Airfoil and Wing Theory

AEM 614

Lecture 1

19th Aug 2015

Upcoming:

Math + Engineering Quiz Friday 21st

- Math, Calculus, ODEs, Complex #s
- Thermodynamics
- Aircraft / Aerospace engr'

The objective of the quiz is to determine the skill levels of students.

Reading

FVA 1
ADTA 1-6

Lesson 1

Nomenclature
and
Reference frames

It's easy to explain how a rocket works, but explaining how a wing works takes a rocket scientist.

— Philippe Spalart

The answer!

2π

Corrected for aspect ratio

Corrected for Mach #

and Reynolds #

But only valid within a linear range of AOA

Except when deccambered through ....

With an unsteady term associated with the instantaneous lift and .....

Local transonic regions change the fundamental solution type.

But only valid within a range of Kn sufficient for continuum approximation.

Don't forget about turbulence and the Kolmogorov scale

Leading edge stall

Corrected for an equivalent roughness

Wing tip shape influences the tip vortex [illegible]

Except in the presence of vortex generators

Standard day vs Hot vs tropic vs.....

Upper surface suction

Flap gain schedules.

lorem ipsum

But air isn't truly an ideal gas.

Ice formation

But past the drag divergence #.....

Boundary layer separation.

Hysteresis.

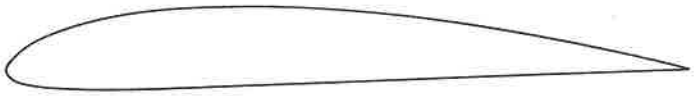
Tunnel turbulence

Crud drag

Eiffel 10 (Wright)

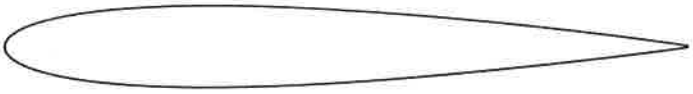
Gottingen 346

CLARK Y

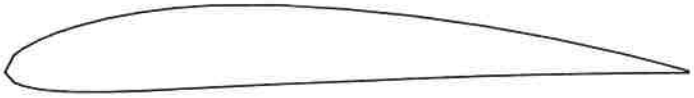


NACA Munk M-4 airfoil

NACA 0012



NACA 4412



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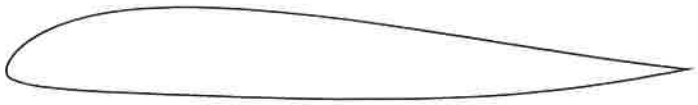
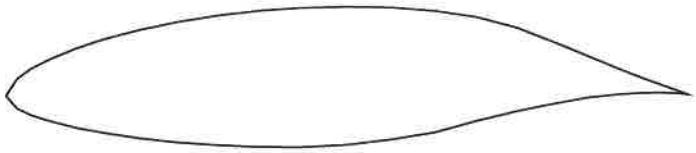


WORTMANN FX 2



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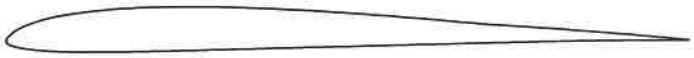
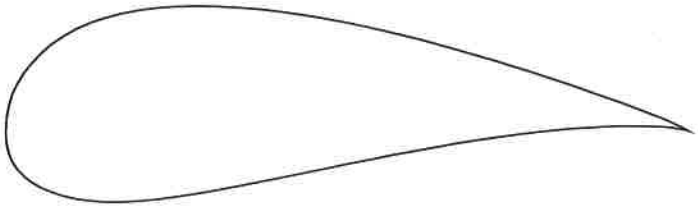
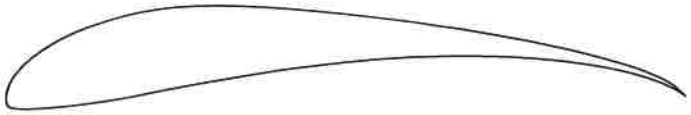


