

## Lesson 15

### Compressible flow

View:

[tiny.cc/ShockWing](http://tiny.cc/ShockWing)

[tiny.cc/ShockCowl](http://tiny.cc/ShockCowl)

[tiny.cc/ShellTransonic](http://tiny.cc/ShellTransonic)

# Definitions:

Critical Mach #: The Mach number where  $M=1$  <sup>first</sup> exists in the flow field.



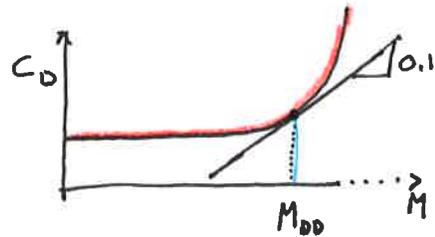
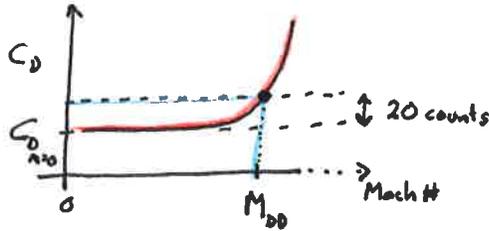
The critical Mach number for an NACA 0012 is  $M 0.77$ .

Drag Divergence Mach #: (multiple definitions)  $M_{DD}$

1) When  $\Delta C_D = 20$  counts from the subsonic value

count  $\equiv 0.0001$

2) When  $\frac{dC_D}{dM} \Big|_{C_L \text{ constant}} = 0.1$

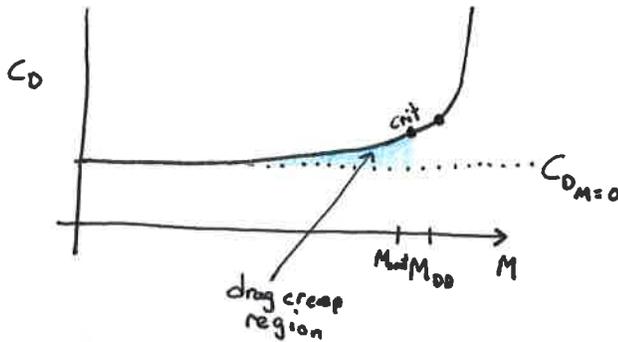


$$M_{DD_{20 \text{ counts}}} \neq M_{DD_{0.1 \text{ slope}}}$$

I prefer this one!

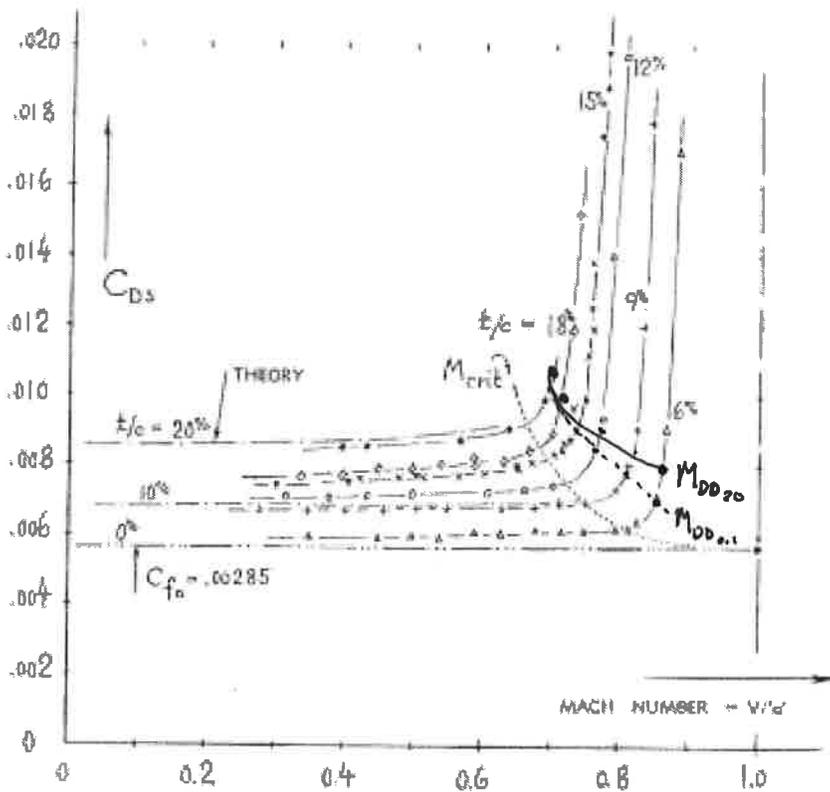
## Drag Creep:

Often, the drag slowly begins increasing below  $M_{crit}$ .



