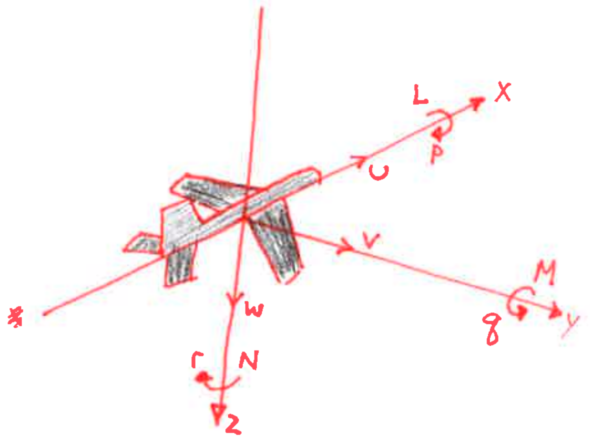
Lesson 18

Directional Stability

Nomenclature / Coordinates (Review)



$$\beta = \mathrm{asin}\left(\frac{V}{|V|}\right)$$

$$= \mathrm{asin}\left(\frac{V}{\sqrt{U^2+V^2+W^2}}\right)$$

$$\approx \frac{V}{U}$$ in radians if $V << U$

Body fixed stability frame:

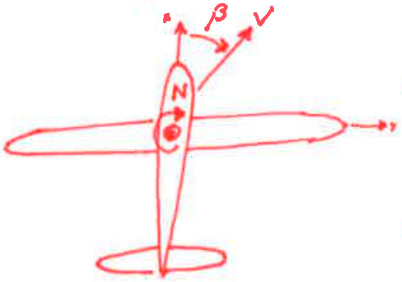
yaw
- Z points down
- N moment in z direction (cw +)
- r angular rate in z direction

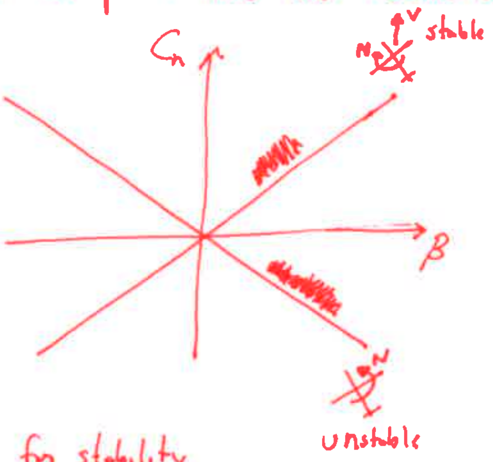
roll
- x points forward
- L moment in x-direction (cw +)
- p angular rate in x direction

pitch
- y point off right starboard wing
- M moment in y direction (nose up +)
- q angular rate

Static directional Stability

The ability of the aircraft to point into the relative wind.



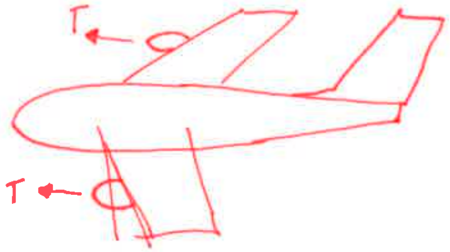


$$C_n = \frac{N}{qSb}$$

↑ notice wing span.

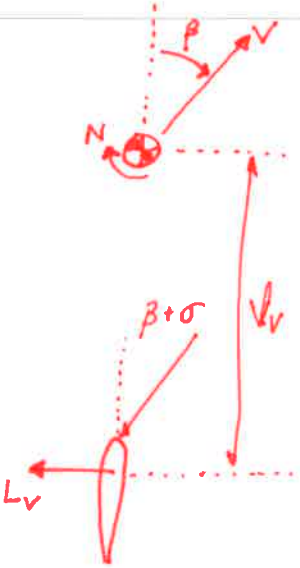
$$C_{n_\beta} \equiv \frac{dC_n}{d\beta} > 0$$ for stability

Major players



- Vertical Stabilizer
- Fuselage
- Offset thrust
- Wing (small to negligible at $\alpha << \alpha_{stall}$)
  exception adverse yaw

Vertical Tail Contribution $C_{n_{tail}}$



$$N = l_v \cdot L_v = l_v \cdot \eta_v \, q \, C_{L_{\alpha_v}} (\beta + \sigma) S_v$$

non dim

$$C_n = \frac{N}{q S_w b} = \eta_v \underbrace{\left(\frac{l_v S_v}{S_w b}\right)}_{\text{Vertical Tail Volume Ratio}} C_{L_{\alpha_v}} (\beta + \sigma)$$

σ is really complicated and ~~often~~ ~~usually~~ maybe? somewhat small. but approximate as $\sigma = \underset{\uparrow}{\sigma_0} + \frac{d\sigma}{d\beta}\beta$

this is probably small unless the aircraft is not symmetric. "Pagin Burt Rutan.... "

$$C_n = \eta_v V_v C_{L_{\alpha_v}} \left(1 + \frac{d\sigma}{d\beta}\right)\beta$$

$$\boxed{C_{n_{v_\beta}} = \eta_v V_v C_{L_{\alpha_v}} \left(1 + \frac{d\sigma}{d\beta}\right)}$$

$\frac{d\sigma}{d\beta}$ is hard to estimate. Use 0 (zero) if you have no other information.

$$\frac{d\sigma}{d\beta} \approx 0$$

Estimate (from Nelson)(DATCOM)

$$\eta_v \left(1 + \frac{d\sigma}{d\beta}\right) \approx 0.724 + 3.06 \frac{S_v/S_w}{1 + \cos \Lambda_{1/4_w}} + 0.4 \frac{Z_w}{d} + 0.009 \, AR_w$$

Z_w is distance from fuselage centerline to wing root quarter chord positive for low-wing aircraft (add to book... typo)

d is the maximum fuselage depth

For a mid mounted unswept wing with $S_v/S \approx 0.1$, $\eta_v \left(1 + \frac{d\sigma}{d\beta}\right) \approx 1.0$

Q: Why? Why does the wing influence the tail?

Why does the wing influence σ and $\frac{d\sigma}{d\beta}$

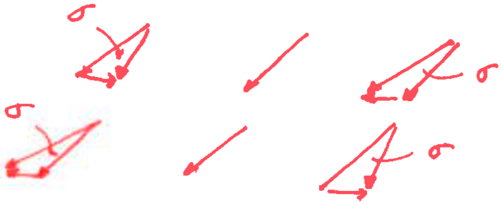
A: Aerodynamics of a wing generating lift

The shed vorticity is

$$\gamma(y) = -\frac{d\Gamma}{dy}$$

So the flow field (cross section) looks like

If we add the free stream to this, there is a sidewash velocity with an angle σ



The wing's vertical location impacts σ

$0.4 \frac{z_w}{d}$ term

- Flow directions σ above the wing turn the velocity vector inwards
- A sideslip angle β places the vertical in a different spanwise location (i.e. increasing β moves the tail into the right (starboard) wing's wake)
- σ increases going outboard above the starboard wing (decreases port above)

Thus $\frac{d\sigma}{d\beta}$ is positive for a lower wing. (the ~~wing~~ vertical "sees" more of the upper wing wake)

Fuselage Contribution

Similar to the pitching Moment resulting from α.