FX 69-PR-281

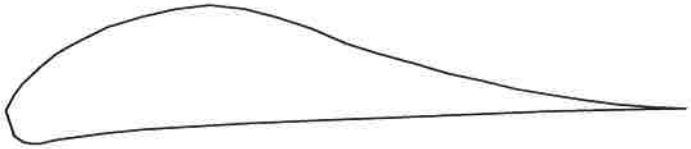


NASA SC(2) 0010

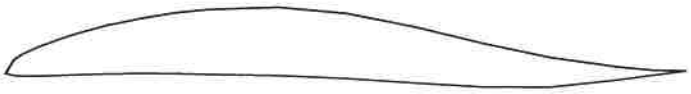
Selig 1223



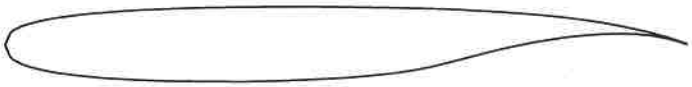
Liebeck L1003



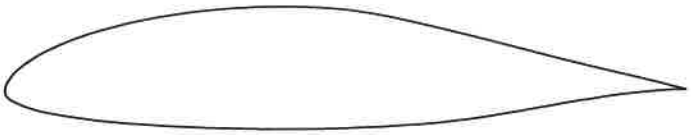
NACA 6-H-10 Rotorcraft



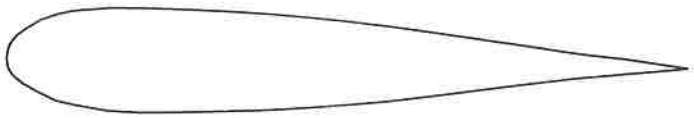
Whitcomb Supercritical



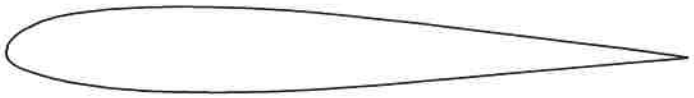
Roncz 1080 Voyager



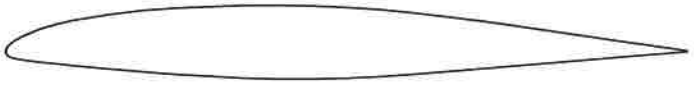
Boeing 737 Root



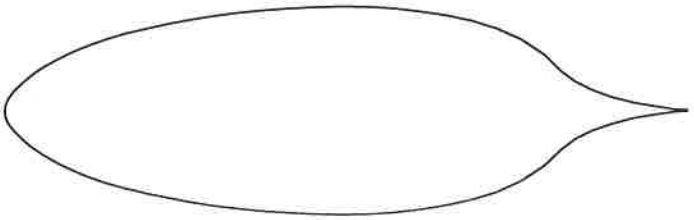
Boeing 737 Midspan



Boeing 737 Tip

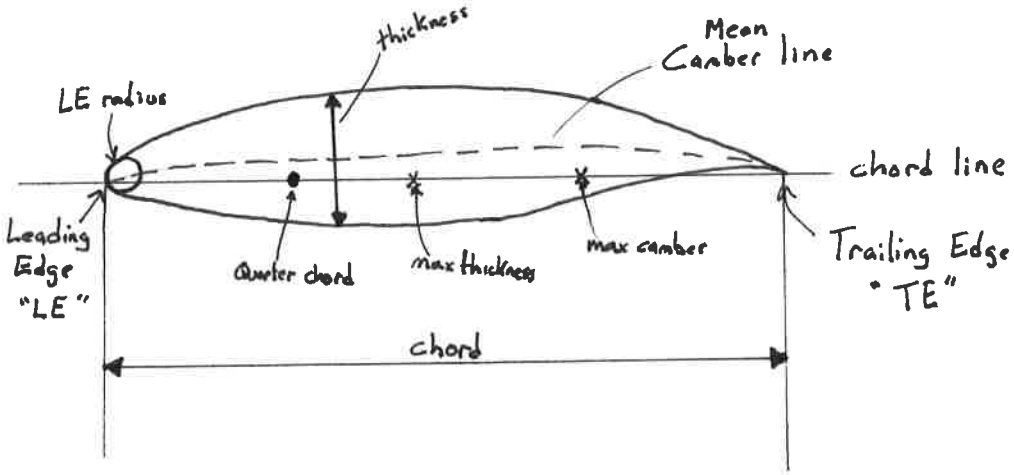


Griffith Suction Airfoil



Airfoil:

- Two-dimensional closed curve
- Cross section of a wing
- Fluid boundary condition (usually air-solid or water-solid)



Thickness ratio:

$$\tau = \frac{\text{Max thickness}}{\text{chord}}$$

usually given in percent (e.g. 12%)

Above FX 2 is $\approx$ 20%

LE radius:

Often, the leading edge is specified as a circular radius to prevent sharp edges in the curve's definition.

Chord line:

The chord line is defined as the line connecting the LE and TE.

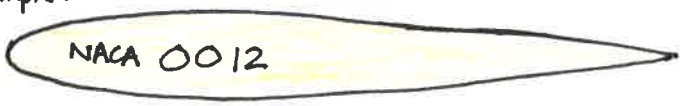
NACA "En - Ay - Cee - Ay"

4 digit airfoils

"When the NACA four digit wing sections were derived, it was found that the thickness distribution of efficient wing sections such as the Göttingen 398 and the Clark Y were nearly the same when their camber was removed ... and they were reduced to the same maximum thickness."

