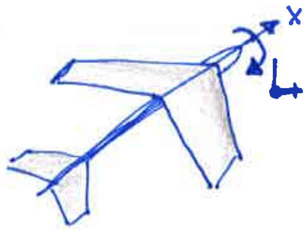
Lesson 19

Lateral S+C
"roll"

Roll Moment



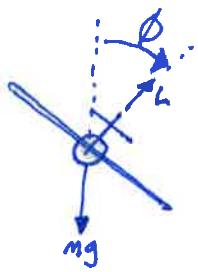
$$C_{L} = \frac{L}{qSb}$$

Warning

L is a moment in this context

L is a lift force in other contexts.

Is there static roll stability?

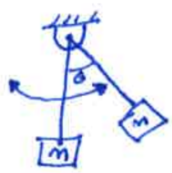


In other words, does ϕ decrease to zero?

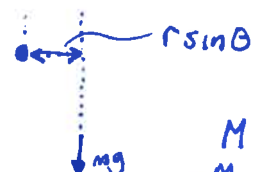
No. Roll angle is statically neutral.

Q: Is there a pendulum effect?

No.

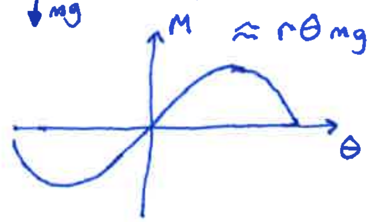


About the rotation point (of a pendulum),

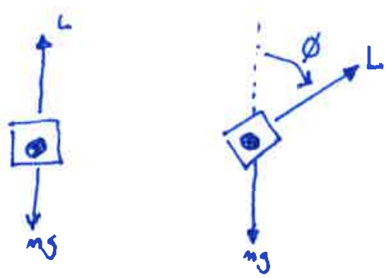


$M = r\sin\theta\, mg$

$\approx r\theta\, mg$ when $\theta \ll 1$

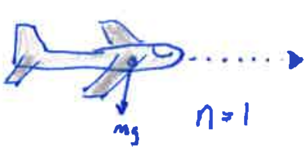


Q: Why is the aircraft different?

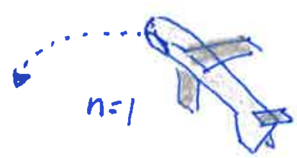


In flight, the roll angle does not impact the lift.

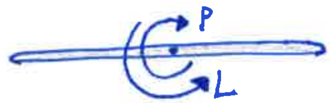
The aircraft (and the pilot) generate aerodynamics force regardless of the direction of Earth. In fact, in flight you can't know down or up by the local body frame forces



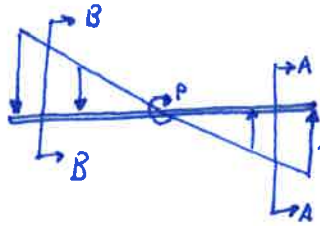
vs



Roll Rate Damping (Nelson 3.6.4) This is a dynamic term.....

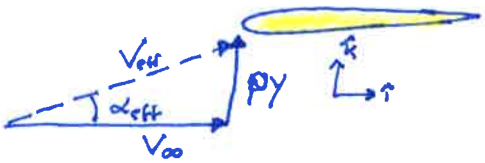


A right rolling wing generates a left (−) roll moment.



