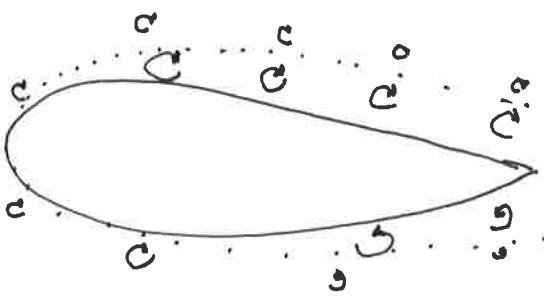


Lesson 12
Panel Method
(Smith-Hess, linear)

Reality

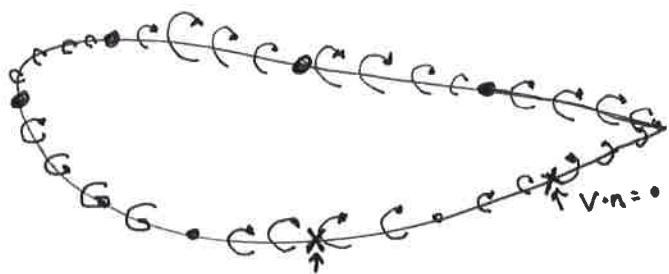


$$\text{Field of } \omega = \nabla \times V$$

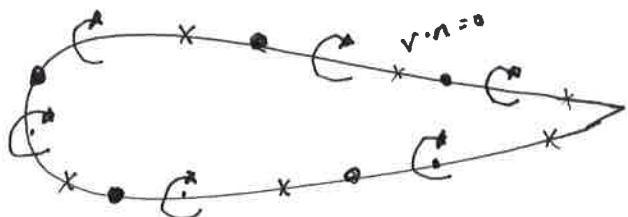
TAT



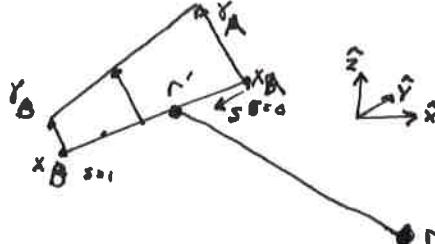
panel method



Vortex



$$V(r) = \frac{1}{2\pi} \int \gamma(\hat{r}) \frac{\hat{y} \times (r - r')}{(r - r')^2} ds$$



Assume a linear distribution of $\gamma(r')$ along the panel from $s=0$ to $s=1$.

$$\gamma(s) = \gamma_A + \left(\frac{\gamma_B - \gamma_A}{l} \right) \cdot s$$

Write r' in terms of s , since the panel is flat and the endpoints are known

$$r'(s) = X_A + (X_B - X_A) \cdot s$$

The cross product term simplifies too. (2D)

$$r = r_x \hat{x} + r_z \hat{z} \quad r' = r'_x \hat{x} + r'_z \hat{z} \Rightarrow r - r' = \Delta r$$

$$\hat{y} \times (r - r') = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 1 & 0 \\ \Delta r_x & 0 & \Delta r_z \end{vmatrix} = \Delta r_z \hat{x} - \Delta r_x \hat{z}$$

The velocity consistent with a vortex sheet is

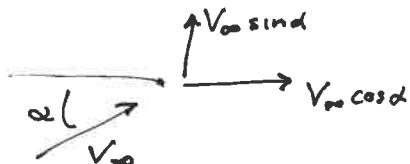
$$V_x(r) = \frac{1}{2\pi} \int_0^l \left(\gamma_A + (\gamma_B - \gamma_A)s \right) \frac{(r_x - X_A - (X_B - X_A)s) \hat{x}}{(r_x - X_A - (X_B - X_A)s)^2 + (r_z - Z_A - (Z_B - Z_A)s)^2} ds$$

$$V_z(r) = \frac{1}{2\pi} \int_0^l \left(\gamma_A + (\gamma_B - \gamma_A)s \right) \frac{-r_z + Z_A + (Z_B - Z_A)s}{(r_x - X_A - (X_B - X_A)s)^2 + (r_z - Z_A - (Z_B - Z_A)s)^2} ds$$

