

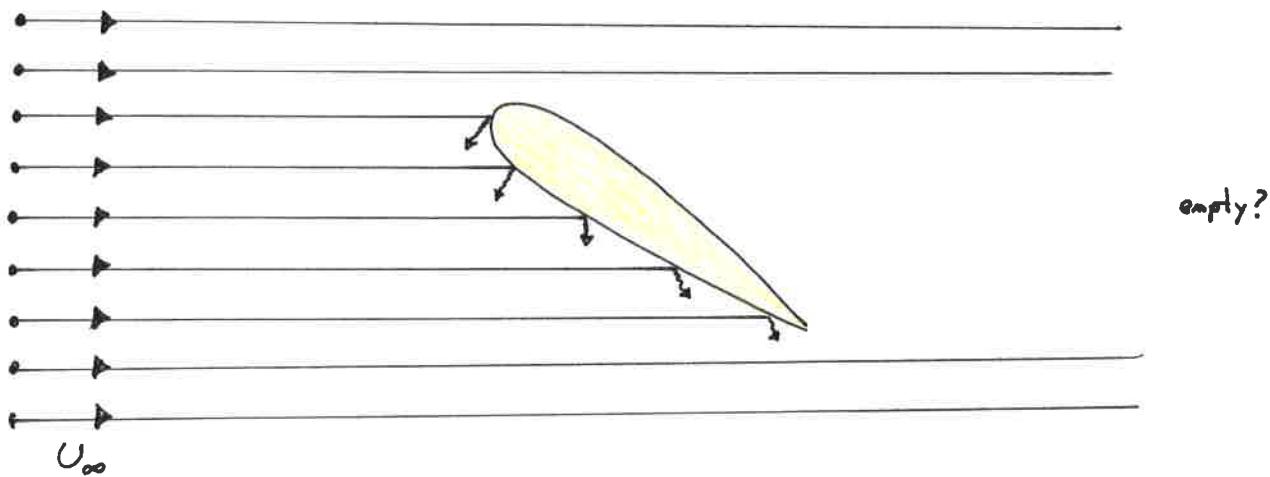
Lecture 2

21st Aug 2015

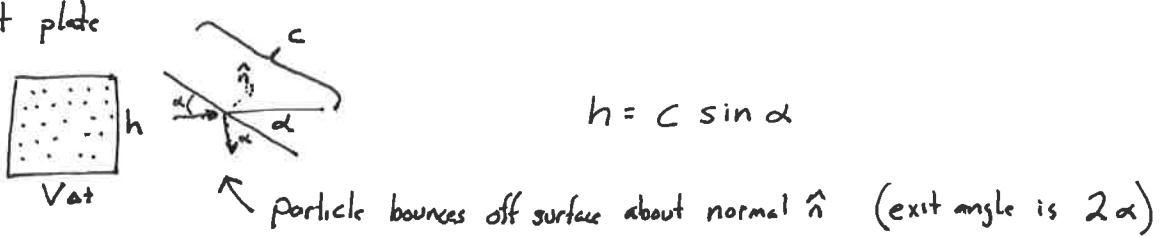
- Ideal Gas

Newton's Lift Model

Non-interacting kinetics



For a simple flat plate



particle bounces off surface about normal \hat{n} (exit angle is 2α)

Impulse ($J = \Delta p = \Delta(mv)$)

$$J = \underbrace{\rho V \Delta t h}_{\text{mass}} \sqrt{ } \left[(\cos 2\alpha - 1) \uparrow - \sin 2\alpha \uparrow \right]$$

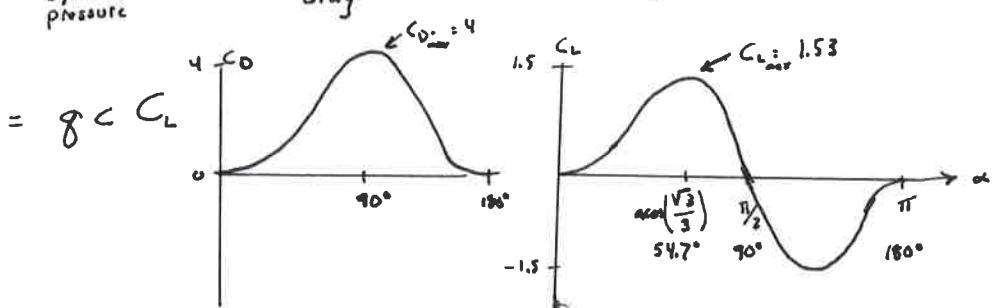
$\underbrace{\qquad\qquad\qquad}_{\text{momentum}}$