Theory of Wing Sections - Abbott and von Doenhoff

Example:



Code:

NACA $(\epsilon)(p)(T\%)$

1 digit, 1 digit, 2 digits

ϵ: max camber (1%)

p: position of max camber (10%)

$T\%$: thickness in percent (1%)

Thickness

$$T = 10\tau c\left(0.2969\sqrt{\frac{x}{c}} - 0.12600\frac{x}{c} - 0.35160\left(\frac{x}{c}\right)^2 + 0.28430\left(\frac{x}{c}\right)^3 - 0.10150\left(\frac{x}{c}\right)^4\right)$$

- This T gives a <u>blunt</u> TE. Some people replace the 2nd order term to give a coincident TE $\left(-0.35160\left(\frac{x}{c}\right)^2 \rightarrow -0.3537\left(\frac{x}{c}\right)^2\right)$, but technically the definition is <u>correct</u> (cf. NACA TN 385).
- Max thickness at 30%

Camber line

$$y_c = \frac{\epsilon x}{p^2}\left(2p - \frac{x}{c}\right) \quad \text{for} \quad 0 < \frac{x}{c} < p$$

$$y_c = \frac{\epsilon(c-x)}{(1-p)^2}\left(1 + \frac{x}{c} - 2p\right) \quad \text{for} \quad p < \frac{x}{c} < 1$$

2 parabolas joined at max camber pt.

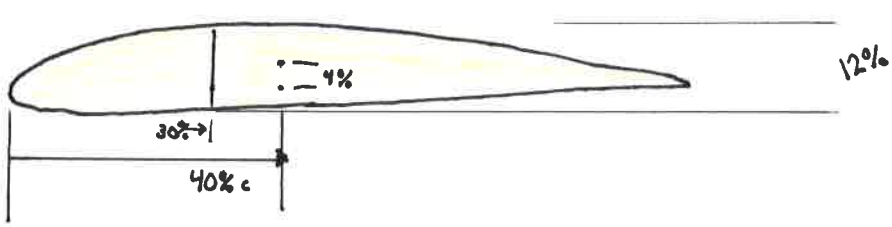
Curves

$$y^+ = y_c + \tfrac{1}{2}T$$

$$y^- = y_c - \tfrac{1}{2}T$$

Example:

NACA 4412: 4 digit NACA airfoil with 4% chord max camber at 40% chord with 12% thickness



Q: Can you have a non-canonical NACA 4 digit airfoil?

$\epsilon = 4.5\%$

$p = 25\%$

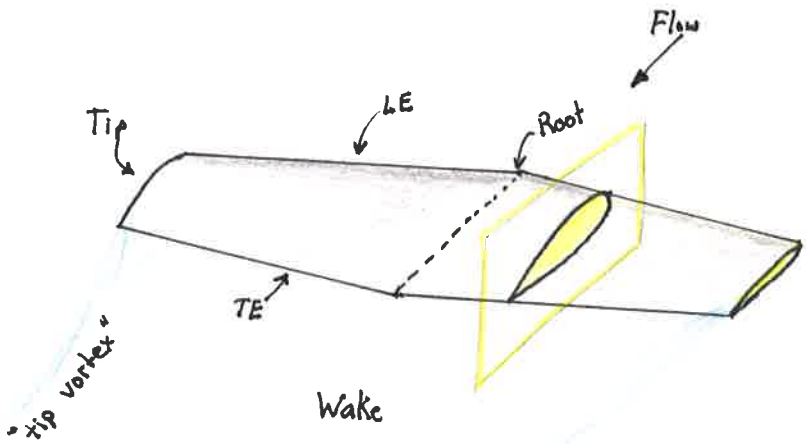
$\tau = 11.25\%$

A: Yes, but it won't be describable by the 4 digit code.

A2: Yes, the shapes are continuous. The code is not.

Wing:

- Three dimensional closed surface
- Generates aerodynamic force
- Cross sections are airfoils

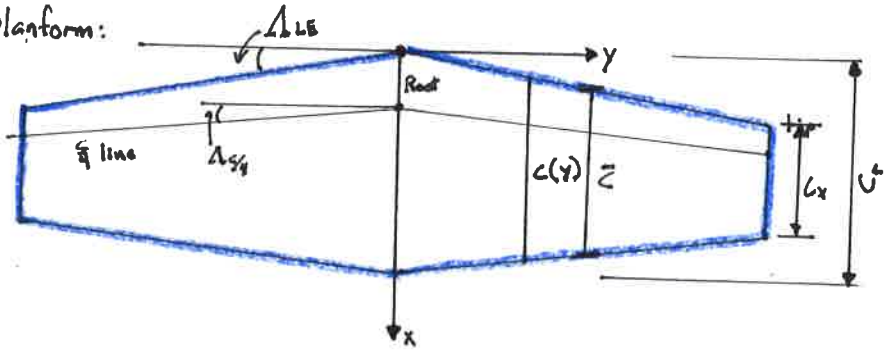


Etymology:

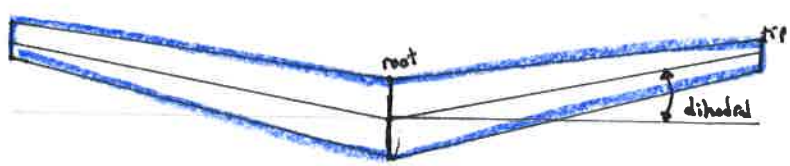
Wing — English
↓
wenge — middle English
↓
vent (english) — Vaenger — Norse
↓ ↓
vent (Latin) — we-ingyaz — Old German
↓
we (to blow) — Indo-European

Interestingly, <u>not</u> from Indo-European word for "fly" which is "peth" from which we get "feather" and "pen".