The Multhop method would work!

- Super quick visual method (qualitative only)
  Compare forward fuselage area and centroid to aft fuselage.

≈
Approx equal ⇒ $C_{n_{\beta_f}} \approx 0$

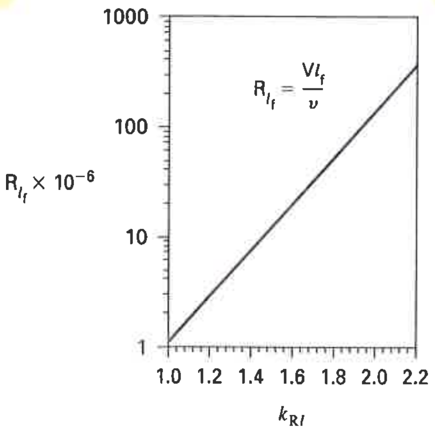
"D. The stability derivative isn't too low, its the wrong sign"
— Mike.

$C_{n_{\beta_f}} > 0$ stable

$C_{n_{\beta_f}} < 0$ unstable

- Nelson's Eq 2.73 and Fig 2.29 and 2.30

$$C_{n_{\beta_{wf}}} = -K_n K_{R_l} \frac{S_{f_s} l_f}{S_w b}$$



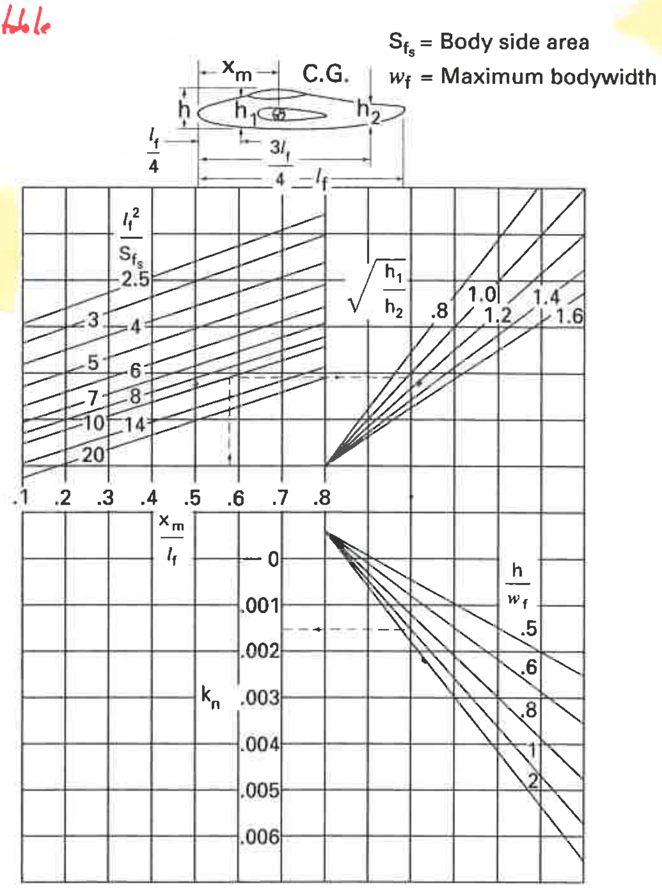
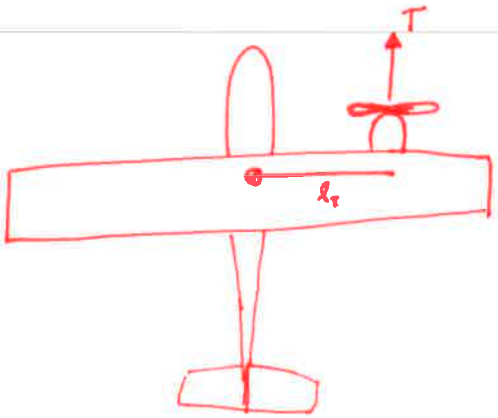


FIGURE 2.29
Wing body interference factor.

Thrust



$$N_T = T \ell_T$$

$$C_{n_T} = \frac{T \ell_T}{q S b}$$

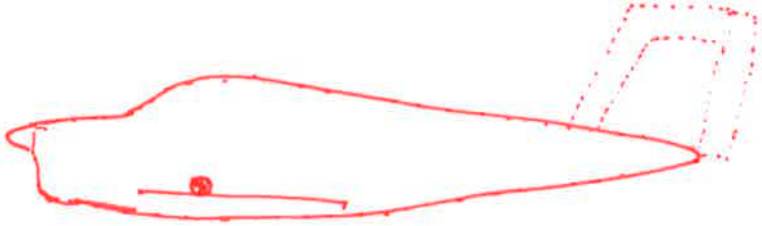
$$= \left(\frac{T}{qA}\right)\left(\frac{A}{S}\right)\left(\frac{\ell_T}{b}\right)$$

= Thrust Coefficient · $\frac{\text{Thrust}}{\text{Wing}}$ Area · Ratio of offset to wing
Disk loading per dyn. pressure.

Worst case (largest C_{n_T})

- High Thrust
- Low speed (low dynamic pressure)
- Large offset (ℓ_T)

Ex: What vertical stab size is necessary for $C_{n\beta} = 0.2$ $^1/_{rad}$ for the following aircraft



Assume.