A rough or thick airfoil (of an older design especially) tends to show this behavior. This also shows up for entire aircraft for similar reasons.

# Fluid Dynamic Drag Hoerner



0012  
 $c = .5 \text{ m}$   
 $t/c = 12\%$  at  $x/c = 30\%$   
 DVL (20, a)  
 $R_e = 6 \cdot 10^6$  at  $M = 0.7$

- 2218 (20, e) at  $2 \cdot 10^7$
- 0015 (22, d) at  $2 \cdot 10^6$
- 0015 (20, m) at  $3 \cdot 10^4$

Figure 13. Drag coefficients (at  $C_L = 0$ ) of a family of symmetrical foil sections (with maximum thickness at 30% of the chord); (a) tested in a large-size wind-tunnel (20, a), and (b) calculated as per equation 18 for  $C_f = 0.00285 = \text{constant}$ .

0015 at transonic speeds, see (36, e) in Chapter XVII.



63A009 AIRFOIL SECTION

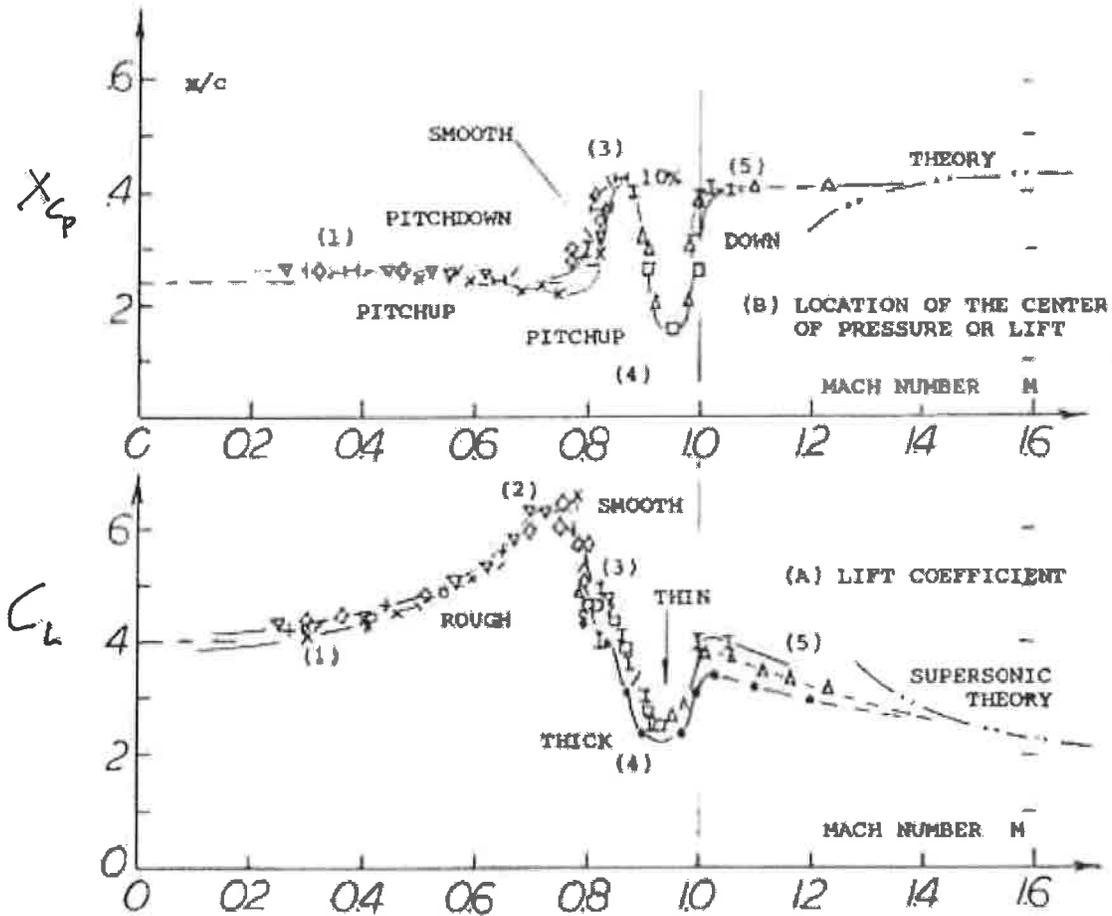
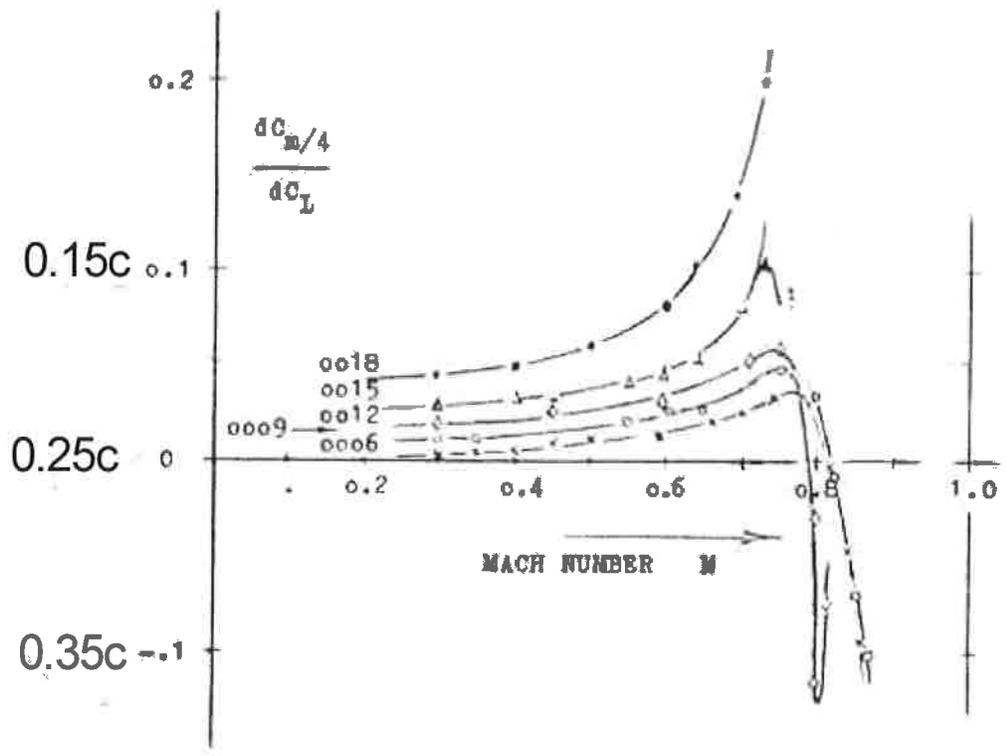