Planform:



Front view



$S \equiv$ Wing Area $[L^2]$

$b \equiv$ Span $[L]$

$c \equiv$ chord

$AR \equiv$ Aspect Ratio $\equiv \frac{b^2}{S}$

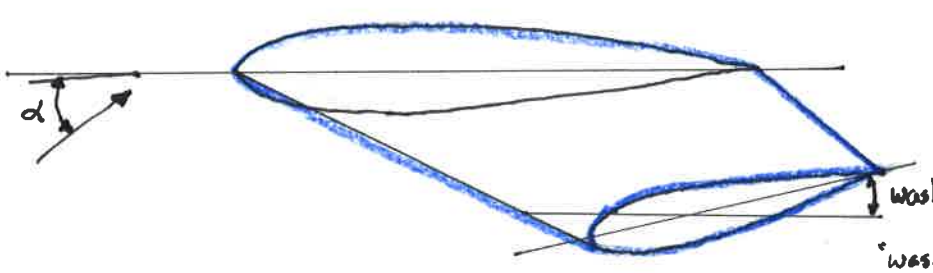
$\lambda \equiv$ taper ratio $\equiv \frac{c_t}{c_r}$, $\left(\frac{\text{tip}}{\text{root}}\right)$

MAC $\equiv$ Mean Aerodynamic Chord

$$\frac{1}{S}\int_{-b/2}^{b/2} c(y)^2\,dy$$

$\bar{c} \equiv$ Average Chord

Side view

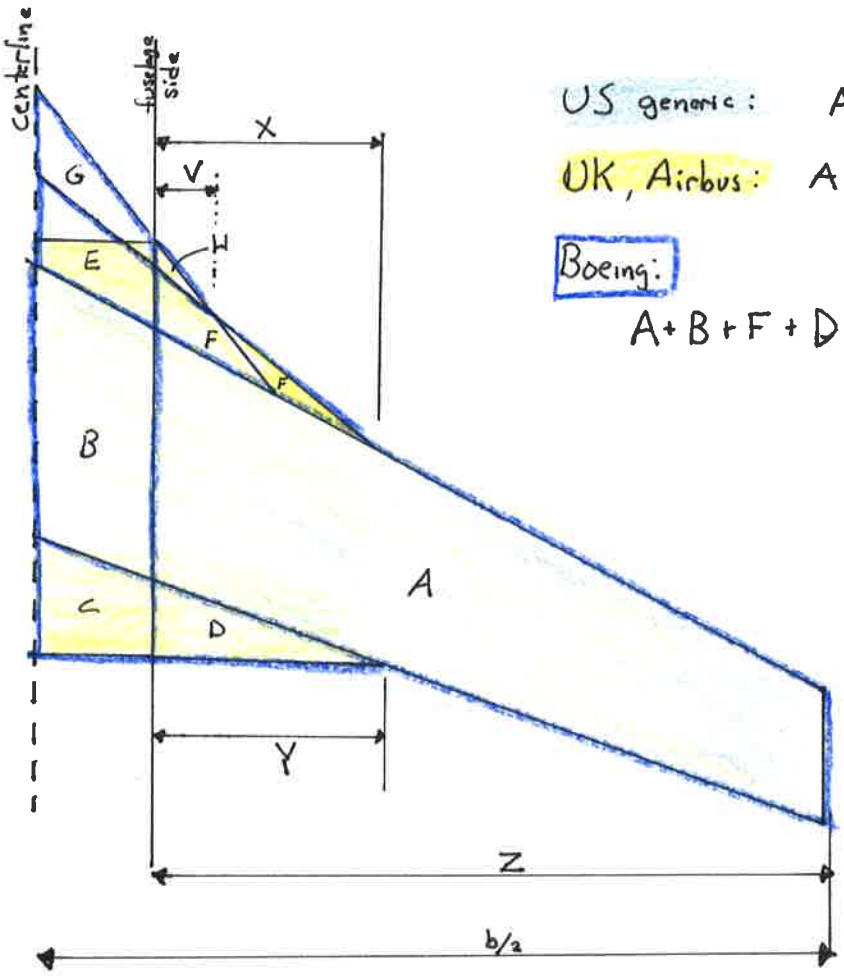


wash out $\equiv$ decrease in incidence angle at tip

"wash in" is a positive increase ... usually a bad idea

Wing Reference Area

ADTA p 269



US generic: A + B

UK, Airbus: A + B + C + D + E + F + H

Boeing:

