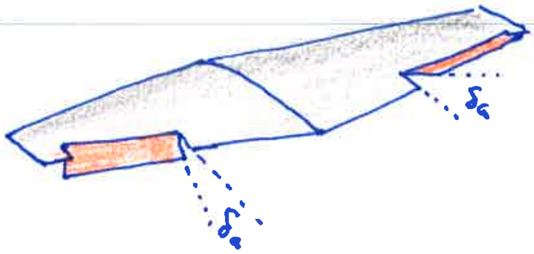
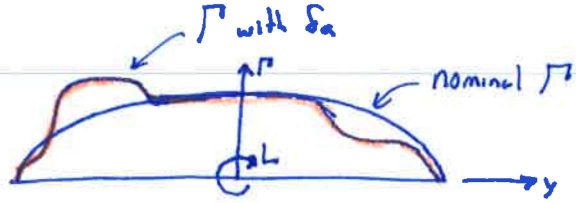
Lesson 20

Roll Control

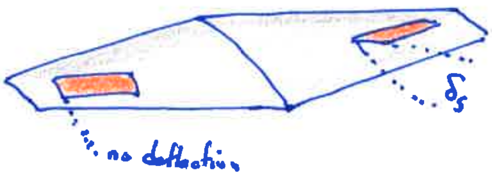
Ailerons





$$\Delta L_{moment} = \Delta L_{lift} \cdot y$$

Spoilers



Roll Moment:

$$dC_L = \frac{dL}{qSb} = \frac{(q C_\ell c)\, y\, dy}{qSb}$$

$$= \frac{C_\ell\, c\, y\, dy}{Sb}$$

Integrate

$$C_L = \int_{-b/2}^{b/2} \frac{C_\ell\, c\, y\, dy}{Sb}$$

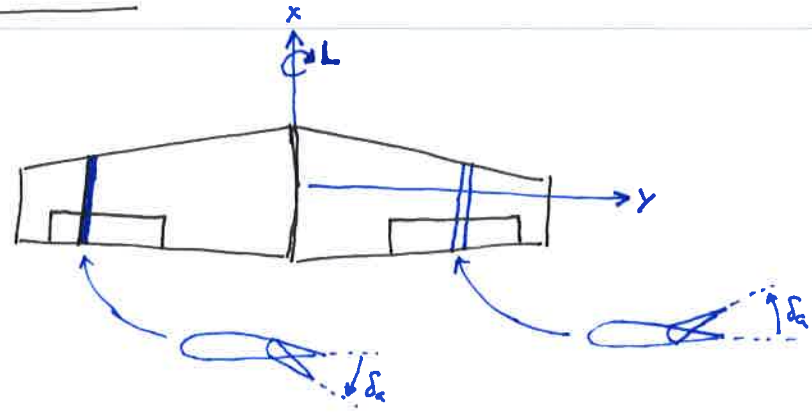
This is called strip theory (assume that C_ℓ depends only on α ... no Γ distribution).

Advantages?

- Control power (ailerons ✓ > spoilers X)
- Near Stall (ailerons $C_{\ell_{\delta_a}}$ ✓)
- C_ℓ distribution (ailerons X)
- Adverse yaw (ailerons X typically)
- Aeroelasticity (spoilers ✓)
- FCS rigging (ailerons ✓)

Roll Control Due to Ailerons

Strip Theory:



$$dL_{\substack{\text{moment}\\\text{ailerons}}} = \text{Force} \cdot \text{distance} = q\,\underbrace{c\,dy}_{\text{area}}\,C_\ell \cdot y$$

$$dC_L = \frac{L}{qSb} = \frac{c\,C_\ell\,y}{Sb}\,dy$$

Assume

$$C_\ell = C_{\ell_\alpha}\alpha = C_{\ell_\alpha}\frac{d\alpha}{d\delta_a}\delta_a = C_{\ell_\alpha}\tau\,\delta_a$$