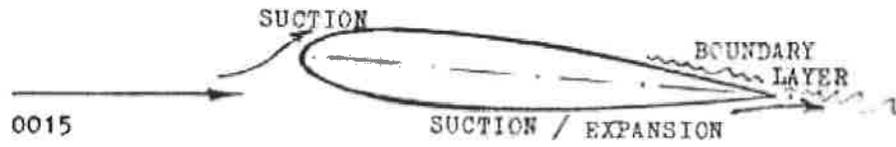


Figure 16. Lift and center of pressure of various symmetrical foil sections from subsonic through transonic M'numbers, at constant angle of attack  $\alpha = 4^\circ$ .



$$X_{ac} = \frac{1}{4} - \frac{dC_m}{dC_l}$$

# Source flow field representation (FVA 8.1.4)

From earlier

$$V_\sigma = \frac{1}{4\pi} \iiint \sigma(r') \frac{r-r'}{|r-r'|^3} dx' dy' dz'$$

From mass continuity

$$\frac{\partial(\rho)}{\partial t} + \frac{\partial}{\partial x_i}(\rho v_i) = 0 \Rightarrow \nabla \cdot (\rho v) = 0 \quad \text{since steady state.}$$

Decompose gradient (chain rule)

$$\nabla \cdot (\rho v) = \nabla \rho \cdot v + \rho \nabla \cdot v$$

~~Take the divergence of mass continuity~~

~~$$\nabla \cdot (\rho \nabla \cdot v) = \nabla \cdot (\rho \nabla \cdot v) + \nabla \rho \cdot \nabla \cdot v$$~~

