Lesson 25

Lateral Motion

- Spiral Mode 
- Rolling Mode 
- Dutch Roll

Lateral Equations of Motion

Side Force, Rolling, Yawing Motions.

Use the same process as with the long' equations of motion and linearization.

$$S = \begin{pmatrix} \Delta v \\ \Delta p \\ \Delta r \\ \Delta\phi \end{pmatrix} \quad \Rightarrow \quad \beta \approx \frac{\Delta v}{U_0} \qquad S = \begin{pmatrix} \Delta\beta \\ \Delta p \\ \Delta r \\ \Delta\phi \end{pmatrix}$$

For a generic aircraft, the equations of motion (linearized) are (table 3.2) with stability derivatives from Table 3.4. (and $\Delta\dot\phi = \Delta p$)

$$\Delta\dot v - Y_v\Delta v - Y_p\Delta p + (U_0 - Y_r)\Delta r - g\cos\theta_0\,\Delta\phi = Y_{\delta_r}\Delta\delta_r$$

$$\Delta\dot p - \frac{I_{xz}}{I_x}\Delta\dot r - L_v\Delta v - L_p\Delta p - L_r\Delta r = L_{\delta_a}\Delta\delta_a + L_{\delta_r}\Delta\delta_r$$

$$-\frac{I_{xz}}{I_z}\Delta\dot p + \Delta\dot r - N_v\Delta v - N_p\Delta p - N_r\Delta r = N_{\delta_a}\Delta\delta_a + N_{\delta_r}\Delta\delta_r$$

We notice that the 2nd and 3rd rows are coupled.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -\frac{I_{xz}}{I_x} \\ 0 & -\frac{I_{xz}}{I_z} & 1 \end{bmatrix} \begin{pmatrix} \Delta\dot v \\ \Delta\dot p \\ \Delta\dot r \end{pmatrix} = \left(\cdots\right) = [P](\dot S) = \left(\cdots\right)$$

This is a "block diagonal" matrix and can be solved as

$$P = \begin{bmatrix} (1)^{-1} & \\ & [2\times2]^{-1} \end{bmatrix} \Rightarrow P^{-1} = \begin{bmatrix} 1 & & \\ & 1/\Gamma & \frac{I_{xz}}{\Gamma I_x} \\ & \frac{I_{xz}}{\Gamma I_z} & 1/\Gamma \end{bmatrix} \quad \text{where } \Gamma = 1 - \frac{I_{xz}^2}{I_x I_z}$$

$$\text{since} \quad A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}}{ad - bc}$$

pre multiply by P^{-1}

$$P^{-1} P \dot{s} = P^{-1}(\cdots\cdot) = \begin{bmatrix} 1 & & \\ & \frac{1}{\Gamma} & \frac{I_{xz}}{\Gamma I_x} \\ & \frac{I_{xz}}{\Gamma I_z} & \frac{1}{\Gamma} \end{bmatrix} \begin{pmatrix} Y_v & Y_p & -(u_0 - Y_r) \\ +L_v & L_p & L_r \\ N_v & N_p & N_r \end{pmatrix} \begin{pmatrix} \Delta v \\ \Delta p \\ \Delta r \end{pmatrix}$$

$$+ \left[P^{-1}\right] \begin{bmatrix} 0 & Y_{\delta_r} \\ L_{\delta_a} & L_{\delta_r} \\ N_{\delta_a} & N_{\delta_r} \end{bmatrix} \begin{pmatrix} \Delta\delta_a \\ \Delta\delta_r \end{pmatrix}$$

$$\begin{pmatrix} \Delta\dot{\beta} \\ \Delta\dot{p} \\ \Delta\dot{r} \end{pmatrix} = \begin{bmatrix} Y_v & Y_p & -(u_0 - Y_r) \\ \frac{L_v}{\Gamma} + \frac{I_{xz}}{I_x}\frac{L_v}{\Gamma} & \frac{L_p}{\Gamma} + \frac{I_{xz}}{\Gamma I_x} N_p & \frac{L_r}{\Gamma} + \frac{I_{xz}}{I_x \Gamma} N_r \\ \frac{I_x}{I_z}\frac{L_v}{\Gamma} + \frac{N_v}{\Gamma} & \frac{I_{xz}}{\Gamma I_z} L_p + \frac{1}{\Gamma} N_p & \frac{I_{xz}}{\Gamma I_z} L_r + \frac{N_r}{\Gamma} \end{bmatrix} \begin{pmatrix} \Delta\beta \\ \Delta p \\ \Delta r \end{pmatrix}$$