Average Force

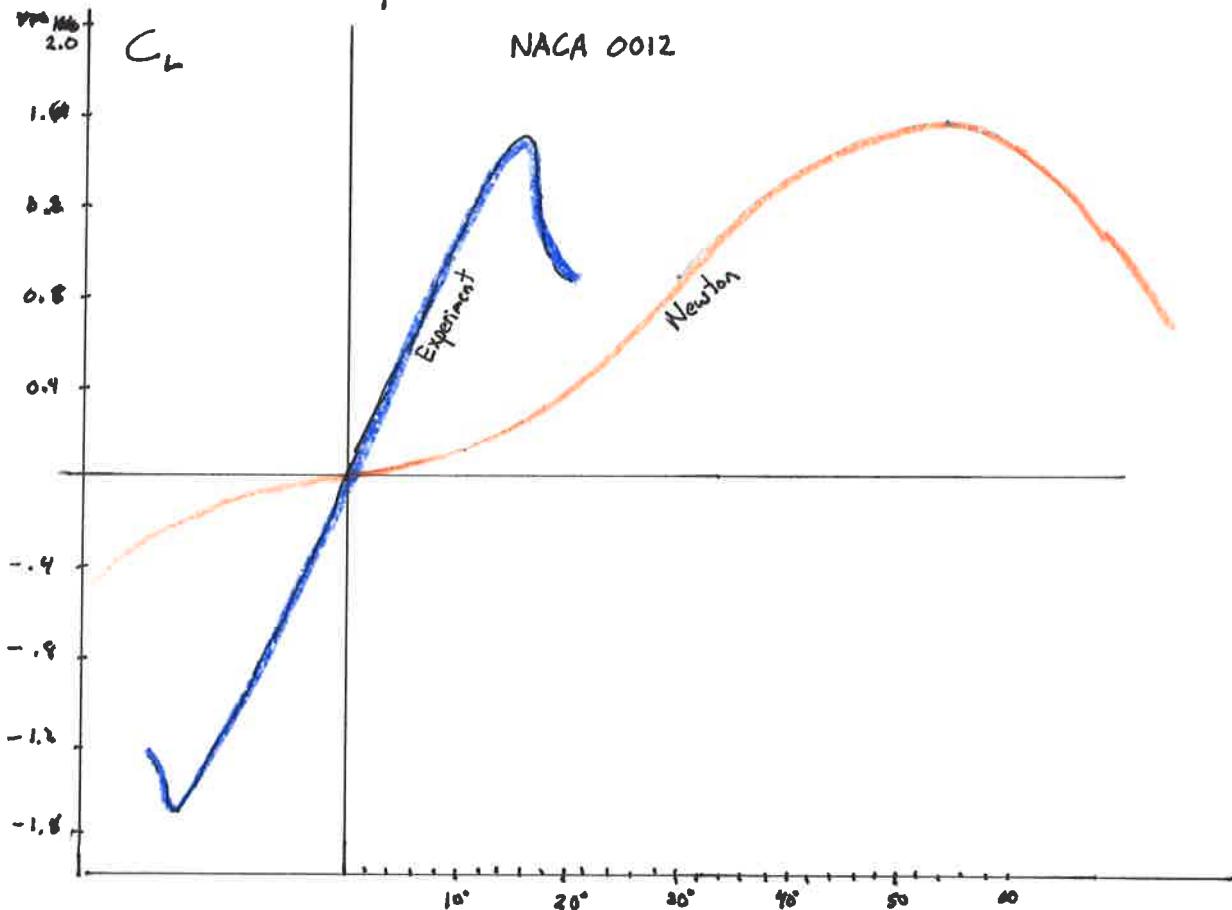
$$F = -\frac{J}{\Delta t} = \rho V^2 C \sin \alpha \left[(\cos 2\alpha - 1) \uparrow - (\sin 2\alpha) \uparrow \right]$$

$$= \frac{1}{2} \rho V^2 C \left[\underbrace{(4 \sin^3 \alpha) \uparrow}_{\text{drag}} + \underbrace{(4 \sin^2 \alpha \cos \alpha) \uparrow}_{\text{lift}} \right]$$

$\underbrace{\qquad\qquad\qquad}_{\text{dynamic pressure}}$



Does this match experimental data?