Rearrange to 
$$\nabla \cdot v = - \frac{\nabla \rho \cdot v}{\rho}$$

From Lesson 10, we found

$$-\frac{\nabla \rho}{\rho} = \frac{1}{2} \frac{\nabla(v^2)}{a^2} \Rightarrow \sigma = \nabla \cdot v = \frac{1}{2} \frac{\nabla(v^2)}{a^2} \cdot v$$

Combine

$$V_\sigma = \frac{1}{4\pi} \iiint \underbrace{\frac{1}{2} \frac{\nabla(v^2)}{a^2} \cdot v}_{\text{Nonlinear and depend on } V(r')} \underbrace{\frac{r-r'}{|r-r'|^3}}_{\text{Volume integral}} dx' dy' dz'$$

Conclusion: Compressible flows are nonlinear and require full-field solutions, which are expensive.

# Review of Compressible Flow Thermodynamics

$$\text{Enthalpy (specific)} \equiv h \equiv \underbrace{e}_{\substack{\text{internal} \\ \text{energy} \\ e(T)}} + \underbrace{\frac{p}{\rho}}_{\substack{= RT \text{ when ideal gas} \\ \text{"pressure term"}}}$$

All of these are "specific" meaning per density

$$E = \rho e$$

$$H = \rho h$$

$$\text{Total Enthalpy} \equiv h_0 \equiv \underbrace{h}_{\substack{\text{internal} \\ \text{energy} \\ + \\ \text{pressure}}} + \underbrace{\frac{V^2}{2}}_{\substack{\text{velocity} \\ \text{"kinetic"} \\ \text{energy}}}$$

Think of enthalpy as measuring the energy at temperature, pressure, and mean velocity. Obviously at a molecular scale, these energies are just molecular motions (squared).

$$h_0 = h + \frac{V^2}{2} = \frac{a^2}{\gamma - 1} + \frac{V^2}{2}$$

$$= h \left( 1 + \frac{\gamma - 1}{2} M^2 \right)$$

$$\text{since } a = \frac{dp}{d\rho} = \sqrt{\gamma RT} = \sqrt{(\gamma - 1)h}$$

And because  $h_0 = h_{0\infty}$  (i.e. at the farfield)

$$h_0 = h_{\infty} \left( 1 + \frac{\gamma - 1}{2} M_{\infty}^2 \right)$$

For isentropic flows

$$\frac{p}{p_{\infty}} = \left( \frac{1 + \frac{\gamma - 1}{2} M_{\infty}^2}{1 + \frac{\gamma - 1}{2} M^2} \right)^{\frac{1}{\gamma - 1}} \quad \text{and} \quad \frac{\rho}{\rho_{\infty}} = \left( \frac{1 + \frac{\gamma - 1}{2} M_{\infty}^2}{1 + \frac{\gamma - 1}{2} M^2} \right)^{\frac{\gamma}{\gamma - 1}}$$

Where did these come from?

See a previous lesson and the Gibbs equation

$$\frac{dp}{p} = \gamma \frac{d\rho}{\rho} = \frac{\gamma}{\gamma - 1} \frac{dh}{h}$$

Egu 1.67 FVA

# Pressure Coefficient in a compressible flow

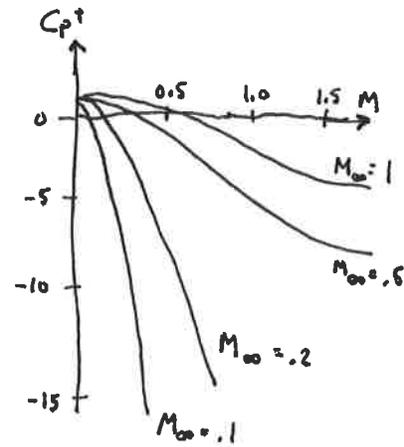
$$C_p = \frac{\Delta P}{\rho} = \frac{P - P_\infty}{\frac{1}{2} \rho V_\infty^2} = \frac{P - P_\infty}{\frac{1}{2} \gamma P_\infty M_\infty^2} = \frac{2 \left( \frac{P}{P_\infty} - \frac{P_\infty}{P_\infty} \right)}{\gamma M_\infty^2}$$

$$= \frac{2 \left( \frac{P}{P_\infty} - 1 \right)}{\gamma M_\infty^2}$$

Substitute isentropic  $P/P_\infty$  ratio

$$C_p = \frac{2}{\gamma M_\infty^2} \left( \left( \frac{1 + \frac{\gamma-1}{2} M_\infty^2}{1 + \frac{\gamma-1}{2} M^2} \right)^{\frac{\gamma}{\gamma-1}} - 1 \right)$$

$$= \frac{2}{\gamma M_\infty^2} \left( \left[ 1 + \frac{\gamma-1}{2} M_\infty^2 \left( 1 - \frac{V^2}{V_\infty^2} \right) \right]^{\frac{\gamma}{\gamma-1}} - 1 \right)$$