~ relative wind due to roll rate p. $V = p \cdot y$

At section A-A:



$$\bar{V}_{eff} = V_\infty \hat{i} + py\,\hat{k}$$

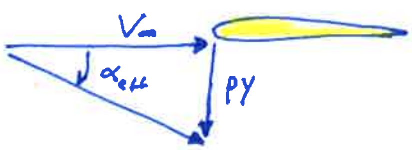
$$\alpha_{eff} = \operatorname{atan}\left(\frac{py}{V_\infty}\right) \approx \frac{py}{V_\infty} \quad \text{when } py \ll V_\infty$$

This section will generate a positive lift due to the roll rate

$$\frac{\Delta L}{\Delta y} \approx q\,\bar{c}\,C_{\ell_\alpha}\,\alpha \approx q\,\bar{c}\,C_{\ell_\alpha}\,\frac{py}{V_\infty}$$

Assume

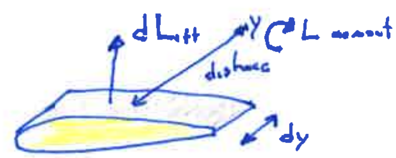
At section B-B



$\alpha_{eff} \approx \frac{py}{V_\infty}$ since $y < 0$ then $\alpha_{eff} < 0$

This section generates negative lift due to roll rate

Roll moment due to roll rate (Strip theory)



$$\frac{dL}{dy} = -\text{Lift} \cdot \text{distance} = -\left(q\,\bar{c}\,C_{\ell_\alpha}\,\frac{py}{V_\infty}\right)\cdot(y)$$

Integrate over the entire span

$$\int dL = \int_{-b/2}^{+b/2} -q\,\bar{c}\,C_{\ell_\alpha}\,\frac{py}{V_\infty}\cdot y\,dy \Rightarrow L = \int_{-b/2}^{+b/2} -q\,\bar{c}\,C_{\ell_\alpha}\,\frac{py^2}{V_\infty}\,dy$$

Non-dimensionalize $C_L = \frac{L}{q S b}$

$$C_L = -\frac{1}{Sb}\int_{-b/2}^{+b/2} \bar{c}(y)\,C_{\ell_\alpha}\,\frac{py^2}{V_\infty}\,dy$$

Constant Chord wing $\quad \bar{c}(y) = \bar{c}$

$$C_L = -\frac{\bar{c}\, C_{\ell_\alpha} p}{S b V_\infty}\int_{-b/2}^{b/2} y^2\, dy = -\frac{\bar{c}\, C_{\ell_\alpha} p}{S b V_\infty} \left.\frac{y^3}{3}\right|_{-b/2}^{b/2} = C \cdot \cancel{\frac{b^3}{8} - \left(-\frac{b^3}{8}\right)}^{\,\frac{b^3}{4}}$$

$$= -\frac{\bar{c}\, C_{\ell_\alpha} p}{S b V_\infty}\, \frac{b^3}{4}$$

Dynamic Simulation:

$$IC \begin{cases} \phi(t=0) = 0 \\ \dot{\phi}(t=0) = 180°/s \end{cases}$$

Mass : M

Span : b :

I : Moment of inertia

$$= \int r^2\, dm = \int_{-b/2}^{b/2} r^2 \rho A\, dr = \rho A \frac{b^3}{4} = \frac{M}{b}\frac{b^3}{4} = \frac{Mb^2}{4}$$

$$\text{Gov Equ 1D} \begin{cases} I\ddot{\phi} = L = qSbC_L \end{cases}$$

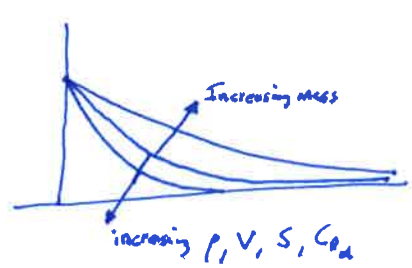
$$\ddot{\phi} = \frac{qSb}{I} C_L$$

$$= \frac{qSb}{I}(-1)\frac{\bar{c}\, C_{\ell_\alpha} \overset{\dot{\phi}}{p}}{\cancel{S}bV_\infty}\frac{b^3}{4}$$

$$\underbrace{\ddot{\phi}}_{\dot{r}} = -\frac{8}{V_\infty}\cdot\bar{c}\cdot\frac{4}{Mb^2}\,\frac{b^3}{4}\, C_{\ell_\alpha}\dot{\phi} = -\frac{8}{V_\infty}\,\bar{c}\,\frac{b}{M}\, C_{\ell_\alpha}\underbrace{\dot{\phi}}_{r}$$

