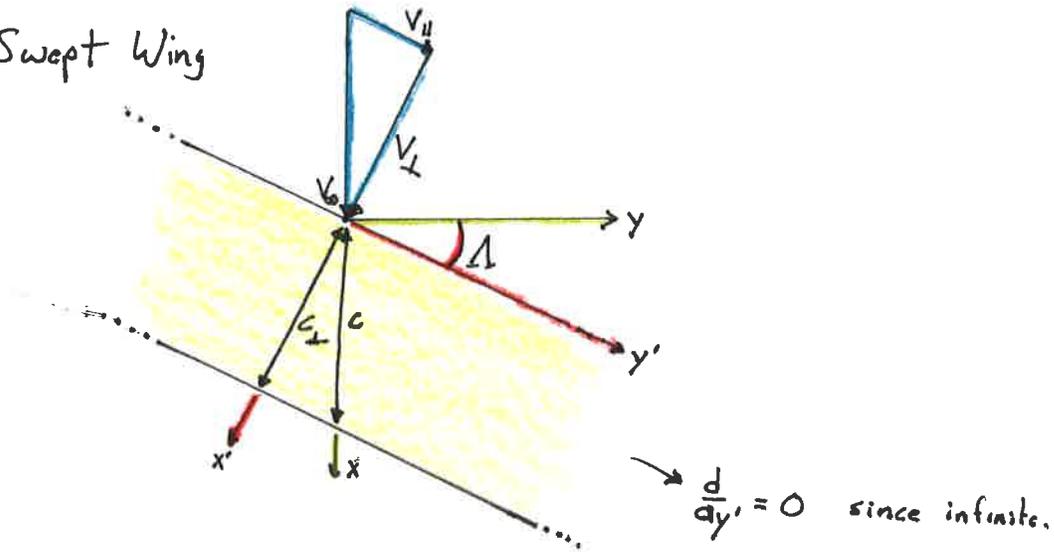


Lesson 18

Swept Infinite Wing

Infinite Swept Wing



Following a particle in the Lagrangian Frame of (x', y', z')

$$\rho \frac{DV'}{Dt} = -\nabla' p \Rightarrow \rho \frac{Du'}{Dt} = -\frac{dp}{dx'}$$

$$\rho \frac{Dv'}{Dt} = -\frac{dp}{dy'} = 0 \Rightarrow \frac{Dv'}{Dt} = 0$$

$$\rho \frac{Dw'}{Dt} = -\frac{dp}{dz'}$$

v' is constant in $x'y'z'$

What is v' ?

Far upstream or anywhere else, $v' = \text{constant}$

$$v' = V_{\parallel} = V_{\infty} \sin \Delta$$

The spanwise velocity is constant
since the wing is infinite

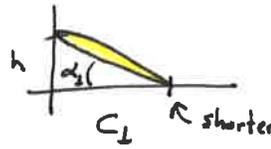
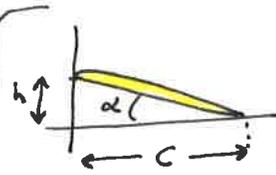
Geometry Terms

$$C_L = C \cos \Lambda$$

$$V_L = V_\infty \cos \Lambda$$

$$\alpha_L = \alpha / \cos \Lambda$$

why?

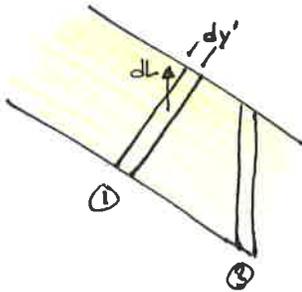


$$\alpha = \arctan\left(\frac{h}{c}\right) \approx \frac{h}{c}$$

$$\alpha_L = \arctan\left(\frac{h}{c_L}\right) \approx \frac{h}{c_L}$$

$$= \frac{h}{c \cos \Lambda} = \frac{h}{c} \frac{1}{\cos \Lambda} = \alpha \frac{1}{\cos \Lambda}$$

Lift



$$dL = \frac{1}{2} \rho V_L^2 \underbrace{C_{L\alpha}}_8 \underbrace{\alpha_L}_{C_L} dS$$

Integrate to give

$$L = \frac{1}{2} \rho V_L^2 C_{L\alpha} \alpha_L S$$

$$= \frac{1}{2} \rho V_\infty^2 \cos^2 \Lambda C_{L\alpha} \alpha \frac{1}{\cos \Lambda} S$$

Lift Coefficient

$$C_{L\alpha} = \frac{L/\alpha}{\rho S} = \frac{\frac{1}{2} \rho V_\infty^2 C_{L\alpha} \cos \Lambda S}{\frac{1}{2} \rho V_\infty^2 \cdot S} = C_{L\alpha} \cos \Lambda$$

Sweep reduces lift by the cosine of sweep.