The isentropic assumption allows this to be used up to around  $M 1.4$  to  $M 1.5$ ,

Also, rearranging allows for finding the local Mach # from the freestream Mach # and the pressure coefficient.

$$M = \sqrt{\frac{2}{\gamma-1} \left[ \frac{1 + \frac{\gamma-1}{2} M_\infty^2}{\left( 1 + \frac{1}{2} \gamma M_\infty^2 C_p \right)^{\frac{\gamma-1}{\gamma}}} - 1 \right]}$$

lowering  $C_p$  increases  $M$ .

Aside:

Vary  $\gamma$ : (combustion, Mars, etc)

Increasing  $\gamma$  shifts  $C_p$  more towards 0.

( $C_p$  increases when  $M > M_\infty$ )

Rate of increasing  $C_p$  is larger for

$M > M_\infty$  than decrease when  $M < M_\infty$ .

# Wave Drag

The total pressure decreases across a shock.  $\frac{v_1 P_{01}}{M_1} \rightarrow \frac{v_2 P_{02}}{M_2}$

$$\frac{P_{02}}{P_{01}} = \left( \frac{(\gamma+1)M_1^2}{2 + (\gamma-1)M_1^2} \right)^{\frac{\gamma}{\gamma-1}} \left( \frac{\gamma+1}{1-\gamma+2\gamma M_1^2} \right)^{\frac{1}{\gamma-1}}$$

$$\approx 1 - \frac{\gamma(M_1-1)^3}{1+2\gamma(M_1-1)}$$

Egu 8.13 FVA

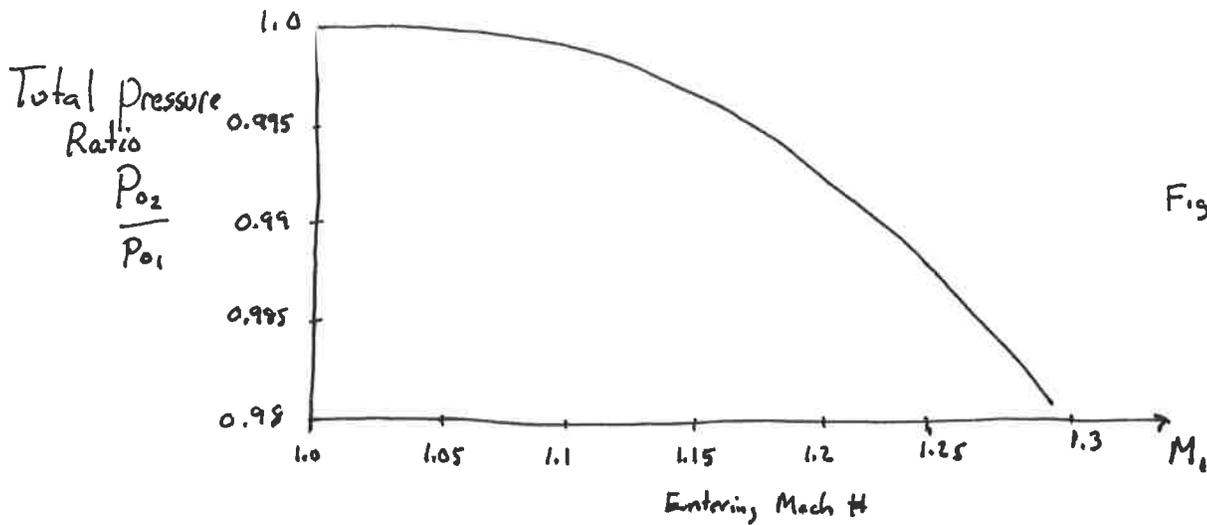


Fig 8.7 FVA

Wave Drag is the drag associated with the velocity defect due to a total pressure loss from a shock.