The boundary conditions are that $V \cdot n = 0$ on a particular location on each panel

The freestream provides

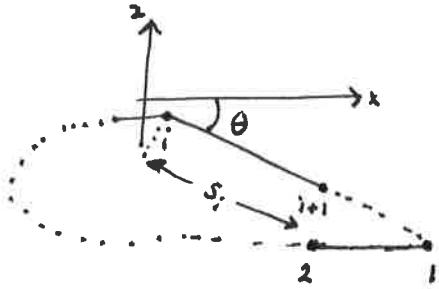
$$V_\infty = V_\infty \cos \alpha \hat{x} + V_\infty \sin \alpha \hat{z}$$



The integral is not trivial.

Kuethe and Chow's "Foundations of Aerodynamics" applies both the $V \cdot n = 0$ condition and the sheet velocity perturbation value to give.

$$\sum_{j=1}^m \left(C_{n1;j} \frac{\gamma_j}{2\pi V_\infty} + C_{n2;j} \frac{\gamma_{j+1}}{2\pi V_\infty} \right) = \sin(\theta_i - \alpha)$$



$C_{n1;j}$ is influence of source at j on panel i

when $i=j$, $C_{1n} = -1$
 $C_{2n} = +1$

$$C_{n1;j} = \frac{1}{2} D \cdot F + G \cdot C - C_{n2;j}$$

$$C_{n2;j} = D + \frac{1}{2} \frac{QF}{S_j} - (AC + DE) \cdot \frac{G}{S_j}$$

$$A = -(x_i - x_j) \cos \theta_j - (y_i - y_j) \sin \theta_j$$

$$B = (x_i - x_j)^2 + (y_i - y_j)^2$$

$$C = \sin(\theta_i - \theta_j)$$

$$D = \cos(\theta_i - \theta_j)$$

$$E = (x_i - x_j) \sin \theta_j - (y_i - y_j) \cos \theta_j$$

$$F = \ln \left(1 + \frac{S_j^2 + 2AS_j}{B} \right)$$

$$G = \arctan \left(\frac{ES_j}{B + AS_j} \right)$$

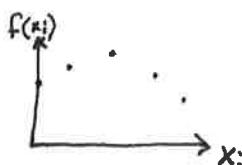
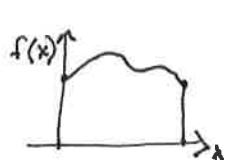
$$Q = (x_i - x_j) \cos(\theta_i - 2\theta_j) - (y_i - y_j) \sin(\theta_i - 2\theta_j)$$

$$P = (x_i - x_j) \sin(\theta_i - 2\theta_j) + (y_i - y_j) \cos(\theta_i - 2\theta_j)$$

Numerical Integration

Approximation of an integral (definite integral) using points ^{and values} within the domain.

$$\int_a^b f(x) dx \approx \sum_{i=1}^n f(x_i) a_i$$



Midpoint



Captures "zero order" terms

Trap'

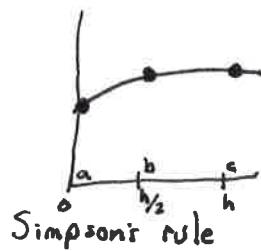


Captures "1st order" term

Higher Order

Fit curve and integrate curve.

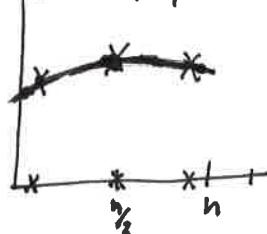
$$f(x) \approx Ax^2 + Bx + C$$



Simpson's rule

$$I \approx \frac{h}{6} (f(a) + 4f(b) + f(c))$$

Gauss-Legendre Quadrature
Legendre polynomials



$$x_1 = 0.112 \quad \text{weights}$$

$$x_2 = 0.5$$

$$x_3 = 0.887$$

$$w_1 = 0.2777$$

$$w_2 = 0.444$$

$$w_3 = 0.2227$$

$$I = \sum x_i w_i$$

The weights and locations are tabulated in most numerical methods books.