$$dC_L = \frac{c\,C_{\ell_\alpha}\tau\,\delta_a}{Sb}\,y\,dy$$

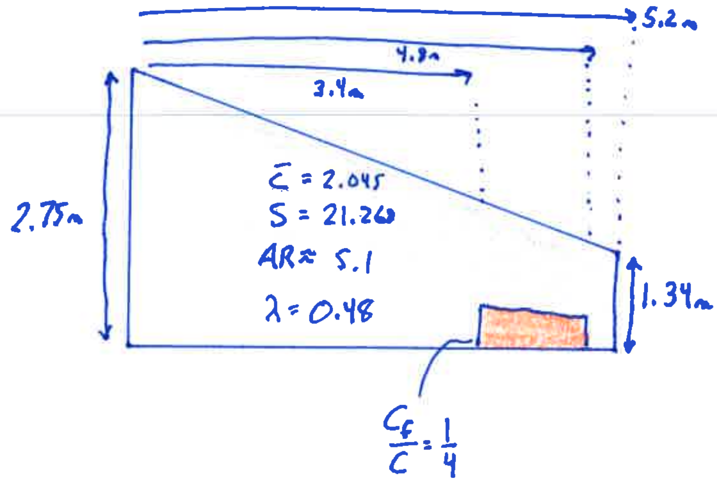
Integrate over one panel (i.e. one aileron) (notice: different from book)

$$\int dC_L = \int \frac{c\,C_{\ell_\alpha}\tau\,\delta_a}{Sb}\,y\,dy$$

$$C_L = \frac{C_{\ell_\alpha}\tau\,\delta_a}{Sb}\int_0^{b/2} c\,y\,dy$$

$$\boxed{C_{L_{\delta_a}} = \frac{C_{\ell_\alpha}\tau}{Sb}\int_0^{b/2} c\,y\,dy}$$ one aileron

Ex: 2.14



a) Use Strip theory.

C_{L_α}

$$C_{L_\alpha} \approx \frac{2\pi}{1 + \frac{2\pi}{\pi AR}} \approx 4.5 \ \frac{1}{rad}$$

Function of chord wrt span

$$c(y) = c_r + \frac{dc}{dy} dy$$

$$\left(\frac{1.34 - 2.75}{5.2}\right) = -0.2712$$

$$= 2.75 - 0.2712\, y$$

Strip theory

$$C_L = \int_0^{b/2} \frac{C_\ell \, c(y)\, y \, dy}{Sb} = \int_0^{b/2} \frac{C_{\ell_\alpha}}{Sb} c(y)\, y\, \tau \delta_a \, dy$$

$C_\ell = C_{\ell_\alpha} \tau \delta_a$

$C_{\ell_\alpha} = 2\pi$ (approximation) $\approx$ Correct for AR ✓ $C_{L_\alpha} = 4.5 \frac{1}{rad}$

look up τ for $c_f/c = 0.25$ $\Rightarrow$ Fig 2.21 $\frac{c_f}{c} = 0.25 \Rightarrow \tau \approx 0.4$

$$C_L = \frac{4.5}{rad} \bigg| \frac{0.4\, \delta_a \,[rad]}{21.268\, m^2} \bigg| \frac{}{10.4\, m} \bigg| \underbrace{\int_0^{5.2m} \left(2.75^{[m]} - 0.2712\, y\right) y^{[m]} dy^{[m]}}_{24.5\,[m^3]}$$

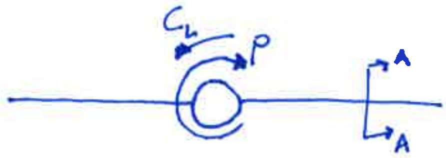
$$= 0.2\, \delta_a$$

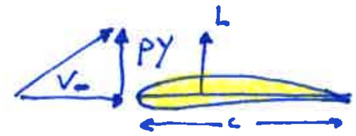
Be careful of the units.

$$\boxed{C_{L_{\delta_a}} \approx 0.2 \ \frac{1}{rad}}$$

Steady roll rate (p)

A steady roll rate generates an opposite roll moment.





Section A-A

$$\alpha_{effective} = \operatorname{atan}\left(\frac{py}{V_\infty}\right) \approx \frac{py}{V_\infty}$$

$$\Delta L_{moment} = y \cdot \Delta L_{lift}$$

$$dC_L = \frac{y\, C_l\, q\, c\, dy}{q S b} = \frac{y\left(C_l \frac{py}{V_\infty}\right) c(y)\, dy}{S b}$$

You can derive that for a linearly tapered wing, $c = \frac{b}{AR}\left(\frac{2}{1+\lambda}\right)\left(1 + \frac{(\lambda-1)2y}{b}\right)$