Vertical $AR \approx 4$

$b = 33.5^{ft}$

$l_f = 27.5^{ft}$

$S_w = 177\ ft^2$

$V_\infty = 300\ ^{ft}/_s$

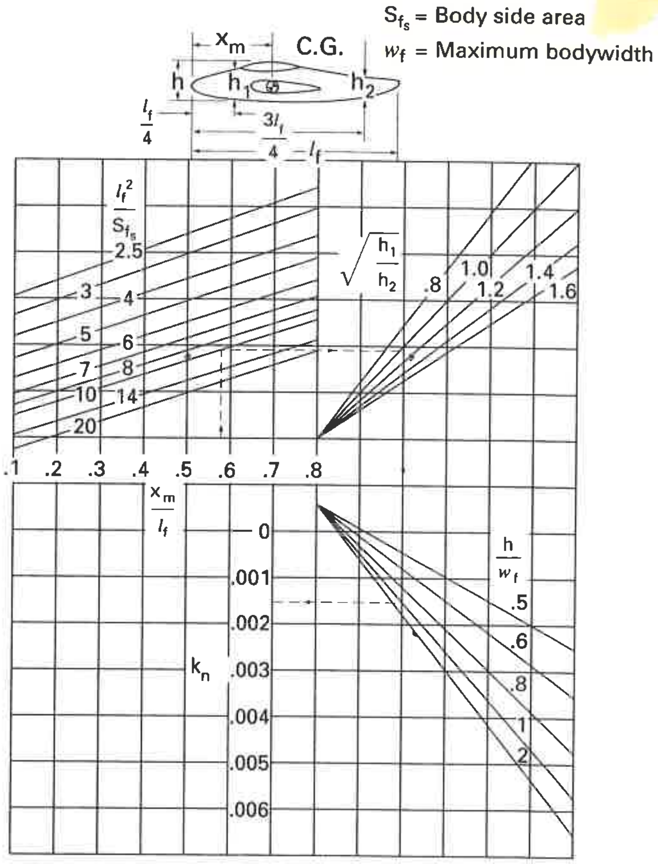
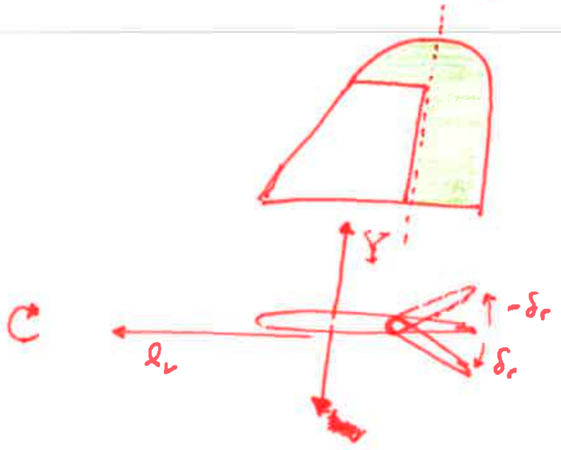
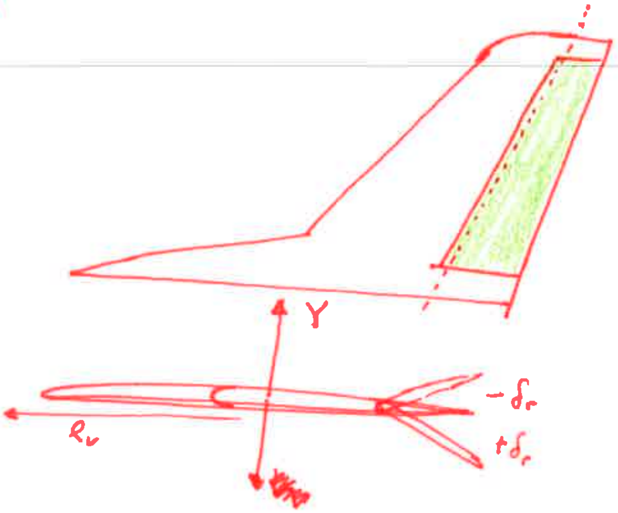


FIGURE 2.29
Wing body interference factor.

Directional Control (Rudder)





$N = -\ell_v Y$ where Y is the sideforce

$$Y = q_v S_v C_{L_v}$$

$$C_n = \frac{N}{q_w S b} = -\frac{q_v}{q_w}\frac{\ell_v S_v}{S_w b}\cancel{C_{L_v}}\frac{dC_{L_v}}{d\delta_r}\delta_r$$

$$= -\eta_v V_v C_{L_{v_{\delta_r}}}\delta_r = -\eta_v V_v \underbrace{\frac{dC_{L_v}}{d\delta_r}}_{\frac{dC_{L_v}}{d\alpha}\frac{d\alpha}{d\delta_r} = C_{L_{\alpha_v}}\tau}\delta_r$$

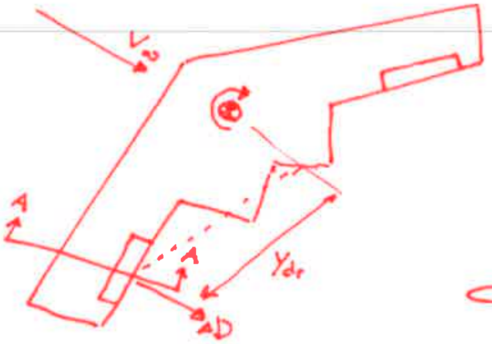
$$\boxed{C_{n_{\delta_r}} = -\eta_v V_v C_{L_{\alpha_v}}\tau}$$

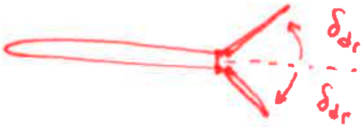
where τ is the flap effectiveness parameter Fig 2.21 in Nelson

Why do we need directional control?

- Adverse Yaw
- Cross wind landing
- Asymmetrical Power/Thrust
- Spin Recovery
- Aerobatics + Fun Maneuvers

Drag Rudder





$$N_{Left} = - y_{dr} \Delta D \qquad \text{with} \quad \Delta D = \frac{dC_D}{d\delta_{dr}} \delta_{dr} q S$$

$$N_{Right} = y_{dr} \Delta D$$

From Roskam,

$$\frac{dC_D}{d\delta_{dr}} \approx 0.025 \frac{b_{dr} c_{dr}}{S} \left[\frac{1}{deg}\right]$$

Approximation only

$$C_{n_{Right}} = \frac{N_{Right}}{qSb} = \frac{y_{dr} \delta_{dr} q S \; 0.025 \; b_{dr} c_{dr}}{S} \cdot \frac{1}{qSb} \delta_{dr}$$

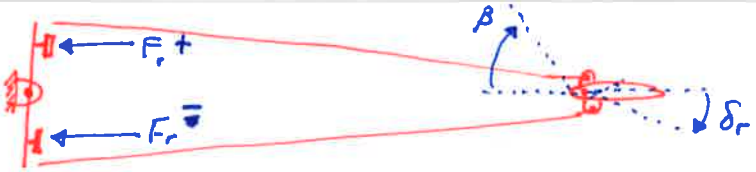
$$= \frac{0.025 \; b_{dr} c_{dr}}{S b} y_{dr} \delta_{dr}$$

$$\boxed{C_{n_{\delta_{r_{right}}}} = \frac{0.025 \; b_{dr} c_{dr}}{S b} y_{dr}} \quad \Rightarrow \quad C_{n_{\delta_{r_{right}}}} = 0.025 \left(\frac{S_{dr}}{S}\right) \left(\frac{y_{dr}}{b}\right)$$

This control method is common on flying wings (B2, YB-49, etc)

Also remember that certain lift distributions are naturally yaw stable (proverse yaw) for aileron deflections. See: Aero-1 / AEM 614 "Bell lift dist"