Fig 8.8 FVA

# Approximate Velocity defect of a normal shock.

From the far field Drag lesson,  $D' = P = \int \rho u (V_\infty - u) dz$

$$C_d = \frac{D'}{\frac{1}{2} \rho_\infty V_\infty^2 c} = \frac{P}{\rho c} =$$

For small "weak" shocks,  $P \approx \int \rho_\infty V_\infty (V_\infty - u_w) dz$

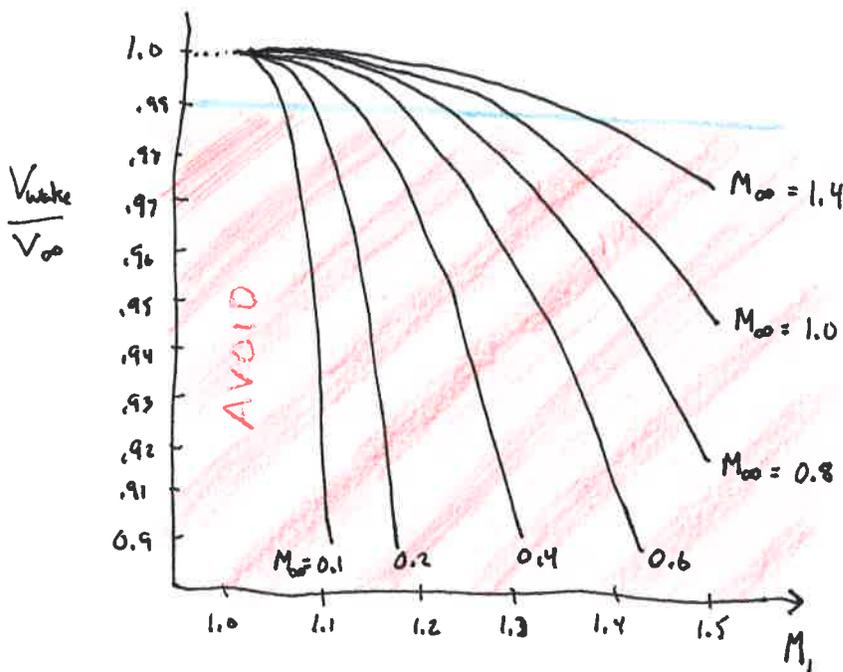
$$C_d = 2 \int \frac{(V_\infty - u_w)}{V_\infty} \frac{dz}{c} = 2 \int \left(1 - \frac{u_w}{V_\infty}\right) \frac{dz}{c}$$

Determine/Approximate  $\frac{u_{wake}}{V_\infty}$  from previous  $\frac{P_{0,2}}{P_{0,1}}$  total pressure ratio and isentropic flow.

$$\frac{V_{wake}}{V_\infty} \approx 1 - \frac{1}{M_\infty^2} \cdot \frac{(M_\infty - 1)^3}{1 + 2\gamma(M_\infty - 1)}$$

or "exactly" as plotted below

$$\frac{V_{wake}}{V_\infty} = \left(1 - \frac{2}{(\gamma-1)M_\infty^2} \left[ \left(\frac{P_{0,2}}{P_{0,1}}\right)^{-\frac{\gamma-1}{\gamma}} - 1 \right]\right)^{1/2}$$



# Wave Drag Conclusions

- Since shocks extend away from the surface a considerable % chord, the integral  $C_d = 2 \int \left(1 - \frac{U_{wake}}{V_{\infty}}\right) \frac{dz}{c}$  requires  $\frac{U_{wake}}{V_{\infty}}$  to be small for drag to be small.

- $C_d$  for airfoils are around the 100 counts region ( $C_d \approx 0.01$ )  
 Keeping  $\frac{U_{wake}}{V_{\infty}} > 0.99$  is necessary to prevent a large drag rise.

- Contrary to intuition, increasing the freestream Mach number increases the allowable local Mach # (pre-shock) for ~~thence~~ a constant wave drag.

Alternatively, geometries creating shocks from low freestream Mach numbers create significantly larger wave drag values than high  $M_{\infty}$ .

- As a design rule, restrict local Mach #s below 1.2 when  $M_{\infty}$  is subsonic.

- This analysis also applies to engine inlets where maintaining a high total pressure ratio is required for high efficiency and avoidance of HCF in fan blades. (plus keeping the engine people "happy")

