

Lesson 8

Lumped Models

Readings

FVA Chapter 2

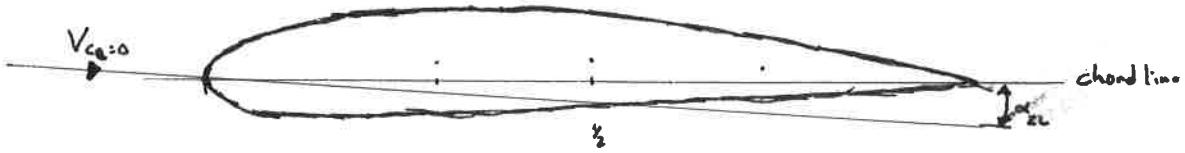
FVA Appendix D

No class on Monday, Labor Day in US. (14th Sept)

Homework due 18th Sept

Ex: Joukowski

Given an airfoil profile, estimate $C_{L\alpha}$, x_{ac} , and $C_{m\frac{1}{4}c}$ and α_{zL}
Also, estimate $C_L(\alpha=0)$



Measure

$$\text{Chord} \approx 102 \text{ mm}$$

Max thickness. $\approx 14 \text{ mm}$ at 30 mm chord

At half chord, the maximum camber is about 3 mm (and importantly, max camber is at $\frac{1}{2}c$)

Non-dimen'.

$$\text{chord} = \frac{\text{Chord}}{102} = 1$$

$$\frac{t_c}{c} = \frac{14}{102} = 13.7\%$$

$$\frac{f_c}{c} = \frac{3}{102} = 3\%$$

Joukowski parameters

$$\epsilon \approx 77\% \frac{t_c}{c} = 0.77 \cdot 0.137 = 0.105$$

$$C_{L\alpha} \approx \frac{2\pi(1+\epsilon)}{1+\epsilon^2} = \frac{2\pi(1.105)}{1.01} = 2\pi \cdot 1.094 = 0.12 \text{ per degree}$$

$$x_{ac} = \frac{1}{4} + \frac{\epsilon^2}{2} = \frac{1}{4} + 0.011 \approx 0.26 \approx 26\% c$$

$$C_{m\frac{1}{4}c} \approx -\pi \frac{f_c}{c} = -\pi \cdot 0.03 = -0.094$$

$$\alpha_{zL} \approx -\frac{2f_c}{c} = -2 \cdot 0.03 = -0.06 \text{ rad} = -3.4 \text{ deg} \Rightarrow C_L(\alpha=0) = 0.12 \cdot 3.4 = 0.41$$

This is the Goe 629.

What is a vortex? What is the relationship between vorticity and vortices?

Vorticity $\equiv \nabla \times F = \text{curl } F \Rightarrow$ measure of the rotation rate at a particular location

Ex. BL

$u=0$

$\frac{du}{dx} = \frac{du}{dy}$

Ex: $\nabla p \times \nabla p$

An infinitesimal vorticity element corresponds to the following velocity field

$$V_\omega = \frac{1}{4\pi} \iiint |\omega| \times \frac{r-r'}{(r-r')^3} dx' dy' dz' \Rightarrow dV_\omega = \frac{1}{4\pi} |\omega| \times \frac{r-r'}{(r-r')^3} dx' dy' dz'$$

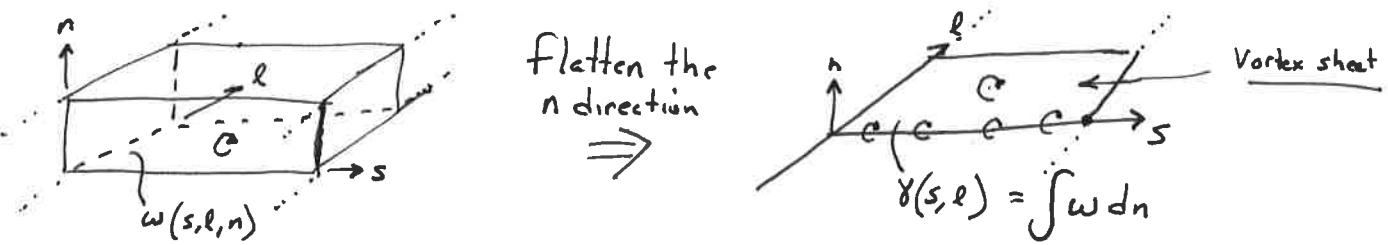
This reverse operation is far too difficult for any practical use in an analytical approach.

A word about "induced" velocity.

A blob of vorticity at a point is not inducing velocity at a distance!