Thus, $\quad \dot{r} = -\frac{1}{2}\rho V \frac{\cancel{\bar{c}b}^{\,S}}{M} C_{\ell_\alpha} r = -\frac{1}{2}\rho V \frac{S}{M} C_{\ell_\alpha} r$

Solution to $\dot{r} = -Cr$ is $\quad r = r_0 e^{-Ct} = 180°/s\; e^{-\frac{1}{2}\rho V \frac{S}{M} C_{\ell_\alpha} t}$

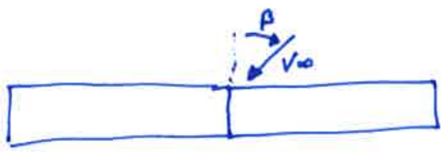


$\boxed{\text{The wing damping drives the rate to zero.}}$

$$r \xrightarrow[t\to\infty]{} 0$$

Dihedral

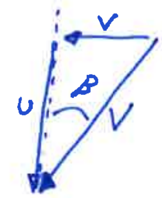
front view



top view

Dihedral ~~gives~~ creates a roll moment given a side slip angle β.

Simplified Analysis



$$\beta = \operatorname{asin}\left(\frac{V}{|V|}\right) \approx \frac{V}{U} \quad \text{when } V \ll U$$

Thus $V \approx \beta V_\infty$

panel has upwash from β and Γ

$V_n \uparrow$

panel has downwash from β and Γ

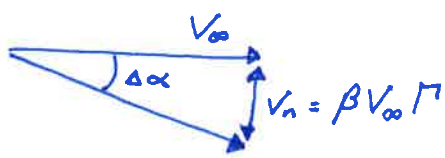
$V_n \downarrow$

In the wing panel frame, there is a normal component of the side velocity V.

$$V_n = V \sin\Gamma \approx V\Gamma$$

$$\approx \beta V_\infty \Gamma$$

Now that the normal component is known, compare to V_∞

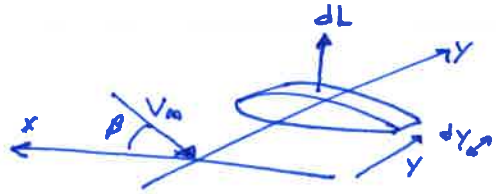


$$\tan(\Delta\alpha) = \frac{V_n}{V_\infty} = \frac{\beta V_\infty \Gamma}{V_\infty}$$

For small angles, $\tan(\Delta\alpha) \approx \Delta\alpha = \beta\Gamma$

The local change in AOA is the product of sideslip angle and the dihedral angle

Integrate over the entire wing.



$$dL_{moment} = -dL_{lift} \cdot y$$

$$q\,c\,C_{l\alpha}\,\Delta\alpha\,dy$$

$+\beta\Gamma$ when $y>0$

$-\beta\Gamma$ when $y<0$

$$dL = -q\,c\,C_{l\alpha}\,\beta\Gamma\,y\,dy$$

Integrate

$$\int dL = \int_{-b/2}^{b/2} -q\,c\,C_{l\alpha}\,(\pm\beta\Gamma)\,y\,dy$$

$$L = \int_0^{b/2} -2q\,c\,C_{l\alpha}\,\beta\Gamma\,y\,dy$$

$$C_L = \frac{1}{qSb}\int_0^{b/2} -2q\,c\,C_{l\alpha}\,\beta\Gamma\,y\,dy = \frac{-2C_{l\alpha}\beta}{Sb}\int_0^{b/2} c\,\Gamma\,y\,dy$$

