

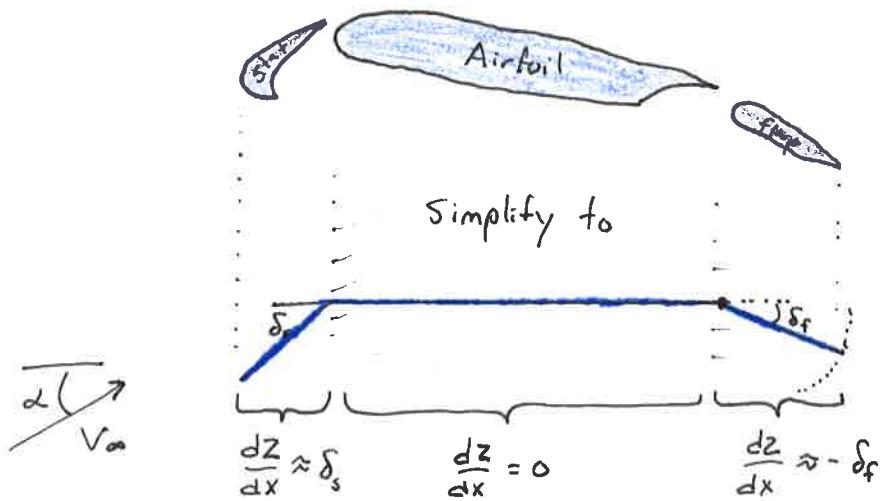
Lesson 11

Panel Methods

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Slats + Flaps in Inviscid Flows

Airfoil with slats + Flaps



$$x = \frac{c}{2} (1 - \cos \theta)$$

$$\theta = \alpha \cos(1 - 2 \frac{x}{c})$$

precisely;

$$\frac{dz}{dx} = \tan \delta$$

$x=0$ $\theta=0$	$x=r_s C$ $\theta=\alpha \cos(1-2r_s) = \theta_s$	$x=r_f C$ $\theta=\alpha \cos(1-2r_f) = \theta_f$	$x=C$ $\theta=\pi$
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From TAT

$$\begin{aligned}
 A_0 &= \alpha - \frac{1}{\pi} \int_0^{\pi} \frac{dz(\theta)}{dx} d\theta = \alpha - \frac{1}{\pi} \int_0^{\theta_s} \delta_s d\theta - \frac{1}{\pi} \int_{\theta_s}^{\theta_f} 0 d\theta - \frac{1}{\pi} \int_{\theta_f}^{\pi} -\delta_f d\theta \\
 &= \alpha - \frac{\delta_s}{\pi} (\theta_s - 0) - \cancel{\frac{1}{\pi} (0)^{\theta_s}} + \frac{\delta_f}{\pi} (\pi - \theta_f) \\
 &= \alpha - \frac{\delta_s \theta_s}{\pi} - \frac{\delta_f \theta_f}{\pi} + \frac{\delta_f \pi}{\pi} = \alpha - \frac{1}{\pi} (\delta_s \theta_s + \delta_f \theta_f) + \delta_f
 \end{aligned}$$

$$\begin{aligned}
 A_n &= \frac{2}{\pi} \int_0^{\pi} \frac{dz}{dx} \cos n \theta d\theta = \frac{2}{\pi} \int_0^{\theta_s} \delta_s \cos n \theta d\theta + \frac{2}{\pi} \int_{\theta_s}^{\theta_f} 0 \cos n \theta d\theta + \frac{2}{\pi} \int_{\theta_f}^{\pi} -\delta_f \cos n \theta d\theta \\
 &= \frac{2}{\pi} \delta_s \int_0^{\theta_s} \cos n \theta d\theta + \cancel{\frac{2}{\pi} 0^{\theta_s}} + -\frac{2}{\pi} \delta_f \int_{\theta_f}^{\pi} \cos n \theta d\theta \\
 &= \frac{2}{\pi} \delta_s \frac{1}{n} \sin n \theta_s - \frac{2}{\pi} \delta_f \frac{1}{n} \sin n \pi + \frac{2}{\pi} \delta_f \frac{1}{n} \sin n \theta_f = \frac{2}{n\pi} (\delta_s \sin n \theta_s + \delta_f \sin n \theta_f)
 \end{aligned}$$