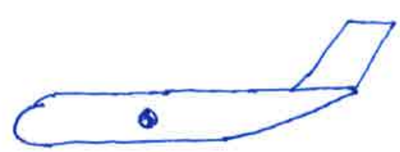
$$(\Delta\dot{\phi}) = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} (\Delta\phi)$$

and $\Delta\dot{\beta} = \frac{g \cos\theta_0}{u_0} \Delta\phi$

$$+ \begin{bmatrix} 0 & Y_{\delta_r} \\ \frac{L_{\delta_a}}{\Gamma} + \frac{I_{xz}}{\Gamma I_x} N_{\delta_a} & \frac{L_{\delta_r}}{\Gamma} + \frac{I_{xz}}{\Gamma I_x} N_{\delta_r} \\ \frac{I_{xz}}{\Gamma I_z} L_{\delta_a} + \frac{N_{\delta_a}}{\Gamma} & \frac{I_{xz}}{\Gamma I_z} L_{\delta_r} + \frac{N_{\delta_r}}{\Gamma} \end{bmatrix} \begin{pmatrix} \Delta\delta_a \\ \Delta\delta_r \end{pmatrix}$$

If $I_{xz} = 0$, $\Gamma = 1$ and the dynamics simplify

$$I_{xz} = \iint x z \, dm$$

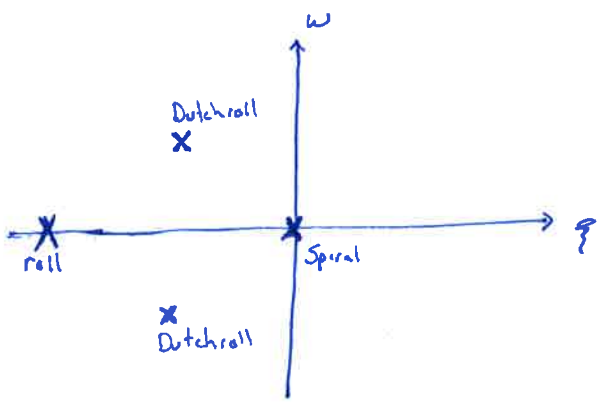


When assuming $I_{xz} = 0$ and $\Delta\beta = \frac{\Delta v}{U_0}$ Table 5.1

$$\dot{S} = \begin{pmatrix} \Delta\dot{\beta} \\ \Delta\dot{p} \\ \Delta\dot{r} \\ \Delta\dot{\phi} \end{pmatrix} = \begin{bmatrix} \frac{Y_\beta}{U_0} & \frac{Y_p}{U_0} & -\left(1 - \frac{Y_r}{U_0}\right) & \frac{g\cos\theta_0}{U_0} \\ L_\beta & L_p & L_r & 0 \\ N_\beta & N_p & N_r & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{pmatrix} \Delta\beta \\ \Delta p \\ \Delta r \\ \Delta\phi \end{pmatrix} + \begin{bmatrix} 0 & \frac{Y_{\delta_r}}{U_0} \\ L_{\delta_a} & L_{\delta_r} \\ N_{\delta_a} & N_{\delta_r} \\ 0 & 0 \end{bmatrix} \begin{pmatrix} \Delta\delta_a \\ \Delta\delta_r \end{pmatrix}$$

There are 4 eigenvalues of this 4 state equation.