$$C_L = \frac{-2C_{l\alpha}\beta}{Sb}\int_0^{b/2} c\,\Gamma\,y\,dy$$

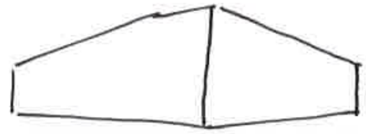
or

$$\boxed{C_{L\beta} = -\frac{2C_{l\alpha}}{Sb}\int_0^{b/2} c\,\Gamma\,y\,dy}$$

We call $C_{L\beta}$ the dihedral effect

A positive β creates a negative roll moment

Linearly Tapered Wing



$$c = c_r\left(1 - (1-\lambda)\left|\frac{2y}{b}\right|\right)$$

$$C_{L_\beta} = \frac{-2C_{L_\alpha}}{Sb}\underbrace{\int_0^{b/2} c_r\left(1-(1-\lambda)\left(\frac{2y}{b}\right)\right)\Gamma\, y\, dy}$$

$$= \frac{-2C_{L_\alpha}}{Sb}\cdot\frac{c_r\, b^2(2\lambda+1)\Gamma}{24}$$

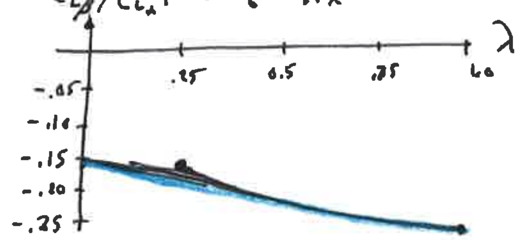
but $AR = \dfrac{2b}{c_r(1+\lambda)}$ verify on own $\Rightarrow c_r = \dfrac{2b}{AR(1+\lambda)}$

$$C_{L_\beta} = \frac{-2C_{L_\alpha}}{S\cancel{b}}\;\frac{2\cancel{b}}{\cancel{AR}(1+\lambda)}\;\frac{\cancel{b^2}}{24}(2\lambda+1)\Gamma\;\cdot\left|\frac{\cancel{AR}\,S}{\cancel{b^2}}\right|$$

cross off terms

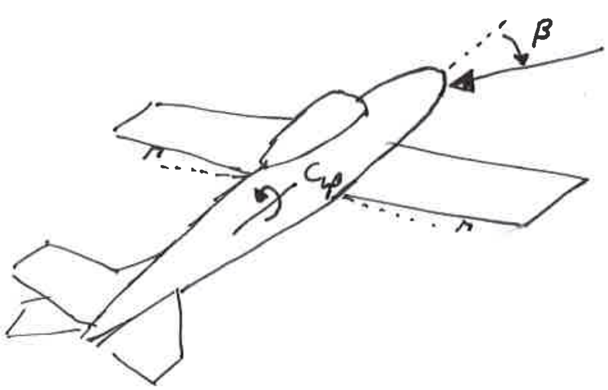
$$\boxed{C_{L_{\beta\,\text{linear taper}}} = -\frac{1}{6}\left(\frac{2\lambda+1}{1+\lambda}\right)C_{L_\alpha}\Gamma}$$

$$C_{L_\beta}/C_{L_\alpha}\Gamma = -\frac{1}{6}\,\frac{2\lambda+1}{1+\lambda}$$



The dihedral effect is linear with respect to dihedral angle

Wings with no taper $(\lambda = 1)$ have stronger dihedral effect than $\lambda \approx 0$.