Rudder Pedal Forces



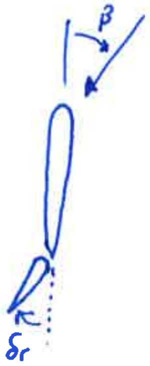
$$F_r = \underset{\substack{\uparrow\\ \text{Gearing}\\ [\text{rad/ft}]}}{G_r}\ \underset{\substack{\uparrow\\ \frac{q_r}{q_\infty}}}{\eta_r}\ \underset{\substack{\uparrow\\ q_\infty}}{q}\ \underbrace{S_r \bar{c}_r}_{\text{rudder geometry}}\ \underbrace{C_{h_r}}_{\substack{\text{rudder}\\ \text{hinge}\\ \text{moment}}} = G_r \eta_r q S_r \bar{c}_r \left(\underbrace{C_{h_{\delta_r}} \delta_r}_{\substack{\text{rudder}\\ \text{deflection}\\ \text{moment}}} + \underbrace{C_{h_{\delta_{r_t}}} \delta_{r_t}}_{\substack{\text{rudder-tab}\\ \text{deflection}\\ \text{moment}}} + \underbrace{C_{h_{\beta_v}} \beta_v}_{\substack{\text{sideslip}\\ \text{moment}}} \right)$$

Free Floating Rudder

$$F_r = 0 \Rightarrow C_{h_{\delta_r}} \delta_r + C_{h_{\delta_{r_t}}} \delta_{r_t} + C_{h_\beta} \beta_v = 0$$

solve for δ_r

$$\delta_r = \frac{-C_{h_\beta} \beta_v - C_{h_{\delta_{r_t}}} \delta_{r_t}}{C_{h_{\delta_r}}}$$



δ_r can be trimmed with δ_{r_t} (or even given an aero boost)

$$\delta_r = -\frac{C_{h_\beta} \beta_v}{C_{h_{\delta_r}}} = -\frac{C_{h_\beta}}{C_{n_{\delta_r}}} \beta_v = -\frac{(+)}{(-)} = +$$

Remember that the yaw moment about the CG was (from a rudder)

$$C_{n_{\delta_r}} = -\eta_v V_v C_{L_{\alpha_v}} \tau \leftarrow \text{flap eff' parameter}$$

So the total C_n is

$$C_n = \cancelto{0}{C_{n_0}} + C_{n_\beta} \beta + C_{n_{\delta_r}} \delta_r$$

but $\delta_{r_{\text{float}}}$ = function of β

$$C_n = \underbrace{\eta V_v C_{L_{\alpha_v}} \left(1 + \frac{d\sigma}{d\beta}\right)}_{C_{n_\beta}} \beta + \underbrace{-\eta_v V_v C_{L_{\alpha_v}} \tau}_{C_{n_{\delta_r}}} \underbrace{\delta_r}$$

$$\delta_{r_{flt}} = -\frac{C_{n_\beta}}{C_{n_{\delta_r}}} \beta$$

$$= \eta V_v C_{L_{\alpha_v}} \left(\left(1 + \frac{d\sigma}{d\beta}\right)\beta + \cancel{-} \tau \left(-\frac{C_{n_\beta}}{C_{n_{\delta_r}}}\right)\beta \right)$$

$$C_n = \eta V_v C_{L_{\alpha_v}} \left(1 + \frac{d\sigma}{d\beta} + \tau \frac{C_{n_\beta}}{C_{n_{\delta_r}}}\right)\beta$$

Thus the free floating rudder directional stability (from the vert + rudder) is.

$$C_{n_\beta} = \underbrace{\eta V_v C_{L_{\alpha_v}}}_{+} \left(\underbrace{1}_{+} + \underbrace{\frac{d\sigma}{d\beta}}_{small} + \underbrace{\tau \frac{C_{n_\beta}}{C_{n_{\delta_r}}}}_{\underbrace{+ \ \frac{(+)}{(-)}}_{-}} \right)$$

The free floating rudder is <u>destabilizing</u>.

Generate Sideslip

$$C_n = 0 \quad \text{at} \quad \beta \neq 0 \quad \Rightarrow \quad C_{n_\beta}\beta + C_{n_{\delta_r}}\delta_r = 0$$

$$\Downarrow$$

$$\delta_r = -\frac{C_{n_\beta}}{C_{n_{\delta_r}}}\beta$$

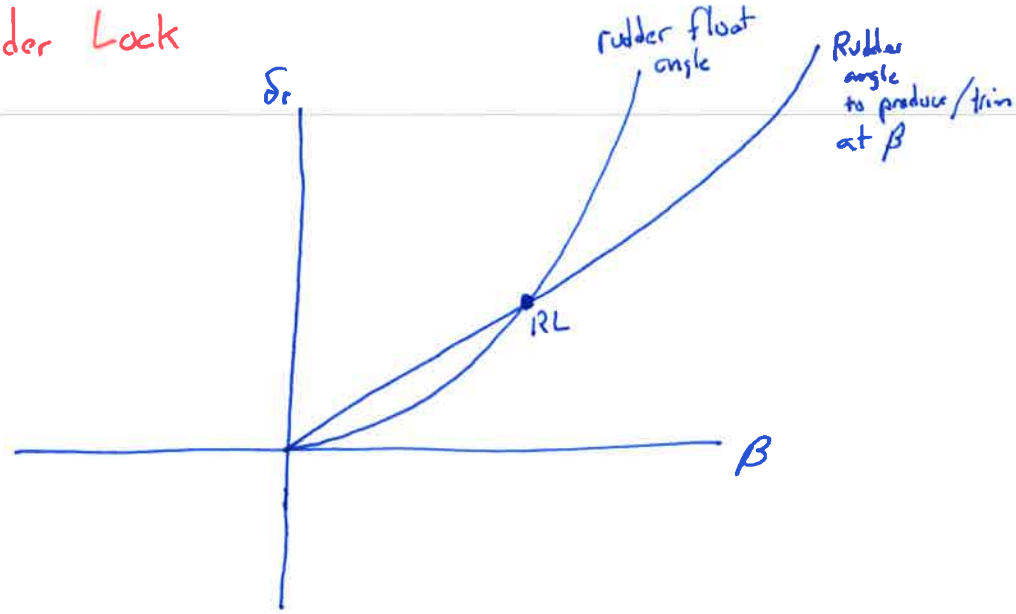
$$F = G_r \eta_r q S_\beta \bar{c}_r \left(C_{n_{\delta_r}}\delta_r + C_{n_{\beta_v}}\beta\right)$$

Pedal Gradients

$$\frac{dF}{d\beta} = \underbrace{G_r \eta q S_r \bar{c}_r}_{+}\left(\underbrace{\underbrace{\underbrace{-\frac{C_{n_{\delta_r}} C_{n_\beta}}{C_{n_{\delta_r}}}}_{-\frac{(-)(+)}{(-)}}}_{-} + \underbrace{C_{n_\beta}}_{+}}_{\text{This can be + or -}}\right)$$

Unfortunately, the hinge moments are usually <u>not</u> linear except in a very small range.