- Spiral Mode (usually real root), low damping/unstable)
- Roll Mode (Roll subsidence) (real root, high damping)
- Dutch roll (complex conjugate pair)



Y_β ≡ Side force due to sideslip

Y_p ≡ Side force due to roll rate

Y_r ≡ side force due to yaw rate

L_β ≡ dihedral effect

L_p ≡ roll damping

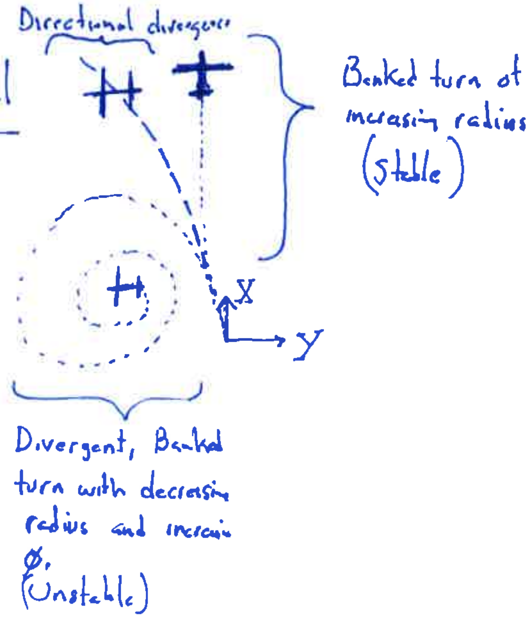
L_r ≡ Roll moment due to yaw rate

N_β ≡ Directional Stability

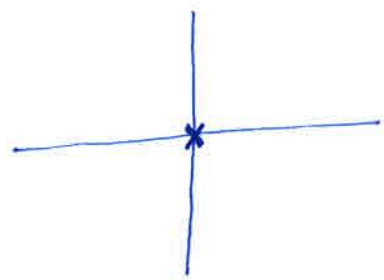
N_p ≡ Yaw moment due to ~~yaw~~ roll rate

N_r ≡ Yaw moment due to yaw rate
Yaw Damping

Spiral



This is a weak, slow mode that is difficult to accurately predict exactly the λ and ζ. Small forces and moments



Approximation

Neglect side force and $\Delta\phi$

Aerodynamics depend on β, p, r

- $\Delta\dot{p}$ is small, so $\cancel{\Delta\dot{p}}^{\,0} = L_\beta \Delta\beta + L_p \cancel{\Delta p}^{\,0} + L_r \Delta r \Rightarrow \Delta\beta = -\dfrac{L_r \Delta r}{L_\beta}$
- Yaw rate, $\Delta\dot{r} = N_\beta \Delta\beta + N_p \cancel{\Delta p}^{\,0} + N_r \Delta r$

Substitute $\Delta\beta$ into $\Delta\dot{r}$

$$\Delta\dot{r} = \frac{-N_\beta L_r}{L_\beta}\Delta r + N_r \Delta r \Rightarrow \Delta\dot{r} = \left(\frac{-N_\beta L_r + N_r L_\beta}{L_\beta}\right)\Delta r$$

The root of this is:

$$\boxed{\lambda = \frac{N_r L_\beta - N_\beta L_r}{L_\beta}} \qquad \zeta \approx 0$$

N_r = Yaw Damping $(-)$
L_β = Dihedral Effect $(-)$
N_β = Directional Stability $(+)$
L_r = Roll moment due to yaw rate $(+)$

For stability, $\lambda < 0$ thus $\underset{\text{Yaw damping}}{N_r}\underset{\text{dihedral}}{L_\beta} > N_\beta L_r$

$(-)(-) > (+)(+)$

$\boxed{\text{Increasing yaw damping and/or dihedral makes the spiral mode stable}}$

$$L_\beta N_r - L_r N_\beta > 0 \quad \text{since} \quad L_\beta < 0$$

or

$$L_\beta N_r > L_r N_\beta$$

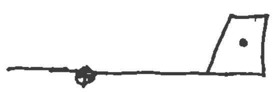
L_β: dihedral — wing + vertical tail above cg

N_r: directional ~~stability~~ yaw damping — vertical size and tail length

L_r: roll damping — natural for aircraft

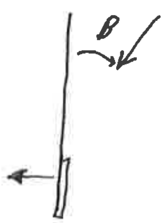
N_β: directional stability — vertical size and tail length

this $Z_{\text{vertical tail}}$ can be changed.