Wrong Physics

What does it get wrong?

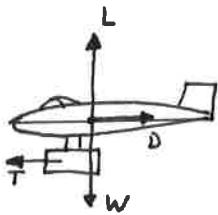
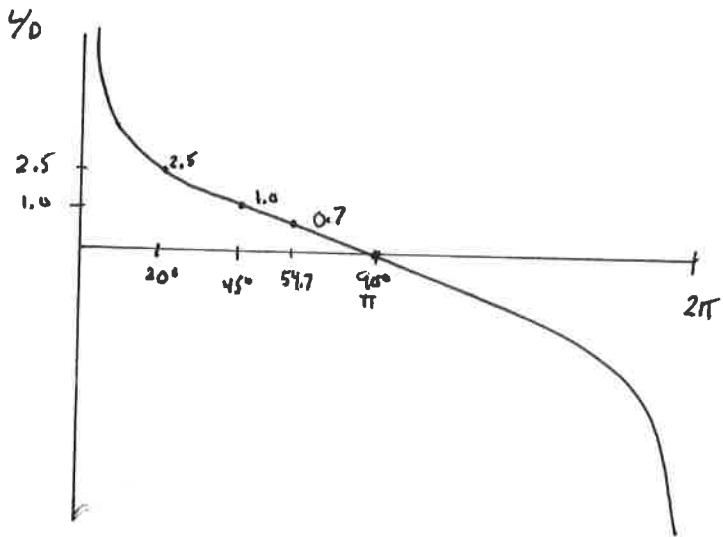
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What does it get right?

-
-

$\frac{L}{D}$ (i.e. how powerful of a propulsion system do we need?)
i.e. Thrust gain / amplifier.

$$\frac{L}{D} = \frac{4 \sin^2 \alpha \cos \alpha}{4 \sin^3 \alpha} = \frac{4 \cos \alpha}{4 \sin \alpha} = \frac{1}{\tan \alpha}$$



$$T = D$$

$$W = L$$

$$T = D = L \left(\frac{L}{D} \right)^{-1} = W \left(\frac{L}{D} \right)^{-1}$$

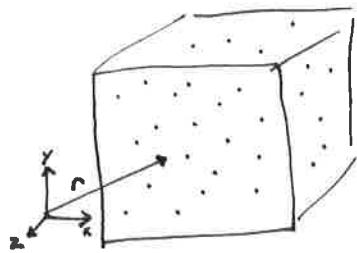
$$\frac{T}{W} = \left(\frac{L}{D} \right)^{-1}$$

$$\therefore \frac{W}{T} = \frac{L}{D}$$

In Newton's world, aircraft would fly fast at low α . Maximum lift would be at $\approx 54^\circ$ with an L/D of 0.7. Engines would need to be gigantic. Aero design is boring...

This world exists, but not at the bottom of our atmosphere.

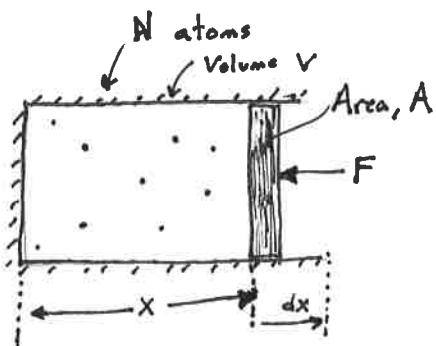
Molecular Dynamics



Molecules have a location and a velocity (6 states)
 "phase space"
 $V = (V_x, V_y, V_z)$
 $X = (x, y, z)$