Rudder Lock



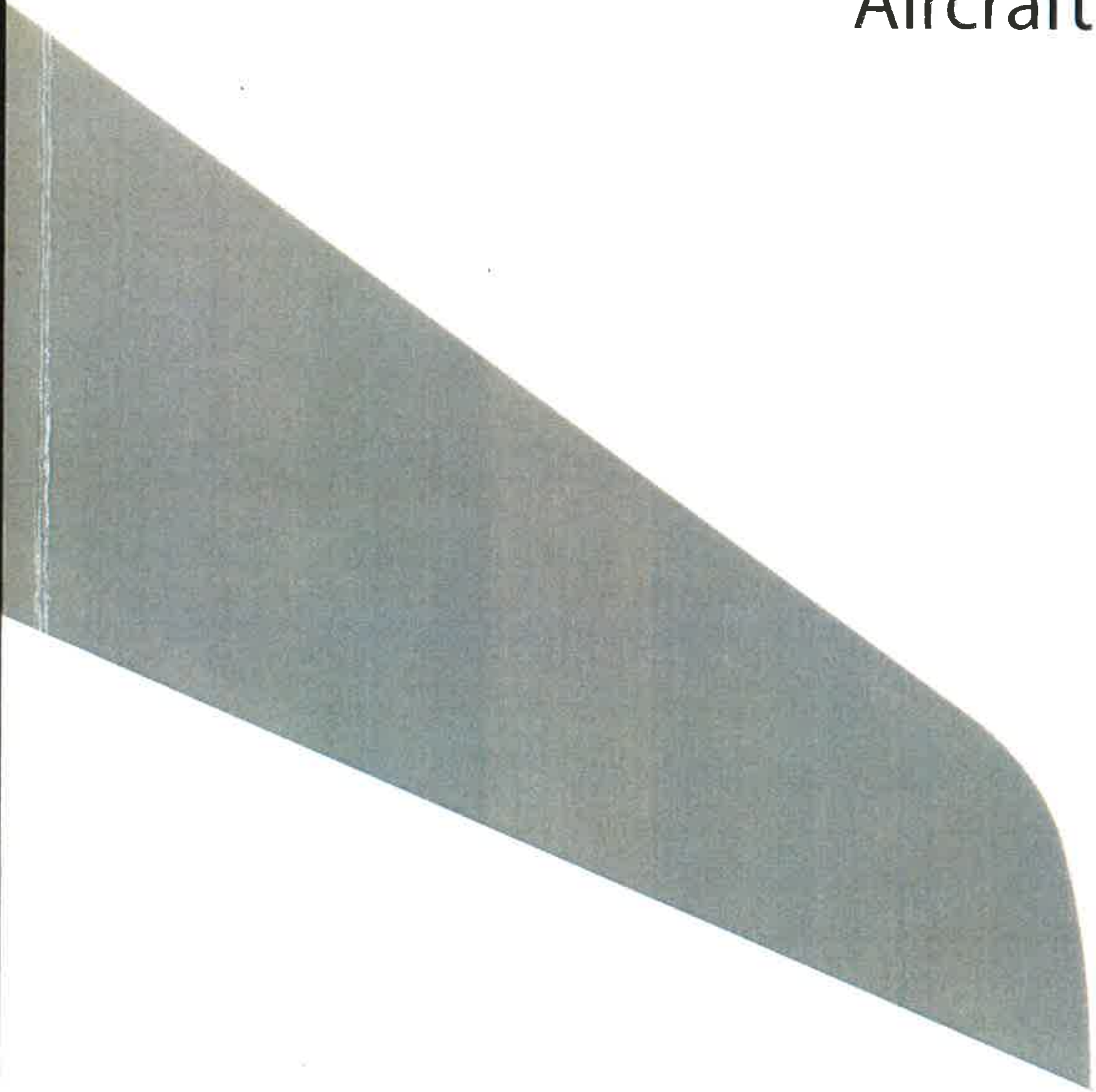
Beyond the point RL, the rudder float angle is more than the rudder angle to trim at that β.

$\Downarrow$

The rudder wants to move <u>all</u> the way to the stops

Dangerous.

Aerodynamic Design of Transport Aircraft



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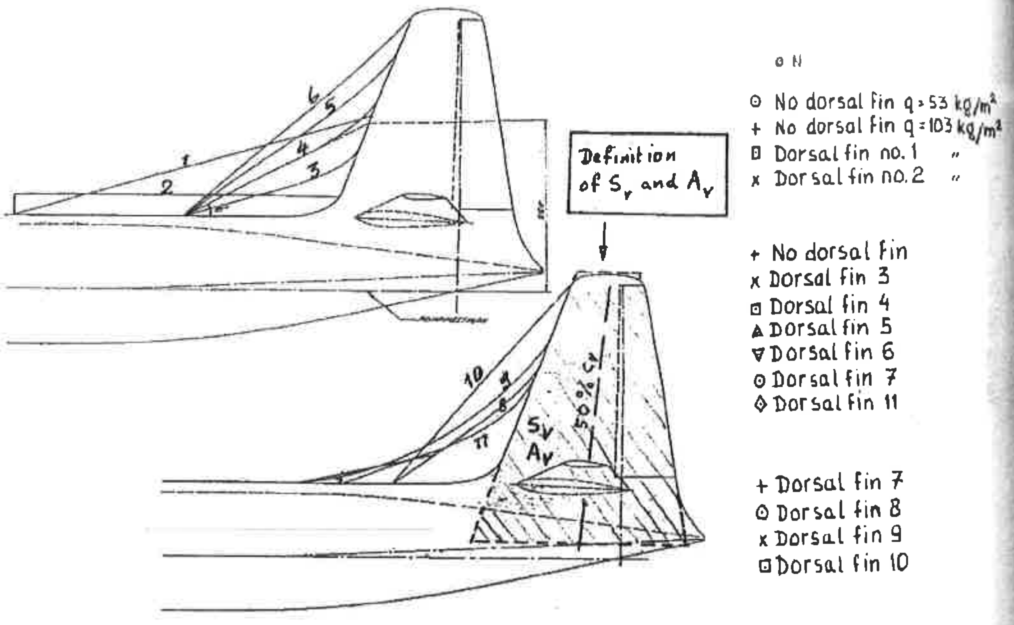


Figure 32.6 - Different dorsal fins. Source: NLL Report A-1374

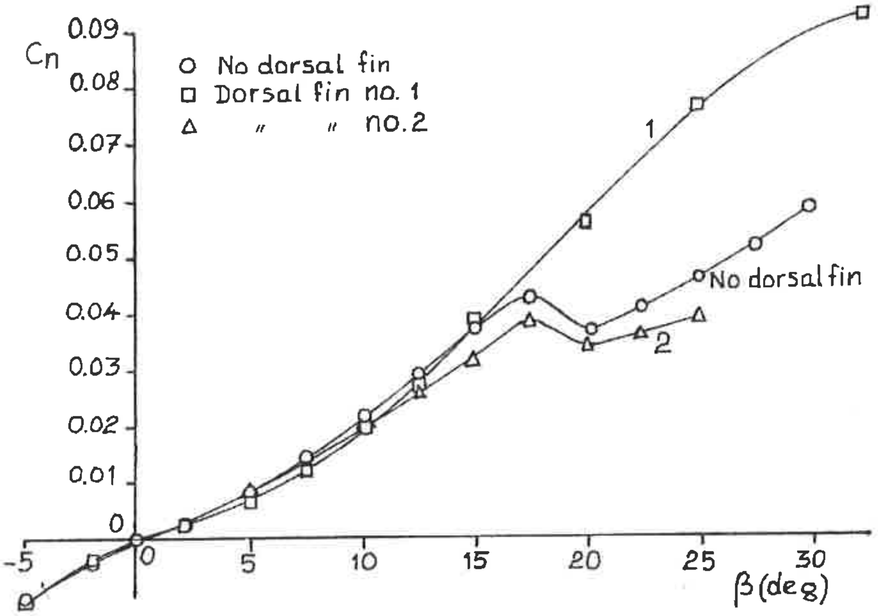


Figure 32.7 - Effect of a dorsal fin on the yawing moment coefficient (1). Source: NLL Report A-1374