$L_\beta < 0$

$L_\beta > 0$

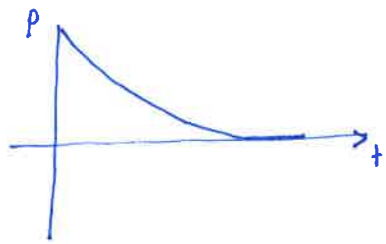


Force above cg creates $-L$ moment

Roll Mode

Highly damped, single-DOF.

Eigenvector mostly p.



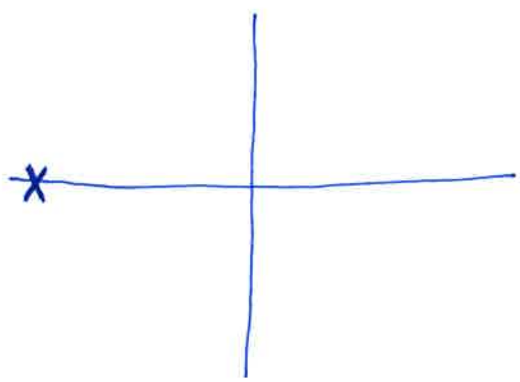
Approx:

$$\Delta\dot{p} = \cancel{L_\beta \Delta\beta} + L_p \Delta p + \cancel{L_r \Delta r} \Rightarrow \Delta\dot{p} = L_p \Delta p$$

$$\lambda = L_p$$

roll damping.

See Lesson 20 for details on L_p



Dutch Roll Approximation — Rough

Track sideslip and yaw (neglect roll)

$$\begin{pmatrix} \Delta\dot{\beta} \\ \Delta\dot{r} \end{pmatrix} = \underbrace{\begin{pmatrix} \frac{Y_\beta}{U_0} & -\left(1-\frac{Y_r}{U_0}\right) \\ N_\beta & N_r \end{pmatrix}}_{A} \begin{pmatrix} \Delta\beta \\ \Delta r \end{pmatrix}$$

eigenvalues are:

$$|\lambda I - A| = 0 \Rightarrow \begin{vmatrix} \lambda - \frac{Y_\beta}{U_0} & \left(1-\frac{Y_r}{U_0}\right) \\ -N_\beta & \lambda - N_r \end{vmatrix} = 0$$

$$\lambda^2 - \left(\frac{Y_\beta + U_0 N_r}{U_0}\right)\lambda + \frac{Y_\beta N_r - N_\beta Y_r + U_0 N_\beta}{U_0} = 0$$

$$\omega_n = \sqrt{\frac{Y_\beta N_r - N_\beta Y_r + U_0 N_\beta}{U_0}} \qquad \zeta = -\frac{1}{2\omega_n}\left(\frac{Y_\beta + U_0 N_r}{U_0}\right)$$

$$= \underbrace{\sqrt{\frac{Y_\beta N_r}{U_0} - \frac{N_\beta Y_r}{U_0} + N_\beta}}_{\text{strong influence from } N_\beta} \qquad = \underbrace{-\frac{1}{2\omega_n}\left(\frac{Y_\beta}{U_0} + N_r\right)}_{\text{strong influence from } N_r}$$

Nr, and Yr

"yaw damping"