$$A + B + F + D + H + \frac{X}{Z}E + \frac{Y}{Z}C + \frac{V}{Z}G$$

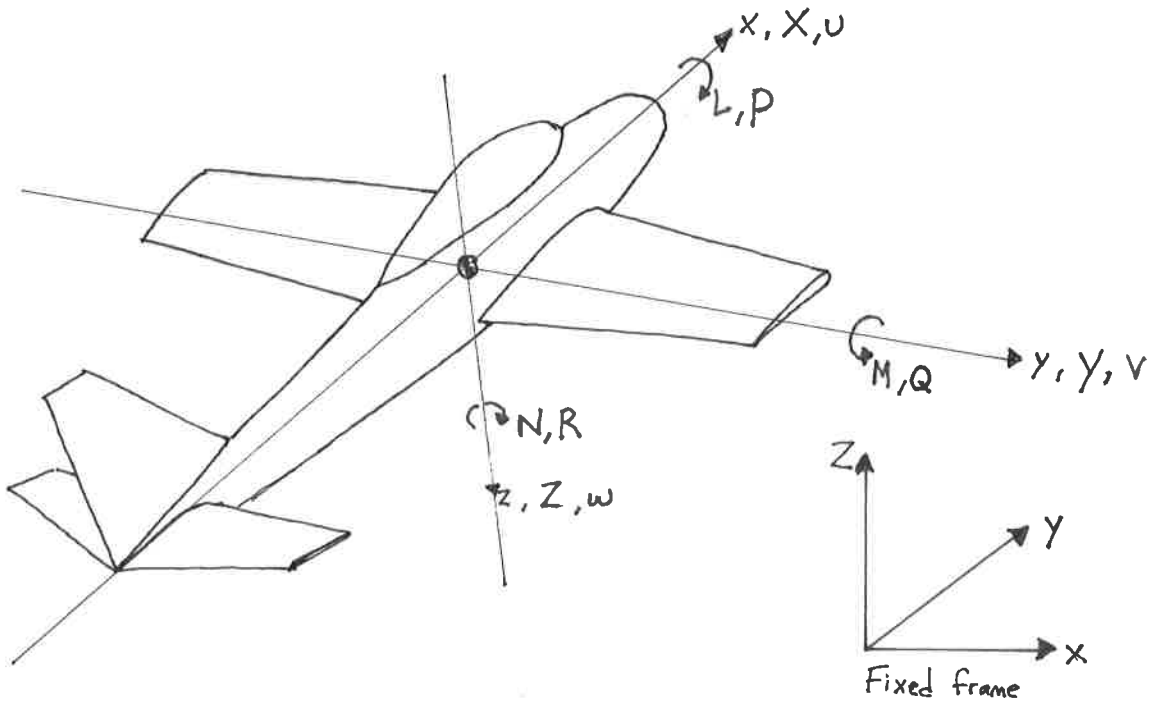
Or, S could be a planform projection from a CAD model.

Or, S could be the reference area of a previous version (i.e. historical).

Why is this important?

- Single most important ref #
- Compare performance

Aircraft Coordinate System



x, y, z	Aircraft "stability" frame location
X, Y, Z	Forces in stability frame
u, v, w	Velocity in stability frame
L, M, N	moment in stability frame
P, Q, R	Angular velocities (roll, pitch, yaw)
ϕ, θ, ψ	Euler angles (orientation)

Warning:

Outside of aerodynamics, the x axis is usually down the length of the a/c.

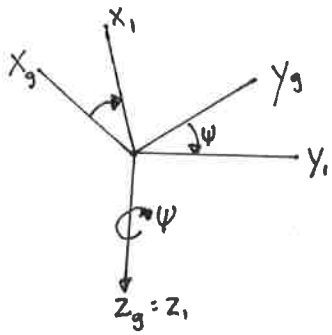
$$X_{loft} = -X_{aero\ stability}$$
$$Y_{loft} = Y_{aero\ stability}$$
$$Z_{loft} = -Z_{aero\ stability}$$

Euler Angles ϕ, θ, ψ (order dependent!)

local to global: yaw(ψ), pitch(θ), roll (ϕ)

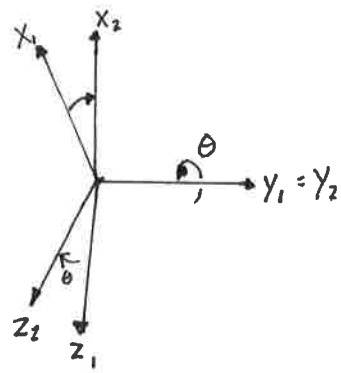
global to local: roll(ϕ), pitch(θ), yaw(ψ)