$$A_1 = \frac{2}{\pi} (\delta_s \sin \theta_s + \delta_f \sin \theta_f)$$

$$= \frac{2}{\pi} \left(\delta_s \sqrt{4r_s + 4r_s^2} + \delta_f \sqrt{4r_f + 4r_f^2} \right)$$

$$\sin(\cos(x)) = \sqrt{1-x^2}$$

$$\text{and } \theta_s = \alpha \cos(1 - 2r_s) \Rightarrow \sin \theta_s = \sqrt{4r_s + 4r_s^2}$$

$$C_e = 2\pi A_0 + \pi A_1$$

$$= 2\pi \left(\alpha - \frac{1}{\pi} (\delta_s \theta_s + \delta_f \theta_f) + \delta_f \right) + \pi \frac{2}{\pi} \left(\delta_s \sin \theta_s + \delta_f \sin \theta_f \right)$$

$$= 2\pi \alpha + 2\pi \left(\delta_f - \frac{\delta_f \theta_f}{\pi} + \frac{\delta_f \sin \theta_f}{\pi} \right) + 2\pi \left(-\frac{\delta_s \theta_s}{\pi} + \frac{\delta_s \sin \theta_s}{\pi} \right)$$

$$= 2\pi \alpha + 2\pi \delta_f \underbrace{\left(1 - \frac{\theta_f}{\pi} + \frac{\sin \theta_f}{\pi} \right)}_{\text{AOA}} + 2\pi \delta_s \underbrace{\left(-\frac{\theta_s}{\pi} + \frac{\sin \theta_s}{\pi} \right)}_{\text{Slat}}$$

Notice the difference between a flap $\left(1 - \frac{\theta_f}{\pi} + \frac{\sin \theta_f}{\pi} \right)$ and a slat $\left(-\frac{\theta_s}{\pi} + \frac{\sin \theta_s}{\pi} \right)$.

Why? The flap directly affects/contributes to C_e with δ_f , but not for the slat!!

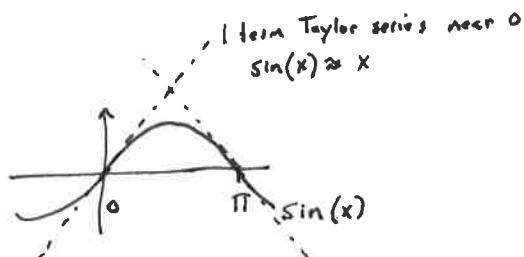
Flaps have the Kutta condition applied. Slat do not.



- Limiting case when slat and flap are small

- Zero order
 - $\sin \theta_s \approx \theta_s$ when θ_s is near 0.
 - $\sin(\theta_f) \approx \theta_f$ when θ_f is near π

$$\underline{C_e = 2\pi \alpha + 2\pi \delta_f}$$



- 1st order

$$\sin \theta_s \approx \theta_s$$

$$\sin \theta_f \approx \pi - \theta_f \text{ near } \pi$$

$$C_e = 2\pi \alpha + 2\pi \delta_f \left(1 - \frac{\theta_f}{\pi} + \frac{\pi}{\pi} - \frac{\theta_f}{\pi} \right) + 2\pi \delta_s \left(-\frac{\theta_s}{\pi} + \frac{\theta_s}{\pi} \right)$$

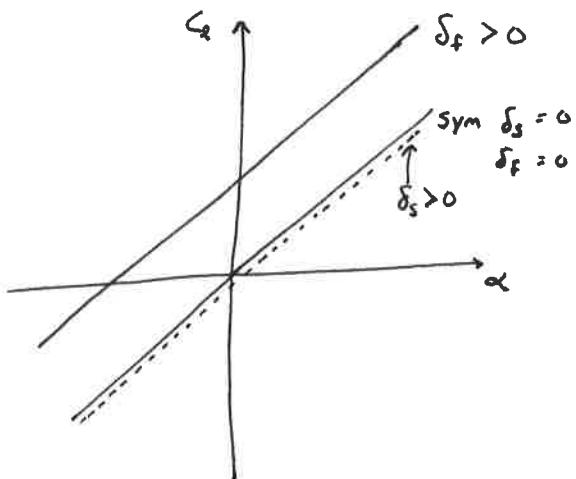
$$\underline{C_e = 2\pi \alpha + 4\pi \delta_f - 4\theta_f \delta_f + 0}$$

AOA !!!!