Dutch Roll

Visualization

tiny.cc/AEM368-DutchRoll-1

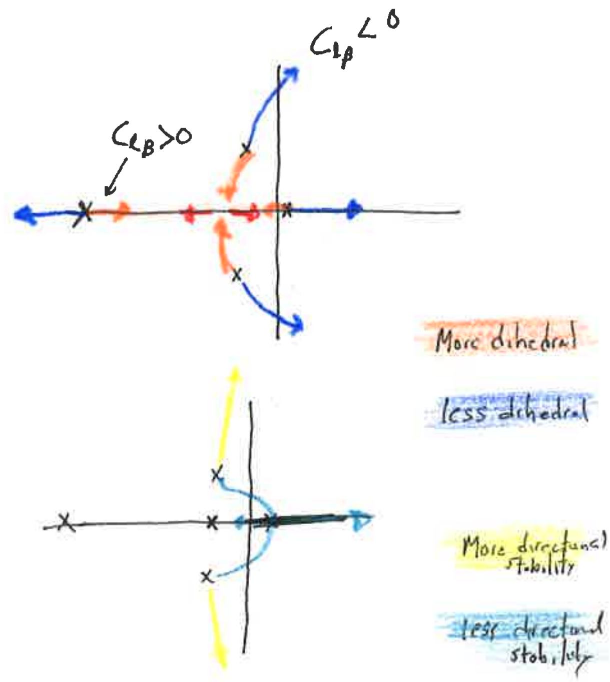
tiny.cc/AEM368-DR2

tiny.cc/AEM368-DR3

tiny.cc/AEM368-DR4

This is a combination of the lateral modes with a phase shift.

Design Parameters on lateral modes



a) Increase Dihedral $C_{l\beta}$

  Improves Spiral mode ✓

  Degrades Dutch Roll ✗

b) Increase Directional stability $C_{n\beta}$

  Improves Dutch Roll ✓

  Degrades Spiral ✗

c) Increase Yaw damping C_{n_r}

  Improves both ✓ ✓

  But we usually can't aerodynamically obtain C_{n_r} without $C_{n\beta}$!

2 typical solutions

1) Vertical below cg

2) Yaw damper stability feedback (AEM 468)

Ex: For the Navion (using derivatives from table 5.2), determine the lateral modes. $U_0 = 176\ \frac{ft}{s}$

$$Y_v = -0.254\ \frac{1}{s} \qquad N_v = 0.025\ \frac{1}{ft\,s} \qquad L_v = -0.091\ \frac{1}{ft\,s}$$

$$Y_\beta = -45.72\ \frac{ft}{s^2} \qquad N_\beta = 4.49\ s^{-2} \qquad L_\beta = -16.02\,s^{-2}$$

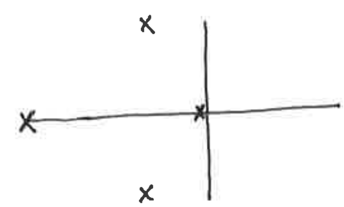
$$Y_p = 0 \qquad N_p = -0.35\ s^{-1} \qquad L_p = -8.4\ s^{-1}$$

$$Y_r = 0 \qquad N_r = -0.76\ s^{-1} \qquad L_r = 2.19\ s^{-1}$$

$$\begin{pmatrix} \Delta\dot\beta \\ \Delta\dot p \\ \Delta\dot r \\ \Delta\dot\phi \end{pmatrix} = \underbrace{\begin{bmatrix} \cancel{\frac{-45.72\ ft/s^2}{176\ ft/s}} \nearrow -0.254\ 1/s & 0 & -(1-0) & \cancel{\frac{32.174\ ft/s^2}{176\ ft/s}} \nearrow 0.182\ 1/s \\ -16.02\ s^{-2} & -8.4\,s^{-1} & 2.19\ s^{-1} & 0 \\ 4.49\ s^{-2} & -0.35\,s^{-1} & -0.76\ s^{-1} & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}}_{A} \begin{pmatrix} \Delta\beta \\ \Delta p \\ \Delta r \\ \Delta\phi \end{pmatrix}$$

eig(A) in Matlab =

$$\begin{aligned} &-8.4779 + 0.0000i \\ &-1.6077 \pm 2.2325i \\ &-0.0066 + 0.0000i \end{aligned}$$

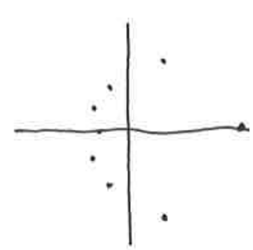


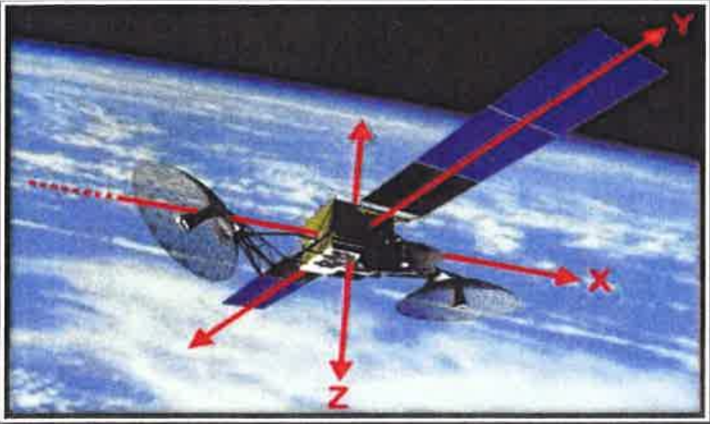
(Stable!)