Yaw



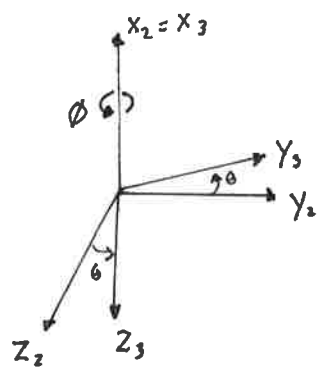
$$\begin{pmatrix} \bar{X}_1 \end{pmatrix} = \underbrace{\begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{T_\psi^{lg}} \begin{pmatrix} \bar{X}_G \end{pmatrix}$$

pitch



$$\bar{X}_2 = \underbrace{\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}}_{T_\theta^{lg}} \bar{X}_1$$

Roll



$$\bar{X}_3 = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}}_{T_\phi^{lg}} \bar{X}_2$$

Global

$$\bar{X}_G = \underbrace{T_\psi^{lg} T_\theta^{lg} T_\phi^{lg}}_{T^{lg}} \bar{X}_L$$

$$T^{lg} = \begin{bmatrix} C_\theta C_\psi & S_\phi S_\theta C_\psi - C_\phi S_\psi & C_\phi S_\theta C_\psi + S_\phi S_\psi \\ C_\theta S_\psi & S_\phi S_\theta S_\psi + C_\phi C_\psi & C_\phi S_\theta S_\psi - S_\phi C_\psi \\ -S_\theta & S_\phi C_\theta & C_\phi C_\theta \end{bmatrix}$$

A beautiful property of transformation matrices is

$$T^{gl} = \left(T^{lg}\right)^{-1} = T^{lg\,T}$$

The inverse is the transpose!

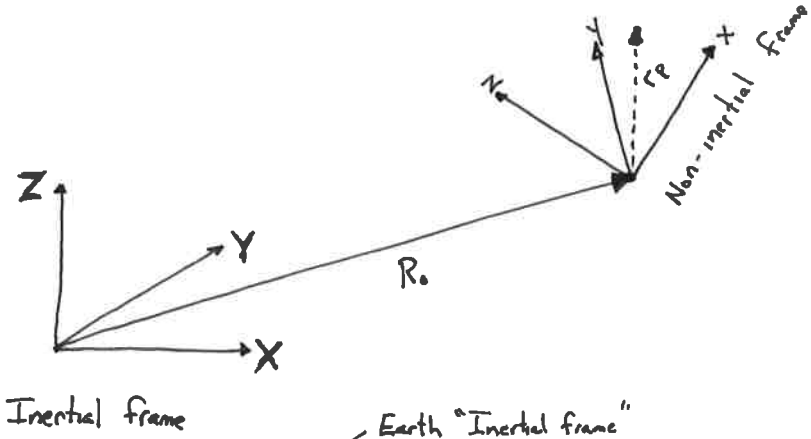
Ex:

Given a global frame velocity of $\bar{V} = (1, 0, 0)$ and the orientation Euler angles of $(\phi, \theta, \psi) = (0, 10°, 10°)$, what is the body frame velocity vector (u, v, w)?

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = T^{gl} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = T^{lg\,T} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} C_\theta C_\psi \\ S_\phi S_\theta C_\psi - C_\phi S_\psi \\ C_\phi S_\theta C_\psi + S_\phi S_\psi \end{pmatrix}$$

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 0.9698 \\ -0.1413 \\ 0.1986 \end{pmatrix}$$

Body Position



Earth "Inertial frame"

this depends on non-inertial frame!

$$R_p^e = R_o^e + r_p^e$$

$$r_p^e = T^{lg} r_p^b$$

$$\boxed{R_p^e = R_o^e + T^{lg} r_p^b}$$

Velocities

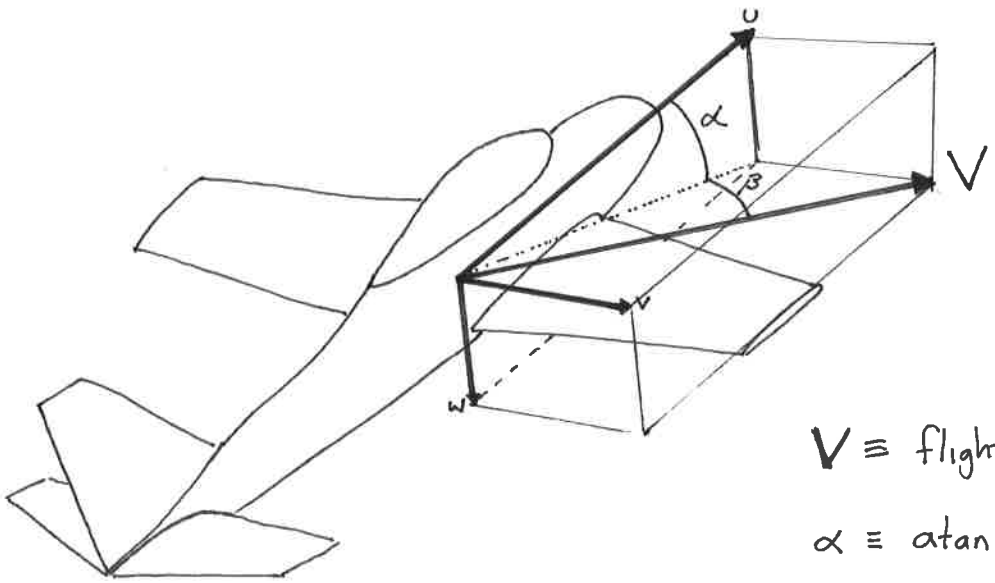
$$\frac{dR_p^e}{dt} = V_p^e = \frac{dR_o^e}{dt} + \frac{dT^{lg}}{dt} r_p^b + T^{lg} \frac{dr_p^b}{dt}$$