Flaps are efficient at increasing lift at a given AOA. Slat are not.

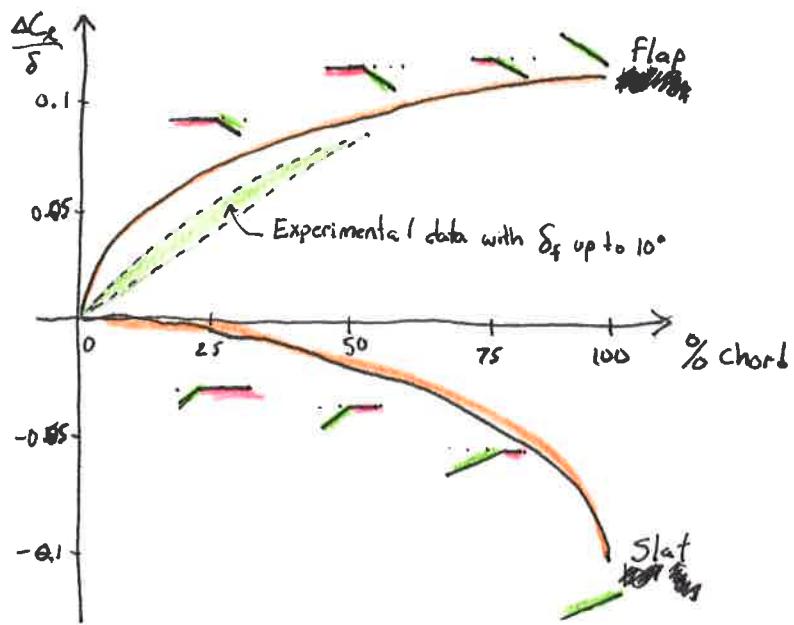
Why use slat? Defer to later lesson after discussing separation. Useless/detrimental!!

For the purely inviscid TAT, how effective are slats and flaps at generating lift?



- For a given configuration (fixed θ_f and θ_s) flaps shift the C_L curve up.
- For a given configuration, slats slightly shift the C_L curve down.

Effectiveness of slats and flaps versus chord



From a configuration design perspective, small chord flaps are surprisingly effective.

A 16% c flap has half the effect as rotating the entire airfoil.

Diminishing returns.

Small chord slats decrease the constant AOA lift only slightly.

A 25% c slat only reduces the lift by 6% compared to rotating the entire airfoil by δ_s .

Would we want negative slats for high lift? No!

It would appear that flaps are considerably better at ~~high~~ generating high lift than slats.

