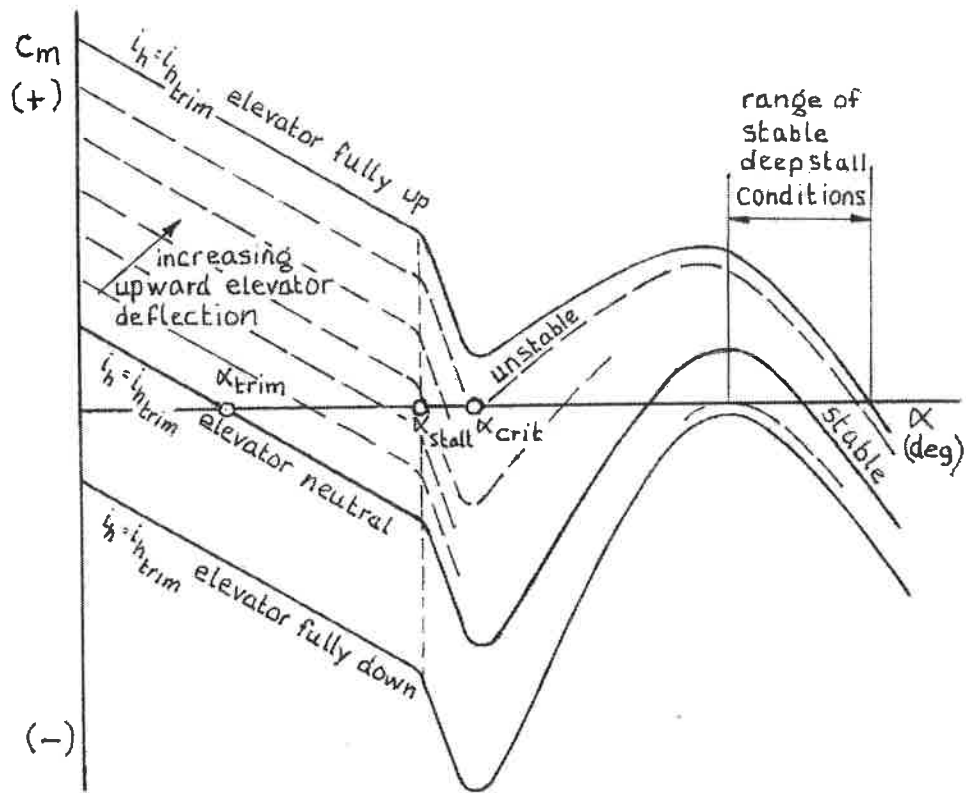


Figure 26.40 - Stability and controllability on an aircraft with a T-tail

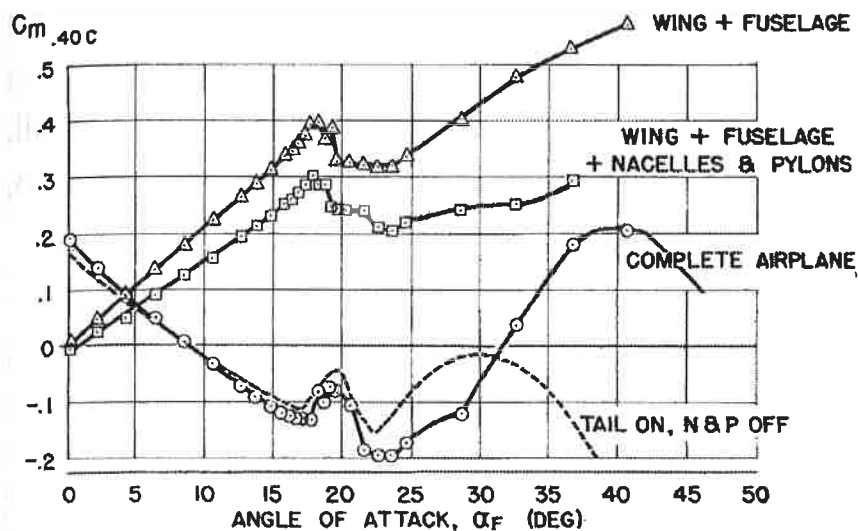


Figure 26.35 - Pitching moment buildup, flaps up, typical model with deep stall problems. Source : AIAA Paper No. 65-738