

# Lesson 10

## Thin Airfoil Theory

HW 2 delay to M<sub>1</sub> 21<sup>st</sup>

# Aerodynamic Center from Experimental Data

a.c.  $\equiv$  the location where  $C_m$  is constant with  $C_e$

$$\frac{dC_m}{dC_e} = 0$$

For an airplane, we call the a.c. by a different name, the Neutral point. The concept is the same.

Non-dim:

$$C_e = \frac{L}{\frac{1}{2} C^2} \quad C_m = \frac{M}{\frac{1}{2} C^2} \Rightarrow \frac{dC_m}{dC_e} \frac{\frac{1}{2} C^2}{\frac{1}{2} C^2} = \frac{dM}{dL} = \text{distance}$$

Visually



If the moment increases with force, where can we move the measurement point such that the increase in  $M$  wrt  $L$  decreases?

$$\text{want } \frac{dM}{dF} = 0 \Rightarrow \frac{d}{dF}(M - F \cdot d) = -d$$

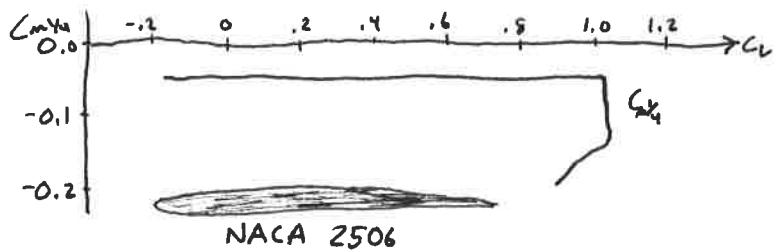
The a.c. is at:

$$a.c. = X_{\text{measured}} - \left. \frac{dC_m}{dC_e} \right|_C C_{\text{measured}}$$

Usually, the measurement point is at the  $\frac{1}{4}c$  ("quarter chord") or MAC if an airplane.  
 $\uparrow$  mean aerodynamic chord.

$$a.c. = \frac{1}{4}c - \left. \frac{dC_m}{dC_e} \right|_C c$$

## Experimental Data



$$C_{m_0} (c_a=0) \approx -0.05$$

$$\frac{dC_m}{dc_a} (c_a=0) \approx 0$$

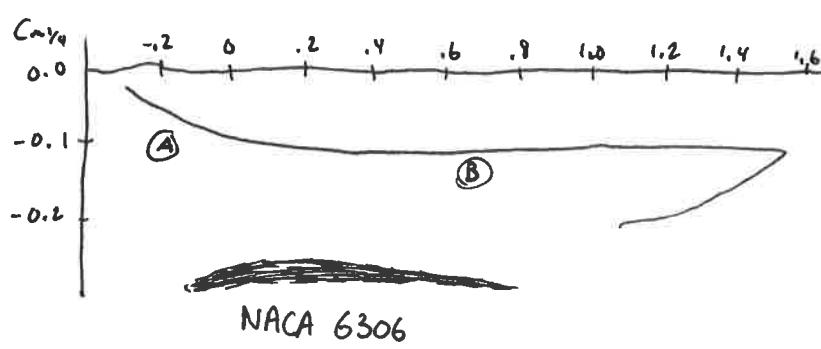
$$a.c. = \frac{1}{4}c - 0 = 0.25$$

At stall,  $\frac{dC_m}{dc_a}$  is strongly negative.

The a.c. at stall shifts aft.

After stall,  $\frac{dC_m}{dc_a}$  is strongly positive!! and  $C_m$  is negative.