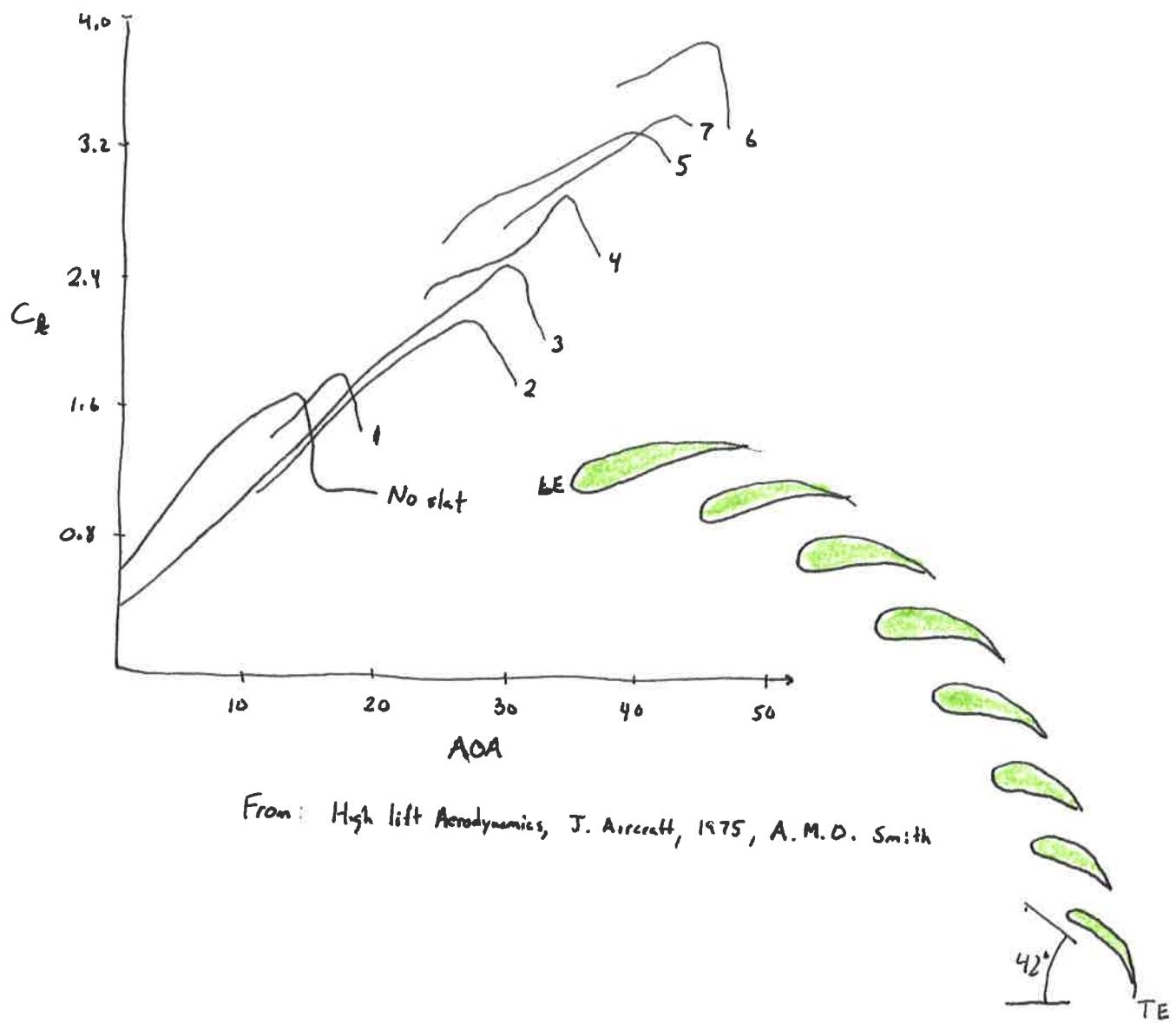
Once we cover viscous flows and separation, we will see why slats are a powerful tool for generating high lift. This occurs not through an upward shift in C_L , but a right shift in the AOA at stall,

We will discuss high lift strategies and configurations later in this class. Since high lift is so dependent on flow separation and attachment, we will delay further discussion until after we study boundary layers and, in general, viscous airfoil flows.

As a teaser, the following figure is why we use slats

No slat $C_{lmax} \approx 1.7$
at $13-14^\circ$

8 element $C_{lmax} \approx 4.0$
at 42°



Lumped Vortex TAT

We derived TAT with source and vortex sheets ($\lambda(x)$ and $\gamma(x)$). Now, we will simplify the physics further by lumping vortex sheets into vortex lines.

$$\text{becomes } \int_a^b \gamma(x) dx \text{ where } \Gamma = \int_a^b \gamma(x) dx$$

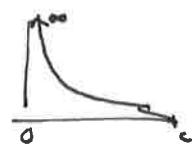
We ignore the source sheet. $\lambda(x) = 0$

You probably know that the vortex line is applied to the quarter chord. From TAT, we know that $C_m \gamma_q = 0$, thus the lift force would act through this γ_q point. However, we can use math to prove this is true.

$$X_{\text{action}} = \frac{\text{Moment}}{\text{Force}} \Rightarrow X_{CG} = \frac{\text{Moment}}{\text{Weight}} \dots \text{etc.}$$

We will do the same operation to $\gamma(x)$

For a flat panel, $\frac{\gamma(b)}{V_\infty} = 2A_0 \frac{1 + \cos\theta}{\sin\theta}$



The integral of γ over the panel is:

$$\int_0^\pi 2 \underbrace{\frac{1 + \cos\theta}{\sin\theta}}_{Y} \underbrace{\sin\theta d\theta}_{dx} = 2\pi$$