Number density $n(r) = \lim_{\Delta r^3} \frac{\Delta^3 N}{\Delta r^3} = \frac{\text{How many in a volume}}{\text{How big is a volume}}$

pressure



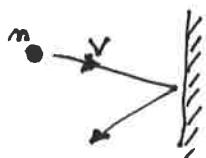
$$P = \frac{F}{A}$$

$$n = \frac{N}{V}$$

Work = Force · distance

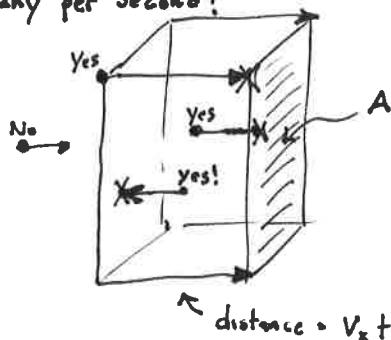
$$dW = F(-dx) = -PA dx = -P dV$$

One atom



$$\begin{aligned} J &= \Delta(mv) \\ &= [m(-V_x)] - [mV_x] = -2mV_x \end{aligned}$$

How many per second?



Total force on A

"Volume" of atoms hitting a wall = $V_x t A$

Number of atoms per second

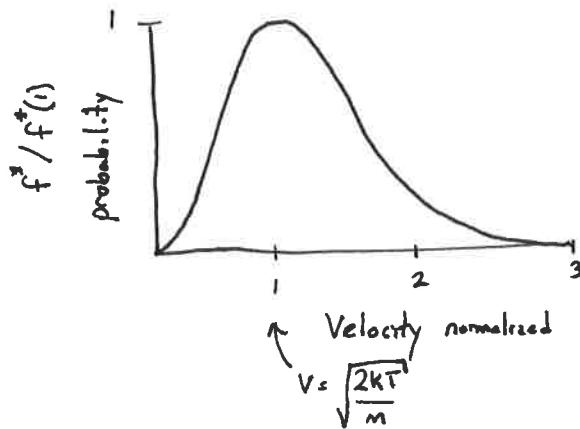
$$N = n V_{\text{box}} = n V_x t A$$

$$F = J \cdot N = -2mV_x \cdot nV_x A \Rightarrow P = \frac{F}{A} = -2mnV_x^2$$

But not all atoms move at same speed !!

Velocity Distribution

Maxwell Boltzmann distribution



$$f^*(v) = 4\pi v^2 \left(\frac{m}{2\pi kT} \right)^{3/2} e^{-\frac{mv^2}{2kT}}$$

For more information, refer to a gas kinetics source. These notes only mention this topic in passing.

The most likely speed is not the average velocity.

Rather, take an average of velocity squared.

$$P = nm \langle V_x^2 \rangle \quad \text{since we only want atoms hitting one wall.}$$

If we make the assumption that the atomic velocities are directionally identical (ok for most cases, ~~most~~ in aerodynamics)

$$P = \frac{2}{3} n \left\langle \frac{mv^2}{2} \right\rangle \quad \text{well, } \frac{mv^2}{2} \text{ is kinetic energy}$$

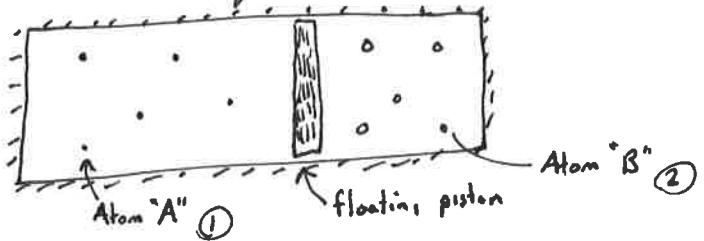
$$= \frac{2}{3} \frac{N}{V} \left\langle \frac{mv^2}{2} \right\rangle$$

$$\nwarrow \text{call this term } (\gamma-1) \frac{N}{V} \quad \gamma = \frac{5}{3} \text{ monatomic gas}$$

warning! physics definition of $(\gamma-1)$

Temperature

Q: What are the equilibrium conditions here?