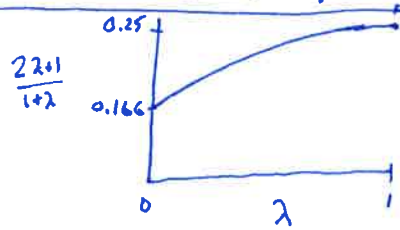
$$C_L = \int_{-b/2}^{b/2} \frac{C_{l_\alpha}}{Sb}\frac{p}{V_\infty} y^2 c(y)\, dy = 2\int_0^{b/2} \frac{C_{l_\alpha}}{Sb}\frac{p}{V_\infty}\frac{b}{AR}\left(\frac{2}{1+\lambda}\right)\left(1+\frac{(\lambda-1)2y}{b}\right) y^2\, dy$$

$$= C_{l_\alpha}\left(\frac{pb}{2V_\infty}\right)\left(\frac{8}{1+\lambda}\right)\frac{1}{b^3}\underbrace{\int_0^{b/2}\left(1+\frac{(\lambda-1)2y}{b}\right)y^2\, dy}_{\frac{b^3(3\lambda+1)}{96}}$$

$$= C_{l_\alpha}\left(\frac{pb}{2V_\infty}\right)\frac{(3\lambda+1)}{(1+\lambda)}\left(\frac{1}{12}\right)$$

dimensional roll rate! Compare with non-dim roll rate in prob 3.7

$$\boxed{C_{L_p} = C_{l_\alpha}\frac{(3\lambda+1)}{(1+\lambda)}\frac{1}{24}\frac{b}{V_\infty}}$$



Roll damping depends on

- taper ratio
- inverse velocity
- C_{l_α}

Q: What is the steady state roll rate? (given δ_a)

$$\cancel{I\ddot{\phi}}^{\,0} = \sum M = M_{damping} + M_{ailerons} \Rightarrow M_{damping} = -M_{ailerons}$$

$$\Downarrow$$

$$C_{L_p} p = -C_{L_{\delta_a}} \delta_a$$

$$\Downarrow$$

$$\boxed{p = -\frac{C_{L_{\delta_a}}}{C_{L_p}} \delta_a}$$

Q: Estimate the maximum roll rate of the wing in 2.14.

$$C_{L_{\delta_a}} = 0.2 \ \frac{1}{rad} \qquad C_{L_p} \approx 2\pi \frac{3(0.48)+1}{1+0.48} \frac{1}{24} \frac{b}{V_\infty} \approx \frac{4.49}{V_\infty}$$

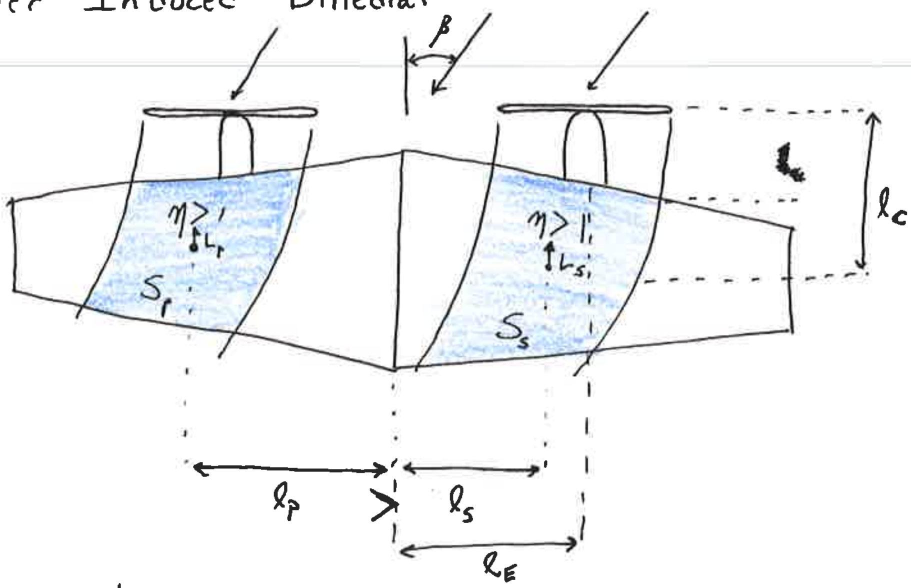
$$p = -\frac{0.2}{rad} \left| \frac{V_\infty \ rad}{4.49 \ s} \right| \delta_a \ rad$$

$$= -0.0445 \ \frac{rad}{s} \text{ per } \delta_a \ rad \text{ per } V_\infty \ ft/s$$

At 200 ft/s and $\delta_a = 20°$

$$p = -0.0445 \cdot \left(\frac{20°}{57.3}\right) 200 \ \frac{ft}{s} = 3.1 \ \frac{rad}{s} \approx 180 \frac{°}{s}$$