Ex: The Facebook Aquila program has developed a high AR swept wings (flying wings).

The aircraft has a sweep angle of about 20° . Estimate $C_{L\alpha}$ if AR is large (ie. 2D flow)



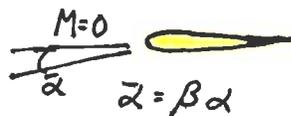
$$C_{L\alpha} \approx C_{L\alpha} \cos \Lambda = 2\pi \cos(20^\circ) = 94\% \cdot 2\pi$$

$$C_{L\alpha} < 5.9 \frac{1}{rad}$$

Compressible Swept Wing

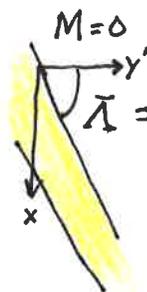
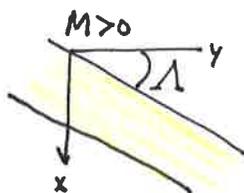
Given a non-zero Mach number, P-G creates an affine transform of y and z by β

AOA:



$$\beta = \sqrt{1 - M_{\infty}^2}$$

Sweep:



$$\bar{\Lambda} = \alpha \tan\left(\frac{1}{\beta} \tan \Lambda\right)$$

The sweep angle increased.
since $y' = \beta y$

Some math to get $\cos \bar{\Lambda}$ from $\tan \bar{\Lambda}$

~~notation~~

$$c^2 + s^2 = 1 \quad \text{and} \quad \frac{s^2}{c^2} = t^2 \Rightarrow \bar{t}^2 + 1 = \frac{1}{\cos^2} \Rightarrow \bar{t}^2 = \frac{1}{\bar{c}^2} - 1$$

$$\bar{c} = \sqrt{\bar{t}^2} = \bar{t} = \sqrt{\frac{1}{\bar{c}^2} - 1} = \frac{1}{\beta} \tan$$

from above

Solve for \bar{c}

$$\frac{1}{\bar{c}^2} - 1 = \frac{1}{\beta^2} t^2 \Rightarrow \frac{1}{\bar{c}^2} = \sqrt{\frac{1}{\beta^2} t^2 + 1}$$

$$\bar{c} = \frac{1}{\sqrt{\frac{1}{\beta^2} t^2 + 1}} \cdot \frac{\beta c}{\beta c} = \frac{\beta c}{\sqrt{t^2 c^2 + \beta^2 c^2}} = \frac{\beta c}{\sqrt{s^2 + \beta^2 c^2}}$$

$$\cos \bar{\Lambda} = \frac{\beta \cos \Lambda}{\sqrt{\sin^2 \Lambda + \beta^2 \cos^2 \Lambda}}$$

From Incompressible case

$$\bar{C}_L = C_{L\alpha} \bar{\alpha} \cos \bar{\Lambda}$$

Apply Goethert's Rule $C_L = \frac{1}{\beta^2} \bar{C}_L$

$$C_L = \frac{1}{\beta^2} C_{L\alpha} \bar{\alpha} \cos \bar{\Lambda} = \frac{1}{\beta^2} C_{L\alpha} \beta \alpha \frac{\beta \cos \Lambda}{\sqrt{\beta^2 \cos^2 \Lambda + \sin^2 \Lambda}}$$

$$= \frac{C_{L\alpha} \cos \Lambda}{\sqrt{\beta^2 \cos^2 \Lambda + \sin^2 \Lambda}} \cdot \alpha$$

$$C_{L\alpha} = C_{L\alpha} \cdot \frac{\cos \Lambda}{\sqrt{\beta^2 \cos^2 \Lambda + \sin^2 \Lambda}}$$

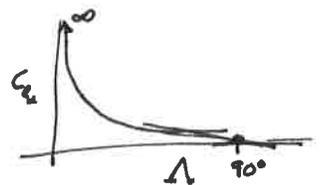
$M=0$: $\beta = \sqrt{1-M^2}$

$$C_{L\alpha} = C_{L\alpha} \cdot \frac{\cos \Lambda}{\sqrt{\cos^2 \Lambda + \sin^2 \Lambda}} = C_{L\alpha} \cos \Lambda \quad \text{recovered previous page's result } \checkmark$$