A: Pressure on piston must be equal

$$P_1 = (\gamma - 1) \frac{N_1}{V_1} \left\langle \frac{m_1 v_i^2}{2} \right\rangle = P_2 = (\gamma - 1) \frac{N_2}{V_2} \left\langle \frac{m_2 v_i^2}{2} \right\rangle$$

$$n_1 \left\langle \frac{m_1 v_i^2}{2} \right\rangle = n_2 \left\langle \frac{m_2 v_i^2}{2} \right\rangle$$

A: But the piston reacts to the atomic impact and thus the KE must be equal.

$$n_1 \left\langle \frac{m_1 v_3^2}{2} \right\rangle = n_2 \left\langle \frac{m_2 v_3^2}{2} \right\rangle$$

Isothermal

where v_3^2 is average of KE_1 and KE_2

Define temperature as a linear function of kinetic energy

$$T = C \cdot \left\langle \frac{m v^2}{2} \right\rangle$$

Rearrange to

$$\Delta U = C_V \Delta T$$

↑ Internal energy ↑ Specific heat ↑ Temp

Ideal Gas

From before, we had the pressure from atomic impact as

$$P = C \cdot \frac{N}{V} \cdot \left\langle \frac{mv^2}{2} \right\rangle$$

↓ ↑
Rearrange to ↓

$$PV = N \cdot C \cdot T$$

Ideal Gas law

$$PV = N R T$$

also

$$\underline{P = \rho R T}$$

R depends on the gas

$$R = \frac{\bar{R}}{M} \quad \text{and} \quad \bar{R} = 8.314 \times 10^3 \frac{\text{J}}{\text{mol K}}$$

$$= 1545.34 \frac{\text{ft lb}}{\text{lb mol}}$$

where ρ is density (aka $\frac{N}{V M}$)

Example:

Compute the pressure of air with density $1.2 \frac{\text{kg}}{\text{m}^3}$ at 300°K

$$P = \frac{1.2 \frac{\text{kg}}{\text{m}^3} \cdot 8.314 \times 10^3 \frac{\text{J}}{\text{mol K}}}{28.97 \frac{\text{J}}{\text{mol K}}} \cdot 300 \text{ K} \cdot \frac{1 \text{ kPa}}{1000 \frac{\text{atm}}{\text{J}}} \cdot \frac{\text{N m}}{\text{J}} \cdot \frac{\text{N}^2 \text{ Pa}}{\text{N}}$$

$$\boxed{P = 103 \text{ Pa}}$$

Compute the pressure of air with density $0.00237 \frac{\text{slug}}{\text{ft}^3}$ at 500°R

$$P = \frac{0.00237 \frac{\text{slug}}{\text{ft}^3} \cdot 1716 \frac{\text{ft}^2}{\text{ft}^2 \text{ R}} \cdot 500 \text{ R} \cdot 1 \frac{\text{lb ft}^2}{\text{slug ft}}} {144 \frac{\text{ft}^2}{\text{in}^2}}$$

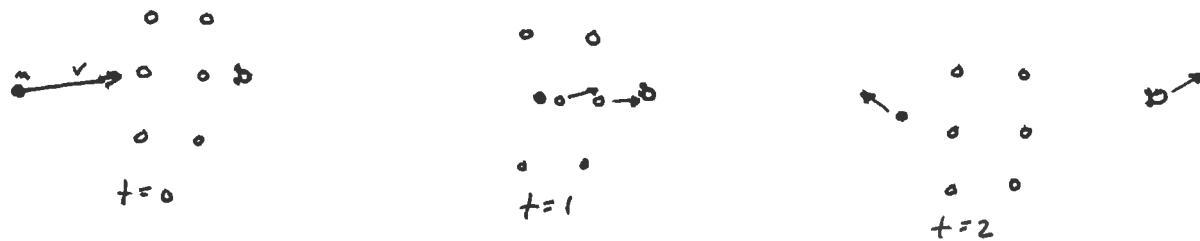
$$\boxed{P = 14 \text{ psi}}$$

Reality

- Not all gases are monatomic.

Air is a mixture.

- Walls are composed of atoms



Heat (aka velocity distributions) "leak"

- Tracking individual atoms is tedious

Too small of scale

$$N \equiv N$$
$$\xrightarrow{109.7 \text{ pm}} = 109700 \text{ nm}$$

How small? The Knudsen # gives an indication of the ratio of the mean free path λ to the geometry scale.

$$\lambda_{SSL} \approx 65 \text{ nm}$$

- Ideal Gasses are common in aerodynamics at moderate temperatures

$$P = \rho R T$$

$$e = C_v T$$

$$h = C_p T$$

$$\gamma = \frac{C_p}{C_v}$$