No warning and nasty behavior... avoid ?!?



$$a.c. = \frac{1}{4}c + \frac{1}{4}c = \frac{1}{2}c$$

$$a.c. = \frac{1}{4}c \quad \text{at } (B)$$

$$\frac{dC_m}{dc_a} \approx \frac{\Delta C_m}{\Delta c_a} = \frac{0.025}{1.4} \approx 0.018$$

$$a.c. = \frac{1}{4}c - 0.018c$$

$$a.c. = 23\% c$$

Joukowski:

$$a.c. \approx \frac{1}{4} + \frac{\epsilon^2}{2}$$

$$\approx \frac{1}{4} + \frac{1}{4} \left(\frac{d}{c}\right)^2$$

$$a.c._{21\% \text{ thick}} \approx 26\%$$

a.c. shift with  $\frac{d}{c}$  for the NACA 4 and 5 digit airfoils, is the opposite of Joukowski!

## Extended Thin Airfoil Theory

Using a combination of a source and vortex sheet, we can form an inviscid solution to a "thin" airfoil of arbitrary camber and thickness.

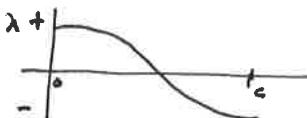
$$\begin{matrix} \uparrow V_\infty \\ \rightarrow U_\infty \end{matrix}$$



To do this, we need a source sheet

$$+ + + + - - -$$

where

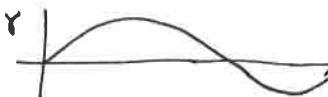


$$\lambda(x) = f(x)$$

and a vortex sheet

$$c c c c c \dots$$

where



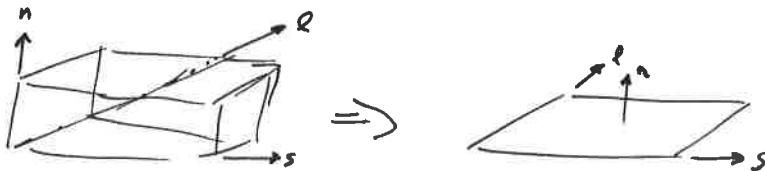
$$\gamma(x) = f(x)$$

such that 1) the flow exactly creates a streamline along the surface  
and

2) The Kutta condition is satisfied.

# Sheet Jumps

Remember, a sheet is a 3D field compressed in one-dimension to give a 2D approximation.



$$V_\sigma = \frac{1}{4\pi} \iiint \sigma(r') \frac{r-r'}{|r-r'|^3} dx' dy' dz' \Rightarrow V_\lambda = \frac{1}{4\pi} \iint \lambda \frac{r-r'}{|r-r'|^3} ds dl$$

$$\lambda = \int \sigma dn$$

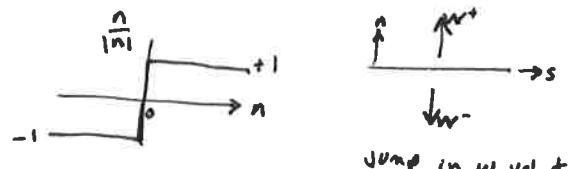
Visually we obtain



Q: What is the velocity jump from  $0+dn$  to  $0-dn$ ?

$$V_\lambda(0,0,n) = \frac{\lambda}{2} \hat{n} \frac{n}{|n|}$$

strength      direction      sign function!



Similarly for a vortex

$$V_\gamma(0,0,n) = \frac{\gamma \times n}{2} \frac{n}{|n|}$$

Strength and orientation      sign function

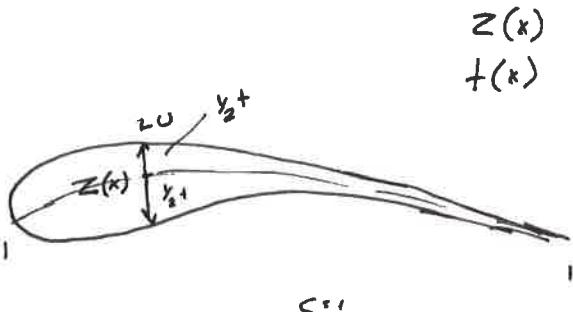
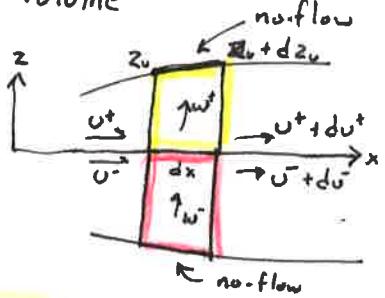
If  $\gamma$  is oriented in the  $l$  direction () then the  $\gamma \times n$  is in the  $s$  direction.