The moment integral of γ is

$$\int_0^\pi 2 \underbrace{\frac{1 + \cos\theta}{\sin\theta}}_{Y} \underbrace{\left(\frac{c}{2}\right) \left(1 - \cos\theta\right)}_{X} \underbrace{\sin\theta d\theta}_{dx} = \frac{\pi}{2} c$$

Thus, the vortex acts at:

$$\frac{\int_0^\pi 2 \frac{1 + \cos\theta}{\sin\theta} \left(\frac{c}{2}\right) \left(1 - \cos\theta\right) \sin\theta d\theta}{\int_0^\pi 2 \frac{1 + \cos\theta}{\sin\theta} \sin\theta d\theta} = \frac{\frac{\pi}{2} c}{2\pi} = \frac{c}{4} \quad \text{The quarter chord.}$$

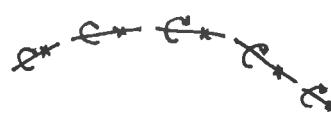
For a single panel, which collocation point gives $C_{\ell\alpha} = 2\pi$?



From before $\Gamma = V_\infty \sin \alpha / 2\pi r$ and $L = \rho V \Gamma$ $\Rightarrow L = \rho V_\infty^2 / 2\pi r \sin \alpha$

$$C_e = \frac{L}{\frac{1}{2} \rho V_\infty^2 c} = \frac{\rho V_\infty^2 / 2\pi r \sin \alpha}{\frac{1}{2} \rho V_\infty^2 c} = 2\pi \sin \alpha / c \Rightarrow r = \frac{1}{2} c \text{ from } \frac{1}{4} c$$

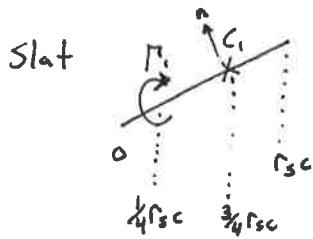
The equivalent collocation point is at $\frac{3}{4} c$.



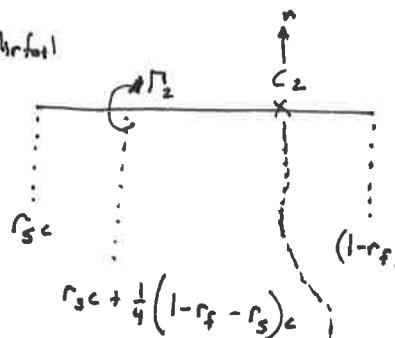
Ex: Slat flap airfoil.



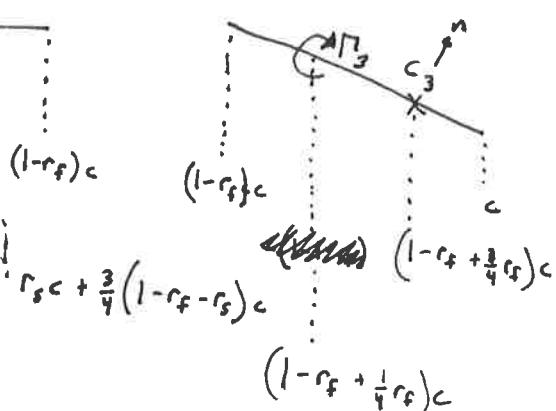
Assume small deflections such that $\Theta = -\alpha$



Airfoil



Flap



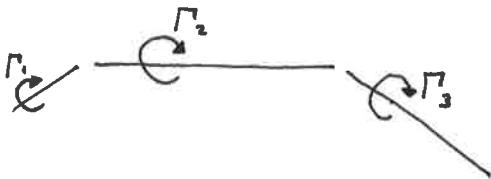
3 unknowns Γ_1 Γ_2 Γ_3

3 equations $V \cdot n = 0$ at C_1 C_2 C_3

Model an airfoil as a series of connected flat panels.



Place a vortex at the $\frac{1}{4}c$ of each panel.