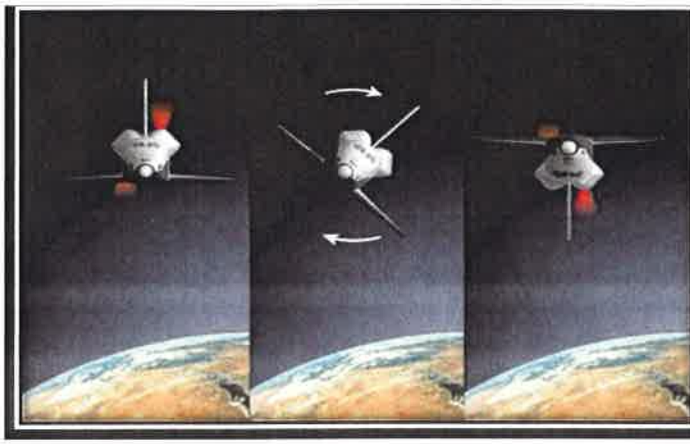
[V, D] = eig(A) gives eigvals and eig. vecs.

$$V = \underbrace{\begin{pmatrix} 0 \\ 1 \\ 0 \\ -0.1 \end{pmatrix}}_{\text{roll}} \quad \underbrace{\begin{pmatrix} -0.27 \pm 0.017i \\ 0.7 \\ 0.18 \pm 0.58i \\ -0.14 \pm 0.20i \end{pmatrix}}_{\text{dutch roll}} \quad \underbrace{\begin{pmatrix} 0.0 \\ 0 \\ 0.1 \\ 0.99 \end{pmatrix}}_{\text{spiral}}$$

complex numbers indicate phase lag







- *Have you ever wondered how spacecraft maintain and change their orientation?*
- *How scientists and engineers make sure that the spacecraft is pointing in the right direction at the right time?*
- *How do you control a spinning spacecraft?*

Fall 2017

AEM 491/591: Spacecraft Attitude Dynamics

For: - Seniors and Graduate Students in AEM
- Seniors and Graduate Students in COE (with instructor's permission)

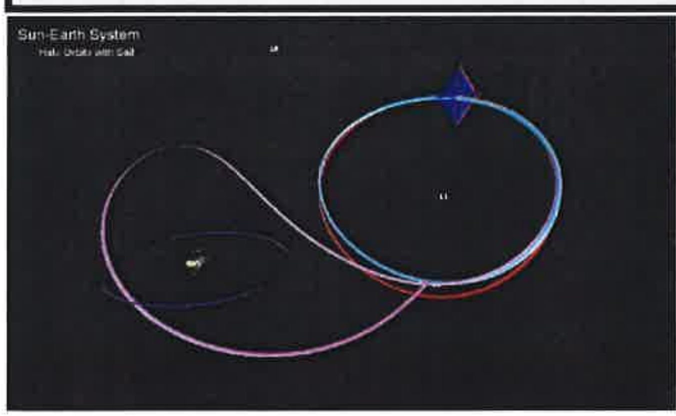
Day/Time: MWF 9:00 AM – 9:50 AM

Credits: 3

Instructor: Dr. Rohan Sood

Description: Formulate, understand, and apply rigid body dynamics to a spacecraft. Determine the orientation and demonstrate the ability to stabilize a spacecraft.

For additional information, send your questions to: rsood@eng.ua.edu



Rohan Sood, PhD
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Areas of Research:

- Astrodynamics and space applications
- Spacecraft trajectory design
- Spacecraft attitude dynamics and control

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