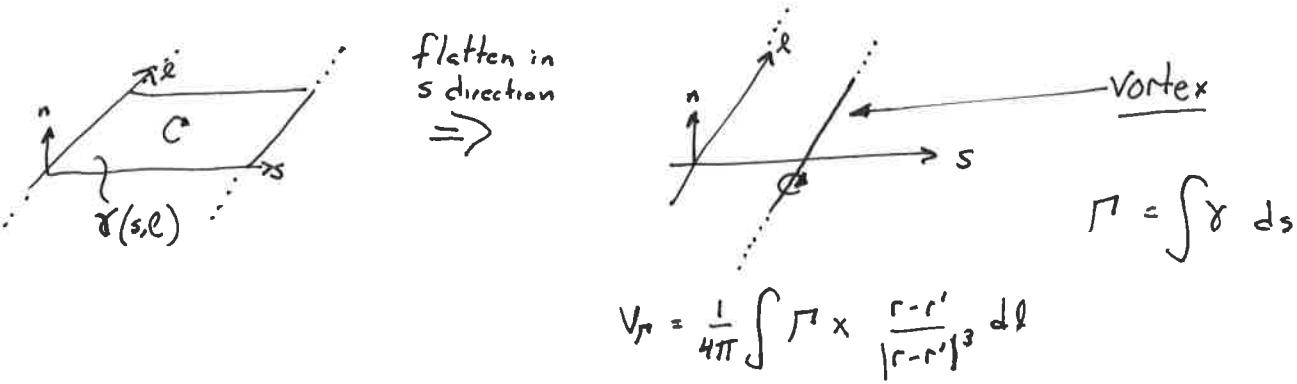
Vorticity is only a point by point measure of the rotation of a fluid particle.

Vorticity Lumping (FVA Fig 2.3)



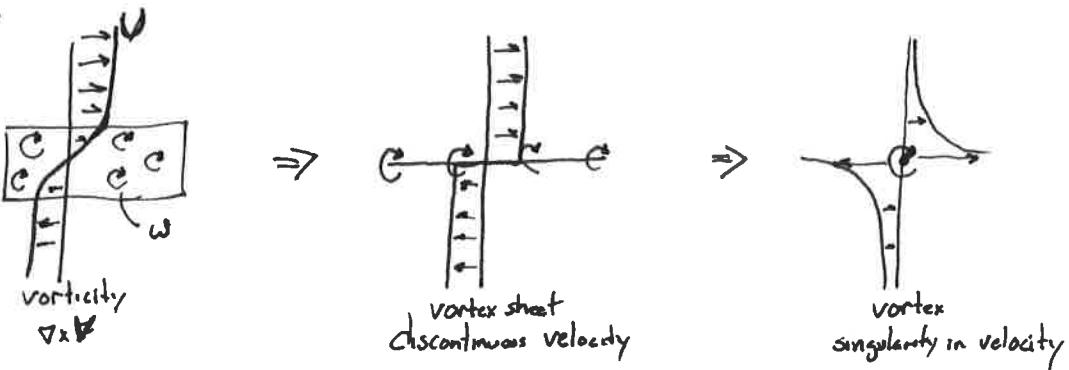
$$V_{\omega} = \frac{1}{4\pi} \iiint \omega(r') \times \frac{r-r'}{|r-r'|^3} dx' dy' dz' \Rightarrow V_{\gamma} = \frac{1}{4\pi} \iint \gamma \times \frac{r-r'}{|r-r'|^3} ds dl$$

We compressed all vorticity along an n line (constant s and l) to a ^{pointwise} γ sheet (s, l).



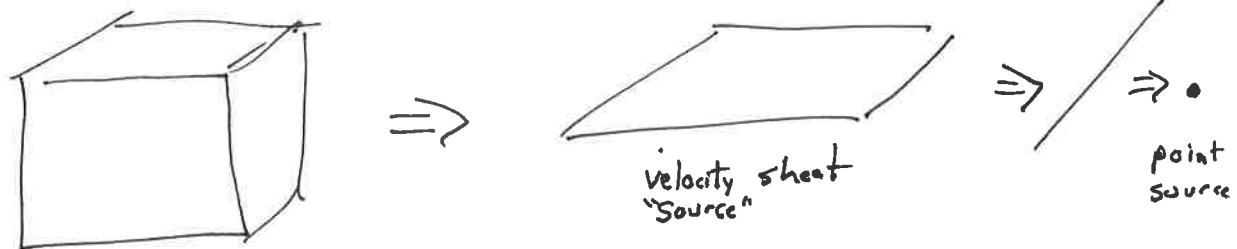
A vortex is a conceptual object generated by lumping vorticity in two directions. Strictly speaking, a vortex is an impossible object (not physically possible).

Why?



$$\nabla \cdot \omega = \nabla \cdot (\nabla \times \mathbf{v}) = 0$$

Lump Source



$$\sigma = \operatorname{div} V = \nabla \cdot V$$