$$= V_o^e + \cdots$$

$$= T^{lg} V^b + \cdots$$

$$= \underbrace{T^{lg} V^b + \frac{dT^{lg}}{dt} r_p^b + T^{lg} \frac{dr_p^b}{dt}}_{\text{All terms in body frame!}}$$

Angle of Attack and Sideslip



$\mathbf{V} \equiv$ flight velocity vector

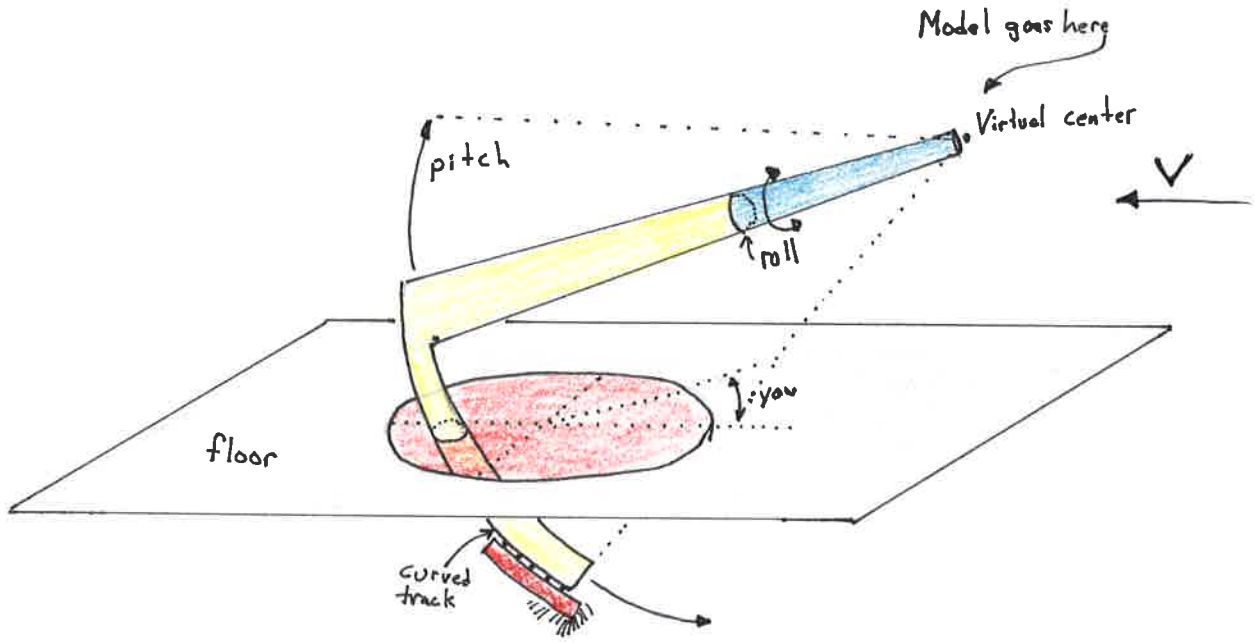
$\alpha \equiv \operatorname{atan}\left(\frac{w}{u}\right)$ Angle of Attack

$\beta \equiv \operatorname{asin}\left(\frac{v}{V}\right)$ Sideslip

α is defined wrt the projection of V onto the body frame (i.e. u)

β is defined wrt the v projection and V.

Wind-Tunnel Mount



What order is the transformation?

Floor → Yaw → pitch → Roll (ie. Euler angles!)