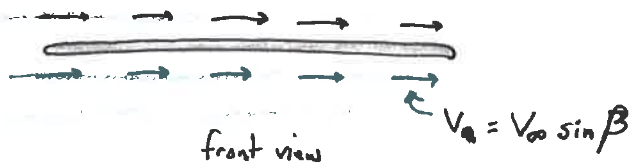
Dihedral allows the rudders to provide roll control

$$\delta_r \Rightarrow \beta = \frac{-C_{n_{\delta_r}}}{C_{n_\beta}}\delta_r \Rightarrow C_L = C_{L_\beta}\beta$$

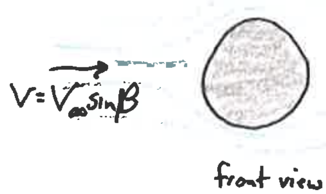
$$\boxed{C_L = -\frac{C_{n_{\delta_r}}}{C_{n_\beta}}C_{L_\beta}\delta_r}$$

Dihedral Effect from Fuselage

The wing position contributes to dihedral effect

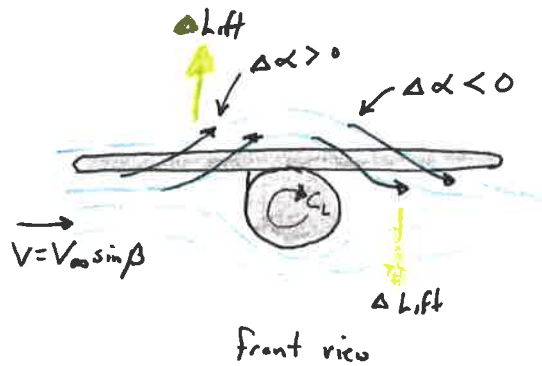


No fuselage, no $C_{l\beta}$



fuselage, no $C_{l\beta}$

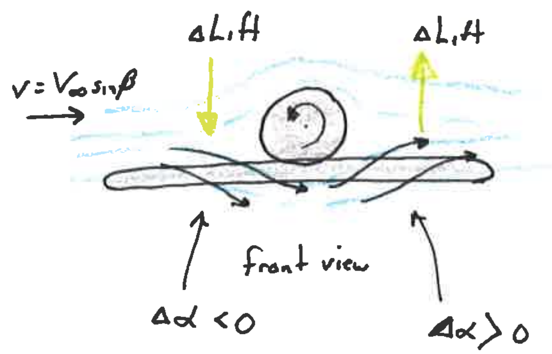
perhaps $C_{Y\beta}$ (sideforce)



fuselage + high wing

In this case, β from left (front view) gives a roll moment in negative x direction

$$C_{l\beta_{\text{high wing + fuselage}}} < 0$$



fuselage + low wing

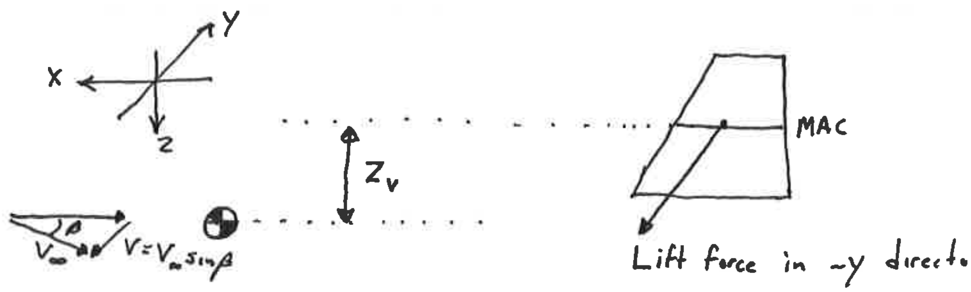
$$C_{l\beta_{\text{low wing}}} > 0$$

McCormick says:

$$\Delta C_{l\beta_{\text{high}}} \approx -0.00016 \ {}^{1}/_{\text{deg}}$$

$$\Delta C_{l\beta_{\text{low}}} \approx 0.00016 \ {}^{1}/_{\text{deg}}$$

Dihedral Effect From Vertical Tail



$$L = -L_v Z_v = -Z_v \eta_v q C_{L_{\alpha_v}} (\beta + \sigma) S_v$$

Non-dim

$$C_L = \frac{L}{qSb} = -\eta_v C_{L_{\alpha_v}} \left(\frac{Z_v S_v}{S\ b} \right) (\beta + \sigma)$$