Why use Gauss quadrature rather than geometric or polynomial integration schemes?

Gauss methods are extendable and robust at high order (i.e. many points)

Disadvantage, points and weights are not trivial to get for multiple dimensions plus, unusual locations.

Since the goal is to determine influence coefficients such that

$$[A\Gamma] \begin{pmatrix} \gamma_1 \\ \gamma_2 \\ \vdots \\ \gamma_m \end{pmatrix} = (B)$$

The boundary condition $\nabla \cdot n = 0$ needs to be reviewed such that each γ_i term can be pulled out of it.

$$\mathbf{V} = (V_x + V_\infty \cos \alpha) \hat{x} + (V_z + V_\infty \sin \alpha) \hat{z}$$

On a segment



the normal points outward $n = (\cos \theta, \sin \theta)$

$$\nabla \cdot n = 0 = V_x \cos \theta + V_\infty \cos \alpha \cos \theta + V_z \sin \theta + V_\infty \sin \alpha \sin \theta$$

Rearrange to

$$V_x \cos \theta + V_z \sin \theta = -V_\infty (\cos \alpha \cos \theta + \sin \alpha \sin \theta)$$

Next, the γ terms need to be separated.

$$V_{x_A} = \frac{1}{2\pi} \gamma_A \int_0^1 (1-s) \frac{(r_x - x_A - (x_B - x_A)s)}{|r - r'|^2} \hat{x} ds = \gamma_A C_{x_A}$$

$$V_{x_B} = \frac{1}{2\pi} \gamma_B \int_0^1 (s) \dots \hat{x} ds = \gamma_B C_{x_B}$$

$$V_{z_A} = \frac{1}{2\pi} \gamma_A \int_0^1 (1-s) \frac{(-r_z + z_A + (z_B - z_A)s)}{|r - r'|^2} \hat{z} ds = \gamma_A C_{z_A}$$

$$V_{z_B} = \frac{1}{2\pi} \gamma_B \int_0^1 (s) \dots \hat{z} ds = \gamma_B C_{z_B}$$

Following through (from $V \cdot n = 0$ expansion)

$$\gamma_A C_{x_A} \cos \theta + \gamma_B C_{x_B} \cos \theta + \gamma_A C_{z_A} \sin \theta + \gamma_B C_{z_B} \sin \theta = -V_\infty (\dots)$$

Rearrange to

$$\underbrace{\gamma_A (C_{x_A} \cos \theta + C_{z_A} \sin \theta)}_{m \text{ term}} + \underbrace{\gamma_B (C_{x_B} \cos \theta + C_{z_B} \sin \theta)}_{n \text{ term}} = -V_\infty (\dots) \quad \underbrace{\qquad}_{B \text{ term}}$$

Notice that the M matrix is constant for a given airfoil geometry.

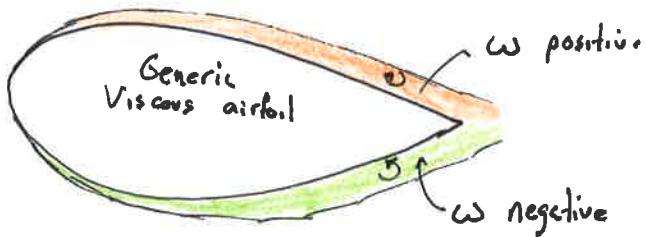
B varies with α

What about the trailing edge?

The Kutta condition must apply.

Does this mean that $\gamma_{TE^+} = \gamma_{TE^-} = 0$?

No!



You must be careful here.

- For an open curve airfoil, the only way to satisfy the Kutta condition is $\gamma(TE) = 0$

- For a closed curve airfoil The upper and lower strengths of vorticity must zero out.

$$\gamma_{TE^+} + \gamma_{TE^-} = 0$$