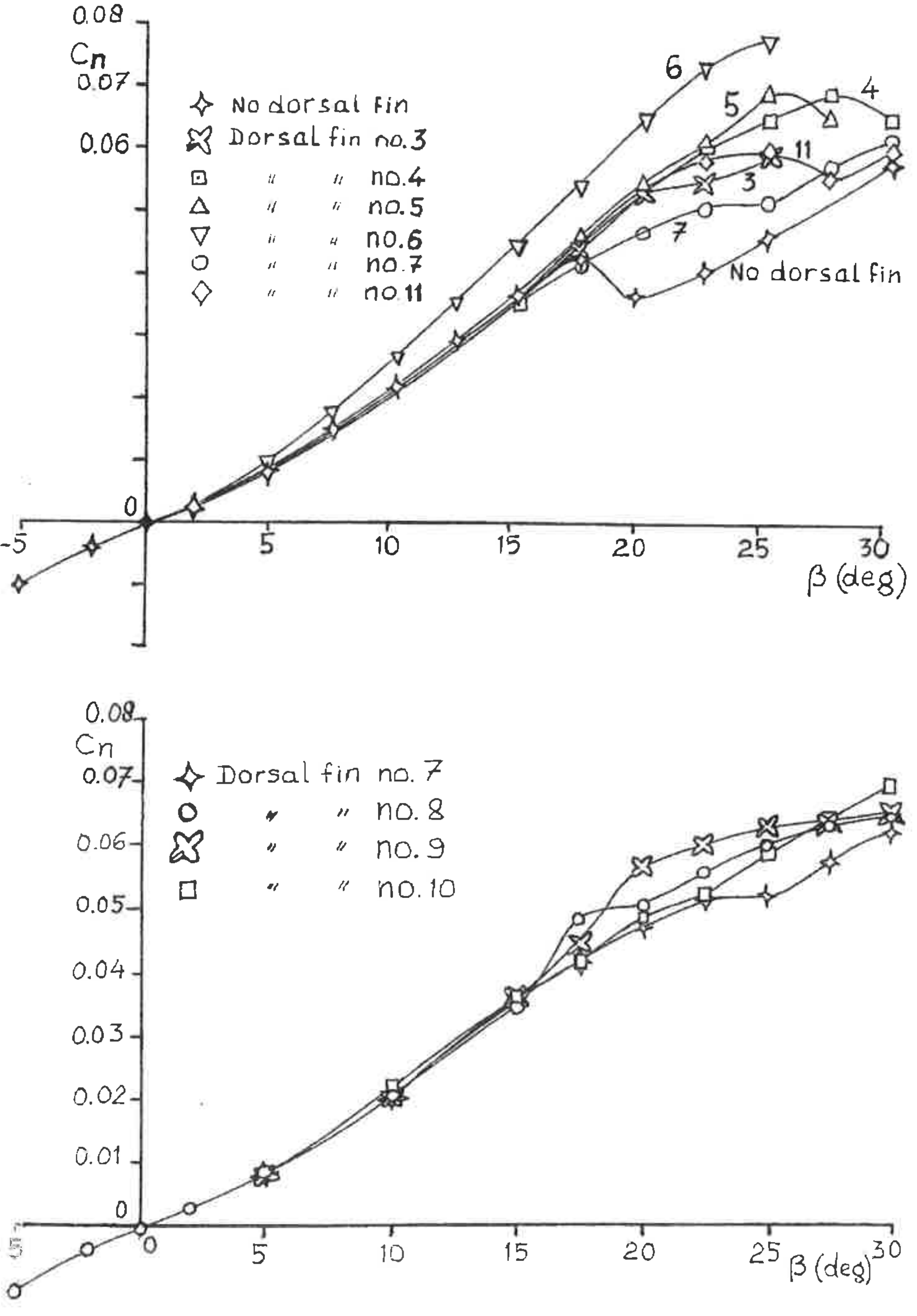
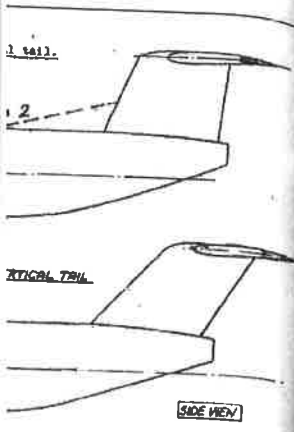


Figure 32.8 - Effect of a dorsal fin on the yawing moment coefficient (2). Source: NLL Report A-1374



ıl fin investigated during

ɪn with a dorsal fin also a
may produce favourable
ed by fin no.6 in figure
hich show test results of
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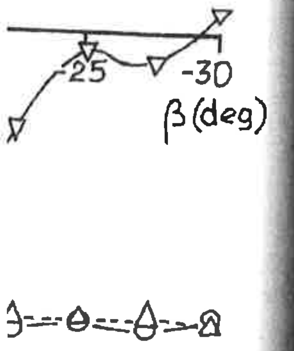
yawing moment curve.



NLR Report A-1582

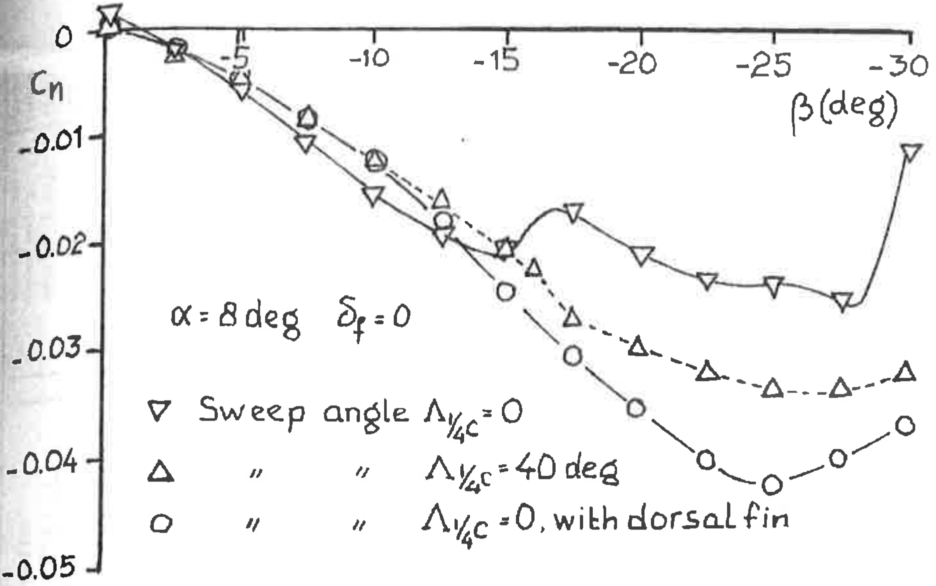


Figure 32.11 - Effect of sweep angle on vertical tailplane lift curve. Source: NLR Report A-1582