Power Induced Dihedral



$$L_{moment} = L_p l_p - L_s l_s$$

For a $\lambda \approx 1$, $S_p \approx S_s$ $\Rightarrow$ $L_{moment} = \eta q S_p C_{l_\alpha} \alpha_{eff} l_p - \eta q S_s C_{l_\alpha} \alpha_{eff} l_s$

$$= \eta q S_{sp} C_{l_\alpha} \alpha_{eff} (l_p - l_s)$$

$$C_L = \frac{L}{qSb} = \eta \frac{S_{sp}}{S} \left(\frac{l_p - l_s}{b} \right) C_{l_\alpha} \alpha_{eff}$$

Also, $l_p \approx l_E + \tan(\beta)\, l_c \approx l_E + \beta l_c$

$$l_s \approx l_E - \tan\beta\, l_c \approx l_E - \beta l_c$$

$$C_L \approx \eta \frac{S_{sp}}{S} \left(\frac{l_E + \beta l_c - l_E + \beta l_c}{b} \right) C_{l_\alpha} \alpha_{eff}$$

$$\approx \eta \frac{S_{sp}}{S} \frac{2\beta l_c}{b} C_{l_\alpha} \alpha_{eff}$$

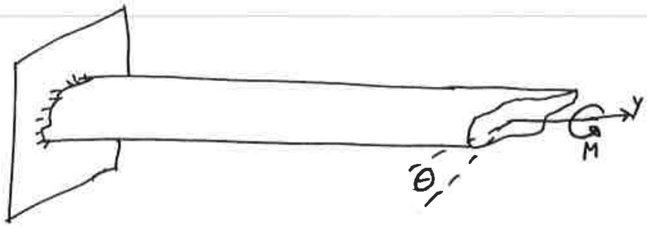
$$\boxed{C_{L_{\beta_{power}}} \approx \eta \left(\frac{S_{sp}}{S} \right) \left(\frac{2}{b} l_c \right) C_{l_\alpha} \alpha_{eff}}$$

power swept area — extended power disk — AOA

positive sign!! rolls into β

At high power settings and ~~high~~ low speeds, high performance aircraft show <u>anhedral</u>. The Martin 202 prototype <u>had</u> to be fixed (add dihedral) to fix this issue (at a <u>great</u> sacrifice of the employees).

Torsional Stiffness.



Applying a y moment to this structure of

- length ℓ
- Modulus G
- Polar Moment of Inertia J

The twist θ is

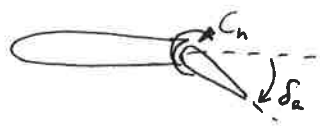
$$\theta = \frac{M\ell}{JG}$$

For an infinitesimal section

$$d\theta = \frac{M\,d\ell}{JG}$$

Control Surface

Deflecting an aileron by δ_a creates a hinge moment.