Thus, a vortex sheet gives a jump in  $u$  velocity



See appendix B in FVA.

# Control Volume



Top volume

$$\underbrace{(u^+ + du^+) (z_u + dz_u)}_{\text{out right side}} - \underbrace{u^+ z_u}_{\text{in left}} - \underbrace{w^+ dx}_{\text{in center}} + \underbrace{0}_{\substack{\text{out top} \\ \text{Steady state}}} = 0$$

Bottom volume

$$\underbrace{(u^- + du^-) (z_e + dz_e)}_{\text{out right}} - \underbrace{u^- z_e}_{\text{in left}} - \underbrace{w^- dx}_{\text{in center}} + \underbrace{0}_{\substack{\text{out bottom} \\ \text{Steady state}}} = 0$$

Rearrange top and divide by  $dx$

$$\frac{d}{dx} \left( u^+ z_u + u^+ dz_u + z_u du^+ + \cancel{du^+ dz_u} - u^+ z_u \right) - w^+ = 0$$

$$\frac{1}{dx} \left( \underbrace{u^+ dz_u + z_u du^+}_{d(u^+ z_u)} \right) - w^+ = 0 \quad \Rightarrow \quad \frac{d}{dx} (u^+ z_u) - w^+ = 0$$

Apply jump for  $u^+$  and definition of  $z_u = Z + \frac{1}{2}t$

$$\frac{d}{dx} \left( (U_\infty + U + \frac{1}{2}\gamma)(Z + \frac{1}{2}t) \right) - (W_\infty + W + \frac{1}{2}\lambda) = 0$$

Similar process from bottom

$$\frac{d}{dx} \left( (U_\infty + U - \frac{1}{2}\gamma)(Z - \frac{1}{2}t) \right) - (W_\infty + W - \frac{1}{2}\lambda) = 0$$

Linear algebra operations (average, subtract)

$$\frac{d}{dx} ((U_\infty + U) + \gamma Z) = \lambda$$

$$\frac{d}{dx} ((U_\infty + U) Z + \frac{1}{4}\gamma t) = W_\infty + W$$

## Traditional "thin airfoil theory" (Glauert)

Small angles and small derivatives

$$\text{Thus, } \frac{d}{dx} \left( U_{\infty} t + u t + \frac{1}{4} \gamma Z \right) = \gamma \Rightarrow U_{\infty} \frac{dt}{dx} = \gamma \Rightarrow \boxed{U_{\infty} \frac{dt}{dx} = \gamma}$$

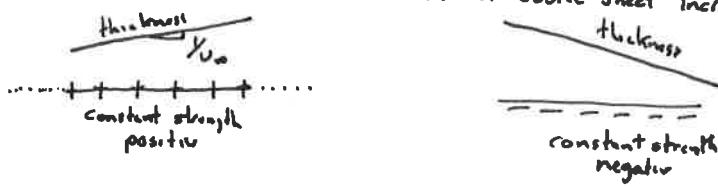
higher order terms

And

$$\frac{d}{dx} \left( U_{\infty} Z + u Z + \frac{1}{4} \gamma t \right) = w_{\infty} + w \Rightarrow \boxed{\frac{d}{dx} (U_{\infty} Z) = w_{\infty} + w}$$

Again, drop these terms

The first equation indicates that a constant source sheet increases the thickness stream by  $1/U_{\infty}$



To generate an airfoil shape, we would need a strongly positive source tapering to zero at the max thickness followed by a negative source to close out the airfoil.



$+++ \dots - - - - -$