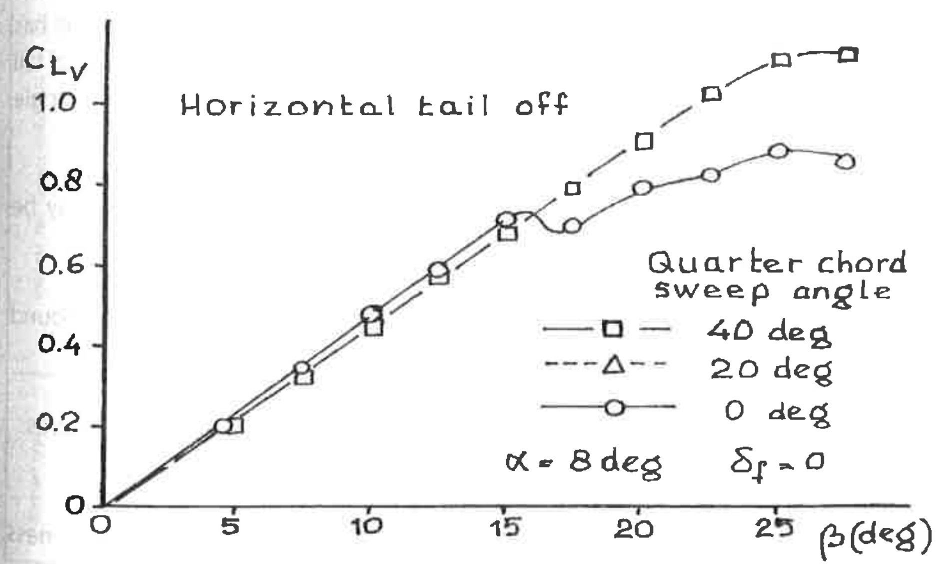


Figure 32.12 - Effect of the sweep angle and of the horizontal tail plane on the lift (side force) of the vertical tailplane in sideslip. Source NLR Report A-1582

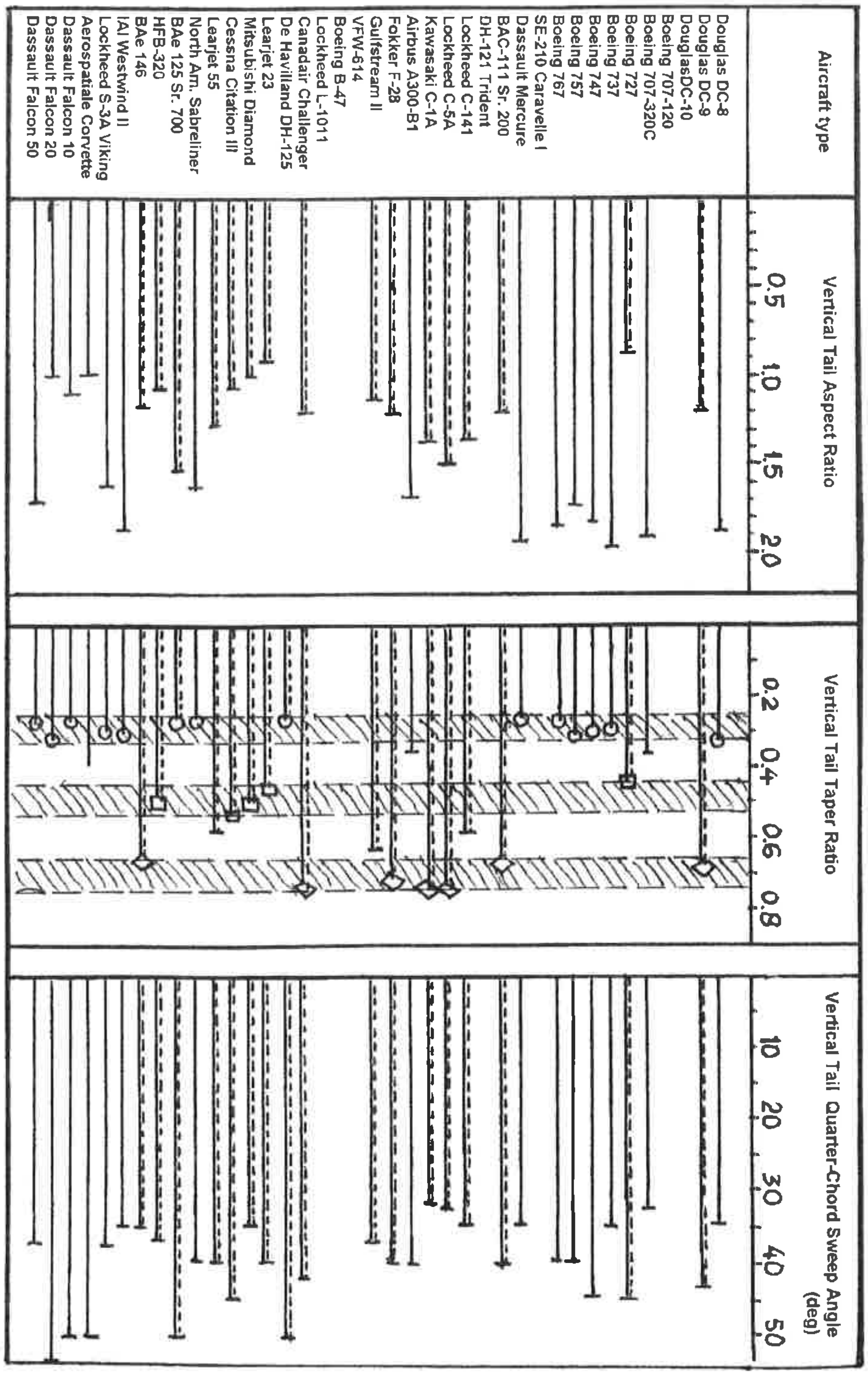
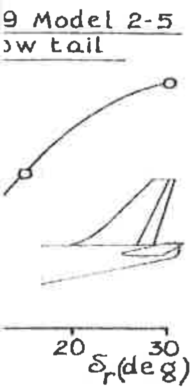


Figure 32.15 - Vertical tailplane data of jet aircraft



rudder deflection – (II)

ed structural element.To
e. The resulting increase
odynamic effectiveness
e has to be made here.
ve at Re = 10^6 than at
n of the rudder brackets
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hinge line and different
nic balances. Two types
e oldest type) and the

ity from geared tabs to

The newest generation of large transport aircraft has irreversible control systems and no aerodynamic balance on the control surfaces (except the Boeing 737 family) to minimise drag. However for small propeller aircraft, business jets and the new class of Very Light Jets (VLJ's) manual control systems with aerodynamic balance on the control surfaces will still remain attractive.