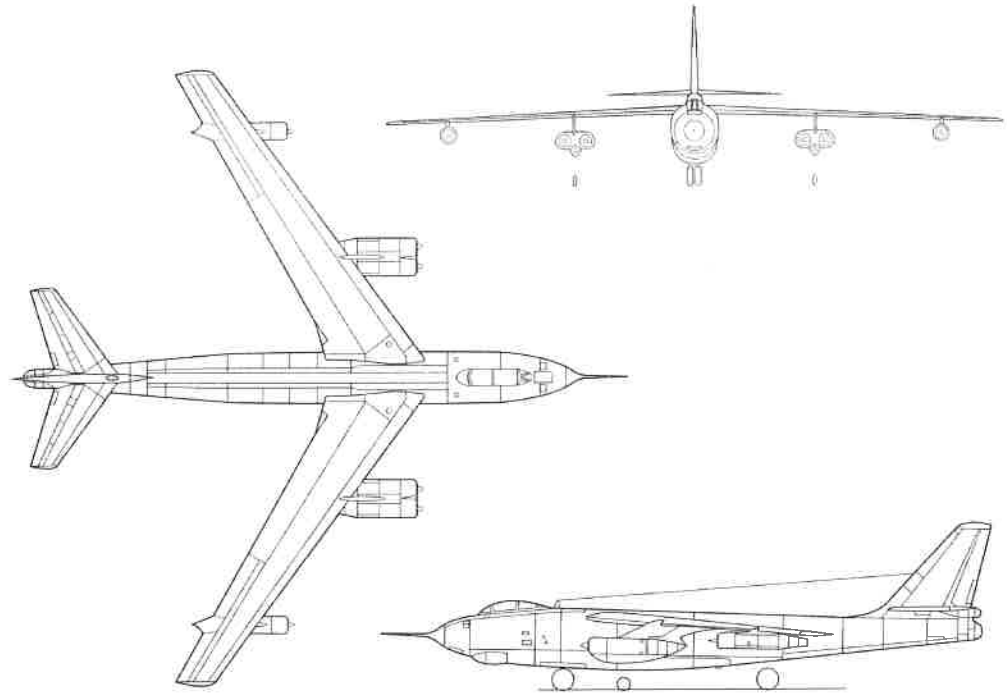


$C_{h_{\delta_a}} < 0$

For a steady deflection, C_h must be matched by an equal and opposite moment on the airfoil.

Also, a flap deflection of angle δ_a creates a LED moment.

$C_m < 0$



Dryden Flight Research Center February 1998
B-47A Stratojet 3-view

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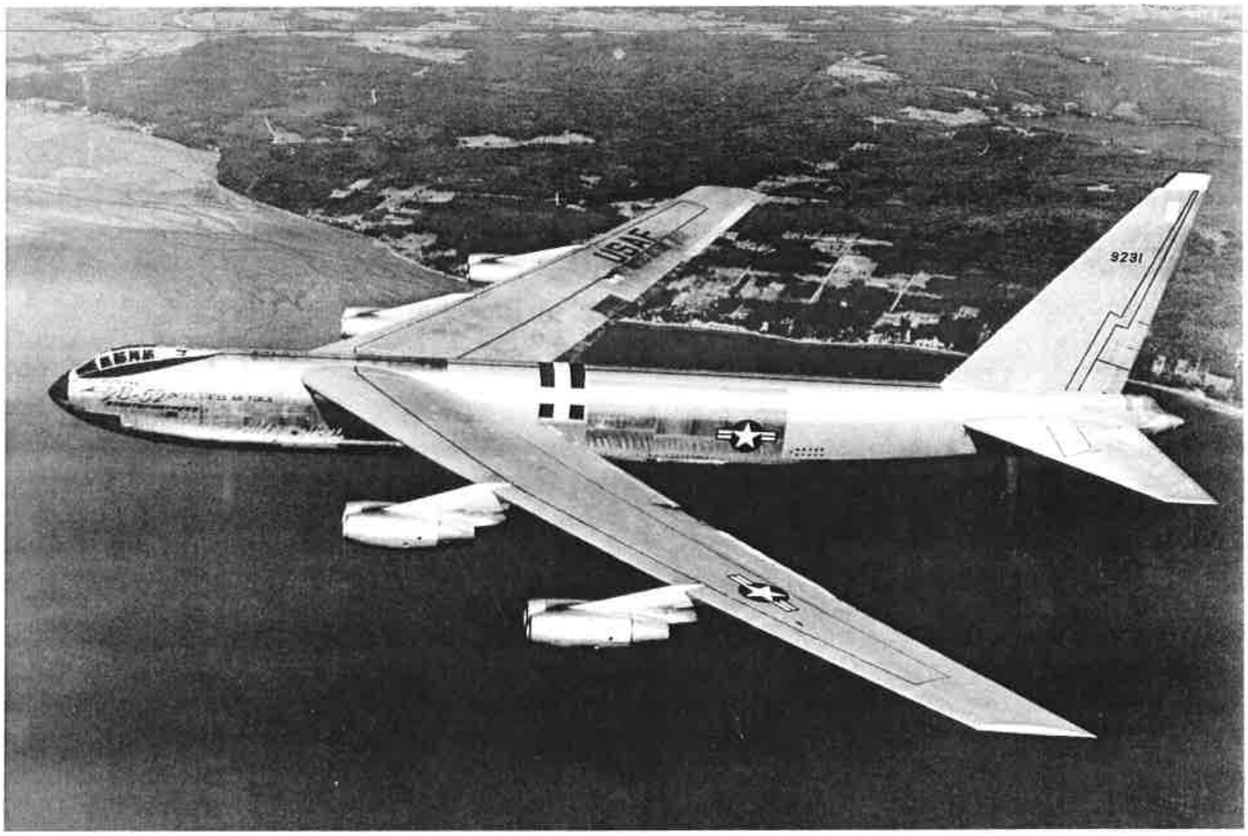
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