Ex: What mount rotations are needed for $\alpha = 30°$, $\beta = 10°$?

$$\alpha = \operatorname{atan}\left(\frac{w}{u}\right) = 30°$$

$$\beta = \operatorname{asin}\left(\frac{v}{\bar{V}}\right) = 10°$$

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = V \begin{pmatrix} C_\theta C_\psi \\ S_\phi S_\theta C_\psi - C_\phi S_\psi \\ C_\phi S_\theta C_\psi + S_\phi S_\psi \end{pmatrix}$$

so, $v = \bar{V} \sin\beta$

and

$$u^2 + \bar{V}^2 \sin^2\beta + u^2 \tan^2\alpha = \bar{V}^2 \Rightarrow u = \bar{V}\sqrt{\frac{1-\sin^2\beta}{1+\tan^2\alpha}} \Rightarrow w = \bar{V}\sqrt{\frac{1-\sin^2\beta}{1+\tan^2\alpha}}\tan\alpha$$

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 0.852869 \\ 0.173648 \\ 0.492404 \end{pmatrix}$$

if $\phi = 0°$ (no roll)

$$\theta = \alpha = 30°$$

$$\psi = -\beta = -10°$$

ϕ is an independent variable

Verify $\theta = \alpha$ $\psi = -\beta$ when $\phi = 0°$ no roll

$$u = V C_\theta C_\psi$$

$$v = V\left(\cancelto{0}{S_\phi} \cdots - \cancelto{1}{C_\phi} S_\psi\right) = -V S_\psi$$

$$w = V\left(S_\theta C_\psi + 0\right) = V S_\theta C_\psi$$

$$\alpha = \operatorname{atan}\left(\frac{w}{u}\right) = \operatorname{atan}\left(\frac{V S_\theta C_\psi}{V C_\theta C_\psi}\right) = \operatorname{atan}\left(\tan(\theta)\right)$$

$$\boxed{\alpha = \theta}$$

$$\beta = \operatorname{asin}\left(\frac{v}{\bar{V}}\right) = \operatorname{asin}\left(\frac{-V S_\psi}{V}\right) = \operatorname{asin}\left(-S_\psi\right) = -\psi$$

$$\boxed{\beta = -\psi}$$

A demonstration of Gimbal Lock.

The transformation matrix when pitch $\theta = 90°$ is

$$T^{lg} = \begin{bmatrix} 0 & S_\phi C_\psi - C_\phi S_\psi & C_\phi C_\psi + S_\phi S_\psi \\ 0 & S_\phi S_\psi + C_\phi C_\psi & C_\phi S_\psi - S_\phi C_\psi \\ -1 & 0 & 0 \end{bmatrix}$$

Remember that

$$\cos(A \mp B) = \cos(A)\cos(B) \pm \sin(A)\sin(B)$$

and

$$\sin(A \pm B) = \sin(A)\cos(B) \pm \cos(A)\sin(B)$$

substitute

$$T^{lg} = \begin{bmatrix} 0 & S(\phi - \psi) & C(\phi - \psi) \\ 0 & C(\phi - \psi) & -S(\phi - \psi) \\ -1 & 0 & 0 \end{bmatrix}$$

Uh-oh! T^{lg} is only a function of $\phi - \psi$

We can not distinguish between ϕ and ψ when $\theta = 90°$!

Apollo 11 transcript

04 08 46 11	CC	Hello, Tranquility Base. Houston. On our DPS venting and that fuel problem, our heat exchanger is cleared up. We heard that the ice is melted, and we are in good shape now. Out.
04 08 49 39	CDR (TRANQ)	Houston. Tranquility is going to put the track modes in POO now.
04 08 59 27	CC	Columbia, Houston. Over.
04 08 59 34	CMP (COLUMBIA)	Columbia. Go.
04 08 59 35	CC	Columbia, Houston. We noticed you are maneuvering very close to gimbal lock. I suggest you move back away. Over.
04 08 59 43	CMP (COLUMBIA)	Yes. I am going around it, doing this CMC AUTO maneuvers to the PAD values of roll 270, pitch 101, yaw 45.
04 08 59 52	CC	Roger, Columbia.
04 09 00 30	CMP (COLUMBIA)	How about sending me a fourth gimbal for Christmas.
04 09 00 40	CC	Columbia, Houston. You were unreadable. Say again please.
04 09 00 46	CMP (COLUMBIA)	Disregard.
04 09 01 21	CC	Columbia, Houston. Several items for you. Over.
04 09 01 28	CMP (COLUMBIA)	Ready to copy.
04 09 01 30	CC	Columbia, Houston. First of all, we'd like a waste-water dump to 10 percent on the backside. Secondly, it does not look like we are going to need any plane change at this time, so we will not be uplinking a new REFSMMAT. Third item, I would like all of your CRYO heaters to AUTO, and we are ready for a battery charge, battery BRAVO; it will last about 7 hours. If you should go to sleep, we will be terminating that BATT charge, but at the moment, we can go ahead and start the BATT charge on BATT Bravo. And a final item, for your SM RCS configuration for your rest period, register 1 for the DAP is 11111; DAP register 2, 01100. And your AUTO RCS select

days hours min seconds

Since liftoff

CC = CAPCOM

CDR = N. Armstrong

CMP = M. Collins