A vortex has a potential of $\phi = -\frac{\Gamma}{2\pi} \theta = -\frac{\Gamma}{2\pi} \operatorname{atan}\left(\frac{y}{x}\right)$

$$U = \frac{d\phi}{dx} = \frac{\Gamma y}{2\pi(x^2+y^2)} = \frac{\Gamma y}{2\pi r^2}$$

$$U_r = \frac{d\phi}{dr} = 0$$

$$V = \frac{d\phi}{dy} = -\frac{\Gamma x}{2\pi(x^2+y^2)} = -\frac{\Gamma x}{2\pi r^2}$$

$$U_\theta = \frac{1}{r} \frac{d\phi}{d\theta} = -\frac{\Gamma}{2\pi r}$$

A freestream U_∞ has the potential $\phi = V_\infty x$

$$U = \frac{d\phi}{dx} = V_\infty \quad \text{and} \quad V = \frac{d\phi}{dy} = 0$$

$$\text{or } \phi = V_\infty \cos \theta r$$

$$U_r = V_\infty \cos \theta \quad U_\theta = -\frac{1}{r} V_\infty \sin \theta r = -V_\infty \sin \theta$$

No flow across panel (at a particular location to be determined)

$$V \cdot n = 0 \text{ implies } U_\theta = 0$$

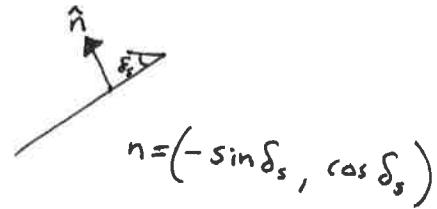
$$-\frac{\Gamma}{2\pi r} - V_\infty \sin \theta = 0 \Rightarrow \Gamma = -V_\infty \sin \theta \frac{2\pi r}{r}$$

$$\begin{aligned} \text{with } \alpha = -\theta, \text{ we get } \Gamma &= -V_\infty \sin(-\alpha) \frac{2\pi r}{r} \\ &= V_\infty \sin \alpha \frac{2\pi r}{r} \end{aligned}$$

At C_1 :

$$U_{C_1} = \frac{\Gamma_1 Y_{11}}{2\pi r_{11}^2} + \frac{\Gamma_2 Y_{21}}{2\pi r_{21}^2} + \frac{\Gamma_3 Y_{31}}{2\pi r_{31}^2} + V_\infty \cos \alpha$$

$$V_{C_1} = -\frac{\Gamma_1 X_{11}}{2\pi r_{11}^2} + -\frac{\Gamma_2 X_{21}}{2\pi r_{21}^2} + -\frac{\Gamma_3 X_{31}}{2\pi r_{31}^2} + V_\infty \sin \alpha$$



$$\nabla \cdot n = 0$$

At C_2 :

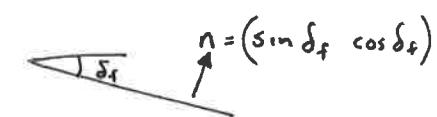
$$U_{C_2} = -\frac{\Gamma_1 X_{12}}{2\pi r_{12}^2} + -\frac{\Gamma_2 X_{22}}{2\pi r_{22}^2} + -\frac{\Gamma_3 X_{32}}{2\pi r_{32}^2} + V_\infty \sin \alpha$$

$$\nabla \cdot n = V_{C_1} = 0$$



$$\text{At } C_3 : U_{C_3} = \frac{\Gamma_1 Y_{13}}{2\pi r_{13}^2} + \frac{\Gamma_2 Y_{23}}{2\pi r_{23}^2} + \frac{\Gamma_3 Y_{33}}{2\pi r_{33}^2} + V_\infty \cos \alpha$$

$$V_{C_3} = -\frac{\Gamma_1 X_{13}}{2\pi r_{13}^2} + -\frac{\Gamma_2 X_{23}}{2\pi r_{23}^2} + -\frac{\Gamma_3 X_{33}}{2\pi r_{33}^2} + V_\infty \sin \alpha$$



$$\nabla \cdot n = 0$$

You notice that we create an influence matrix from the collocation equations

$$\begin{bmatrix} M & & \end{bmatrix} \begin{pmatrix} \Gamma_1 \\ \Gamma_2 \\ \Gamma_3 \end{pmatrix} = \begin{pmatrix} F \end{pmatrix} V_\infty$$

For a complicated geometry, use a computer to populate and invert M .

Solving for Γ_i requires inverting M (or an equivalent process).

What is the lift?

$$L = \rho V_\infty \sum \Gamma_i = \rho V_\infty M$$