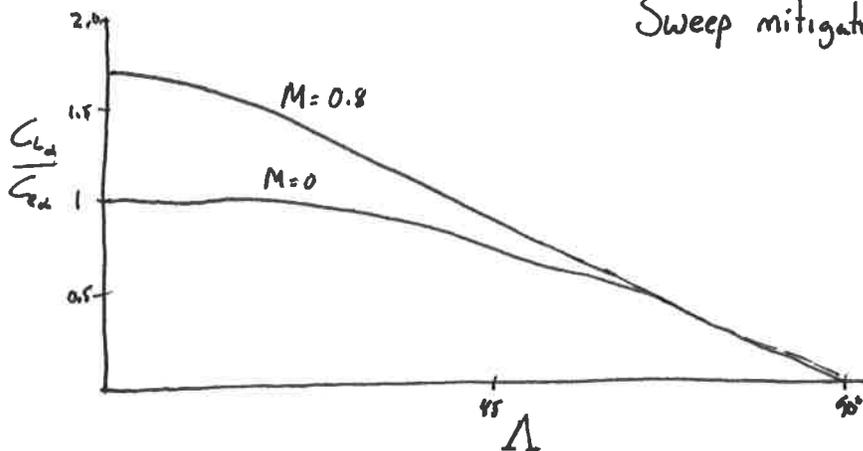
$M \approx 1$: PG is nonsense.

$$C_{L\alpha} = C_{L\alpha} \cdot \frac{\cos \Lambda}{\sqrt{0 \cdot \cos^2 \Lambda + \sin^2 \Lambda}} = C_{L\alpha} \frac{\cos \Lambda}{\sin \Lambda} = C_{L\alpha} \frac{1}{\tan \Lambda}$$

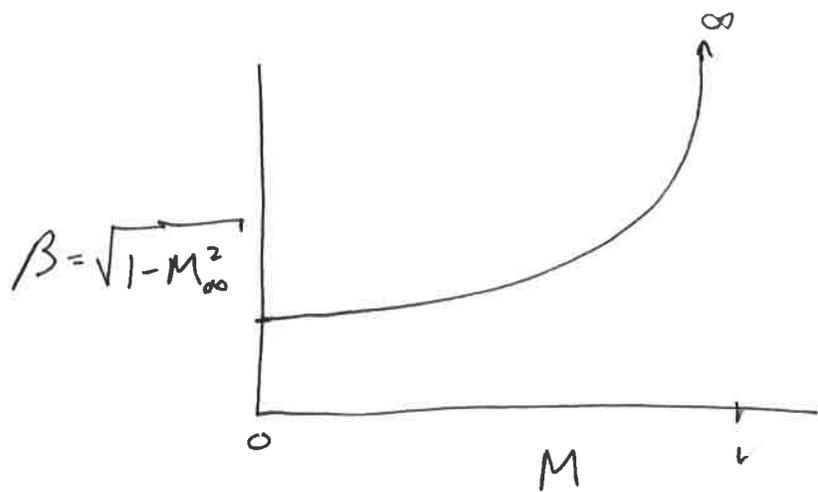
$$\Lambda = 0 \Rightarrow C_{L\alpha} \text{ is } \infty \quad X$$



From ~~AFDD~~ Fig 8.18
FVA



"Sweep mitigates Compressibility"



What is the C_p for sonic conditions given M_∞ ?

Isentropic:

$$M^2 = \frac{2}{\gamma-1} \left(\frac{1 + \frac{\gamma-1}{2} M_\infty^2}{\left(1 + \frac{1}{2} \gamma M_\infty^2 C_p\right)^{\frac{\gamma-1}{\gamma}}} - 1 \right)$$

Solve for C_p

$$\frac{\gamma-1}{2} + 1 = \frac{1 + \frac{\gamma-1}{2} M_\infty^2}{\left(1 + \frac{1}{2} \gamma M_\infty^2 C_p\right)^{\frac{\gamma-1}{\gamma}}}$$

Rearrange + invert

$$\frac{1 + \frac{\gamma-1}{2} M_\infty^2}{\frac{\gamma-1}{2} + 1} = \left(1 + \frac{1}{2} \gamma M_\infty^2 C_p\right)^{\frac{\gamma-1}{\gamma}}$$

Power of $\frac{\gamma}{\gamma-1}$

$$1 + \frac{1}{2} \gamma M_\infty^2 C_p = \left(\frac{1 + \frac{\gamma-1}{2} M_\infty^2}{\frac{\gamma-1}{2} + 1} \right)^{\frac{\gamma}{\gamma-1}}$$

$$C_p = \frac{\left(\frac{1 + \frac{\gamma-1}{2} M_\infty^2}{\frac{\gamma-1}{2} + 1} \right)^{\frac{\gamma}{\gamma-1}} - 1}{\frac{1}{2} \gamma M_\infty^2}$$

$$C_p = \frac{2}{\gamma M_\infty^2} \left(\frac{1 + \frac{\gamma-1}{2} M_\infty^2}{\frac{\gamma-1}{2} + 1} \right)^{\frac{\gamma}{\gamma-1}} - \frac{2}{\gamma M_\infty^2}$$

What is C_p at $M=1$ for M_∞ ?