The type of aerodynamic balance on control surfaces has a large effect on the linearity of the hinge moment coefficient vs. angle-of-attack and vs. control surface deflection.

$$C_h = C_{h_0} + \frac{\partial C_h}{\partial \alpha}\alpha + \frac{\partial C_h}{\partial \delta}\delta \qquad \text{(seldom applies at large angles)}$$

Linearity of the hinge-moment relations is particularly important for rudders because the rudder may be deflected to its maximum angle both to the weather side (during flight with a failed engine) and to the lee side (during side slips and cross-wing take-offs and landings). Non-linearities in hinge moment coefficients may cause large variations in control forces.

Large overhang balances also limit the maximum control surface deflections. If the leading edge of the balance nose protrudes outside the section contour overbalance and control lock may occur.

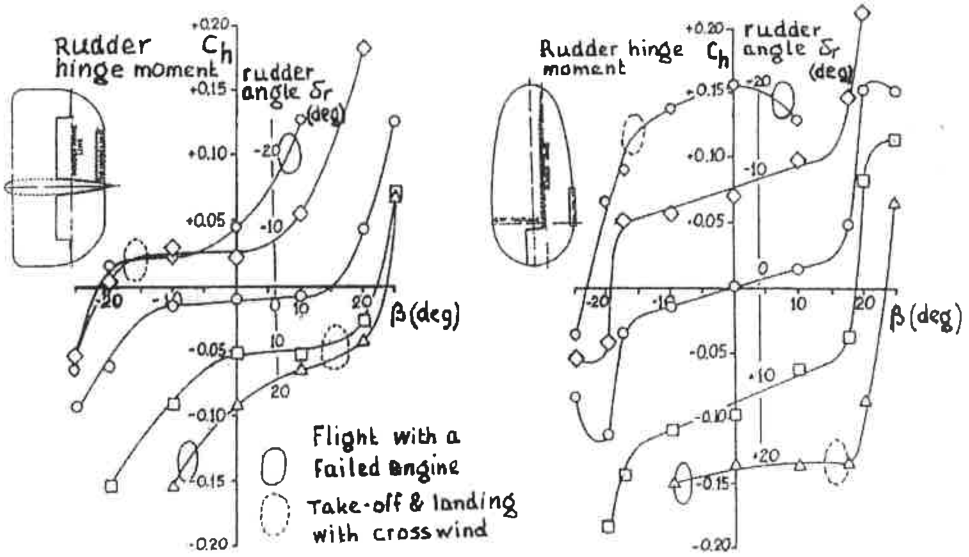

Figure 33.3 - The effect of two types of rudder balancing over the range of rudder hinge moments. Left: Handley-Page Halifax, Right: Avro Lancaster. Source: ARC R&M 2479

nt coefficients are
the Handley Page

flight with a failed
ınding (the dotted

Source: Ian Nightingale

r leading to rudder
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is shown in figure
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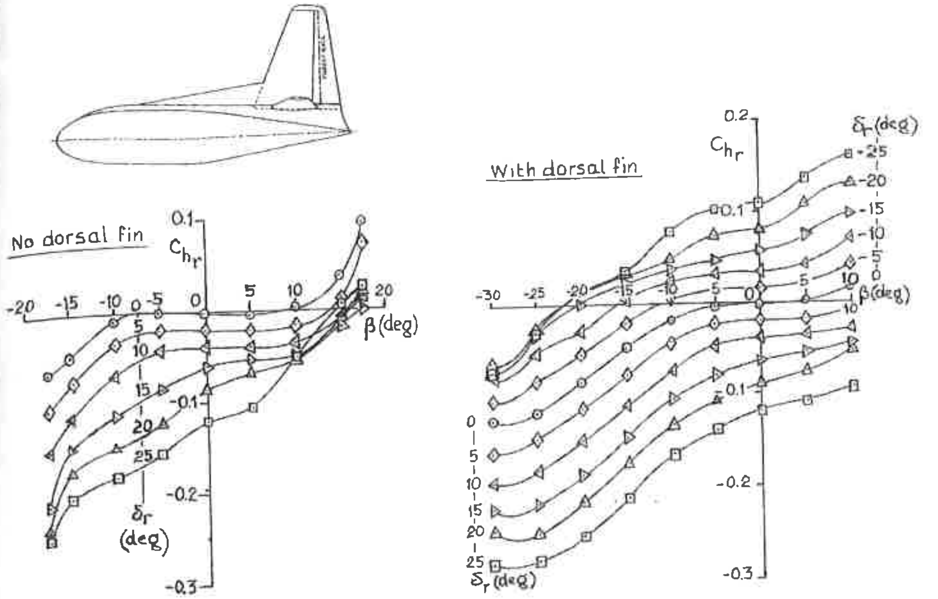

Figure 33.5 - Effect of a dorsal fin on the rudder hinge moment - Fokker F-27. Source: NLL Report A - 1394

Some significant characteristics of elevators were already discussed in chapter 31. Also on horizontal tail surfaces large variations occur in angle-of-attack and control deflection. In particular when some ice accretion is taken into consideration this may heavily influence the choice of primary design parameters for the horizontal tailplane and may also effect the elevator control force characteristics.

Contrary to elevator and rudder, which are attitude controls, ailerons are primarily rate controls. Therefore their effect increases linearly with speed unless compressibility effects or aeroelastic deformation causes a decrease in achievable roll rates. This is illustrated in figures 33.6 and 33.7 where peak roll rates for aileron-alone roll manoeuvres are presented for the Fokker F-28 Mk 1000.

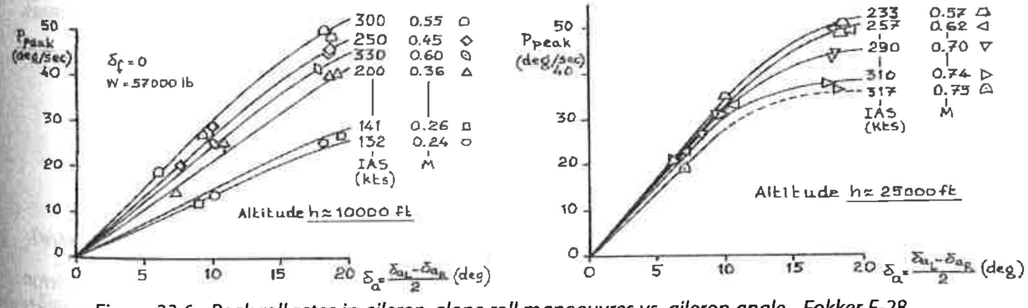

Figure 33.6 - Peak roll rates in aileron-alone roll manoeuvres vs. aileron angle - Fokker F-28. Source: Fokker Report V - 28 – 75.