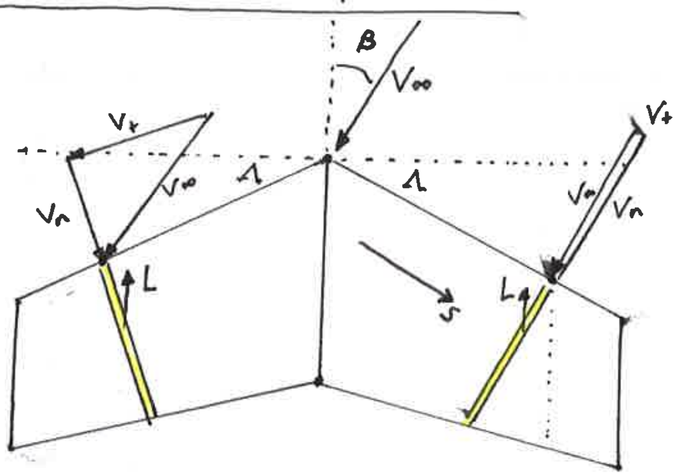
with $\sigma = \sigma_0 + \frac{d\sigma}{d\beta}$

$$\boxed{C_{L_\beta} = -\eta_v C_{L_{\alpha_v}} \left(\frac{Z_v S_v}{S\ b} \right) \left(1 + \frac{d\sigma}{d\beta} \right)}$$

A vertical above the cg (common!) tends to provide dihedral effect.

$C_{L_\beta} < 0$

Dihedral Effect Due to Swept Wings



The normal velocity on the left wing is $V_n = V_\infty \cos(\Lambda + \beta)$

The normal velocity on the right wing is $V_n = V_\infty \cos(\Lambda - \beta)$

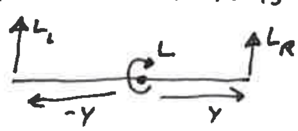
The lift is

$$L = q_n c_n C_\ell = q_\infty \cos^2(\Lambda+\beta)^* \cdot \cos\Lambda\, c\, C_\ell$$

The right wing lift is

$$L = q_n c_n C_\ell = q_\infty \cos^2(\Lambda-\beta)^* \cos\Lambda\, c\, C_\ell$$

The differential roll moment is



$$dL = q c C_\ell\, y \left(\cos^2(\Lambda+\beta) - \cos^2(\Lambda-\beta) \right) \cos\Lambda\, ds$$

and $dy = \cos\Lambda\, ds$

$$dL = q c C_\ell\, y \left(\cos^2(\Lambda+\beta) - \cos^2(\Lambda-\beta) \right) dy$$

Integrate over half span (since we already took differential)

$$L = q C_\ell \left(\cos^2(\Lambda+\beta) - \cos^2(\Lambda-\beta) \right) \int_0^{b/2} c\, y\, dy$$

The total lift is

$$C_{L_{lift}} = C_\ell \cos^2\Lambda \qquad \text{(approximately for swept wings)}$$

The roll moment is thus

$$C_L = \frac{L}{qSb} = \frac{C_L}{Sb\cos^2\Lambda}\left(\cos^2(\Lambda+\beta) - \cos^2(\Lambda-\beta)\right)\int_0^{b/2} c\, y\, dy$$

$$C_{L_\beta} = \left.\frac{dC_L}{d\beta}\right|_{\beta=0} = \frac{-4C_L\tan\Lambda}{Sb}\int_0^{b/2} c\, y\, dy$$

Or for a linearly tapered wing

$$C_{L\beta} = -\frac{1+2\lambda}{3(1+\lambda)}\, C_L \tan\Lambda$$

This is only an approximate result.

- The dihedral effect in swept wings depends on the lift

$$C_L \uparrow \Rightarrow C_{L\beta} \downarrow \quad \text{more dihedral effect!}$$

- " depends on tan of sweep angle
- " stronger for larger λ