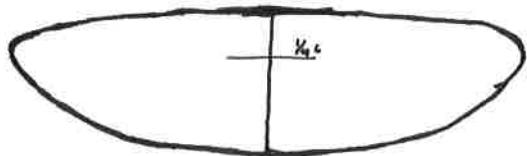


Elliptical Wing



Spitfire

P-47 \approx except for late models

A5M

Crows, sparrows, ... some birds

Chord

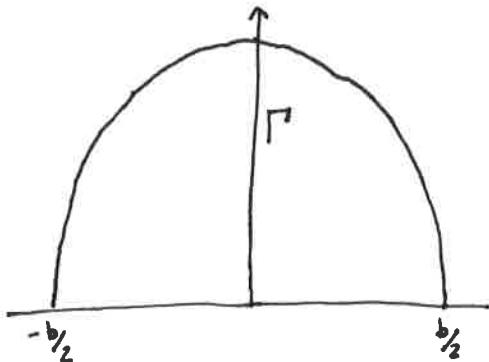
$$C(\theta) = C_0 \sin \theta$$

$$C(y) = C_0 \sqrt{1 - \left(\frac{y}{b}\right)^2}$$

Flat Elliptical

$$A_i = \frac{C_{\alpha} / \pi AR}{1 + C_{\alpha} / \pi AR} (\alpha + \alpha_{aero})$$

\uparrow Not a function of y here.



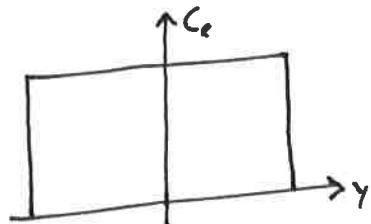
$$\Gamma(\theta) = \Gamma_0 \sin \theta$$

Sectional lift coefficient

$$C_L = \frac{L'}{\frac{1}{2} \rho V_\infty^2 c} \quad \text{from Kutta-Joukowski} \quad L' = \rho V \Gamma$$

$$= \frac{\rho V \Gamma}{\frac{1}{2} \rho V_\infty^2 c} = \frac{2 \Gamma(y)}{C(y)}$$

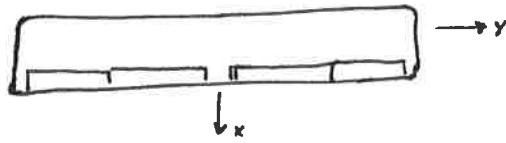
Constant across span



Entire wing theoretically stalls all at once!

Hershey Bar Constant Chord Wing

Piper Cherokee
 " Tomahawk
 and many others



Chord

$$c(y) = C_0$$

Using the collocation method described in Flight Vehicle Aerodynamics,

$$a_{mn} = \sin(n\theta_m) + \frac{c(\theta_m)}{4b} C_{l_m} n \frac{\sin(n\theta_m)}{\sin(\theta_m)}$$

$$\theta_m = \pi \frac{n}{N+1}$$

$$r_m = \frac{c(\theta_m)}{4b} C_{l_m} (\alpha + \alpha_{aero}(\theta))$$

$$x = \frac{b}{2} \cos(\theta)$$

$$\begin{bmatrix} a_{mn} \\ A_n \end{bmatrix} = \begin{bmatrix} r_m \end{bmatrix}$$

How many terms of A_n ?

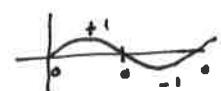
Worst case: How many terms of $c(\theta) = \sum_{n=1}^{\infty} c_n \sin(n\theta)$ are necessary to represent a step/pulse? Infinite....

$$c(\theta) = \sum c_n \sin(n\theta)$$

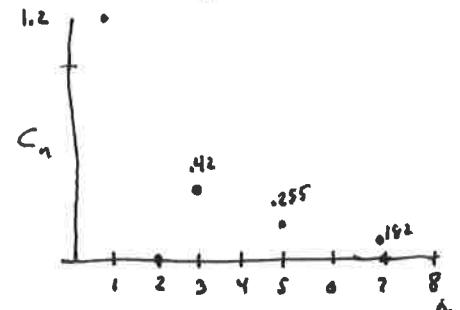
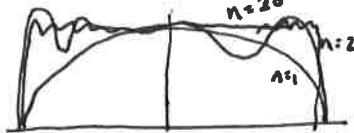
Integrate and multiply by $\sin(m\theta)$

$$\int_0^\pi c(\theta) \sin(m\theta) d\theta = \int_0^\pi C_n \sin(n\theta) \sin(m\theta) d\theta$$

$$C_n = \frac{2}{\pi} \int_0^\pi c(\theta) \sin(n\theta) d\theta$$



$$= \frac{4}{\pi n} \sin^2 \left(\frac{\pi n}{2} \right) = \frac{4}{\pi n} \text{ when } n \text{ is odd}$$



"Induced" Drag: Drag due to Lift.

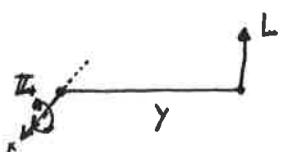
$$\begin{aligned}
 D_i &= -\rho \int_{-\frac{b}{2}}^{\frac{b}{2}} \Gamma w_{wake} dy = \rho \int_0^{\pi} \Gamma(\theta) w_{wake}(\theta) \frac{b}{2} \sin \theta d\theta \\
 &= \rho \int_0^{\pi} 2b V_{\infty} \sum_{n=1}^{\infty} A_n \sin(n\theta) \cdot V_{\infty} \sum_{n=1}^{\infty} n A_n \frac{\sin(n\theta)}{\sin \theta} \cdot \frac{b}{2} \sin \theta d\theta \\
 &= \rho 2b V_{\infty}^2 \frac{b}{2} \underbrace{\int_0^{\pi} n A_n^2 \sin(n\theta)^2 \frac{\sin \theta}{\sin \theta} d\theta}_{\frac{\pi b}{2}} \\
 &= \frac{1}{2} \rho V_{\infty}^2 \cdot \pi b^2 \sum n A_n^2
 \end{aligned}$$

$$C_{D_i} = \frac{D_i}{\frac{1}{2} \rho V_{\infty}^2 S} = \frac{\frac{1}{2} \rho V_{\infty}^2 \cdot \pi b^2 \sum n A_n^2}{\frac{1}{2} \rho V_{\infty}^2 S} = \pi AR \sum n A_n^2$$

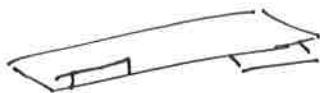
FVA typo

$$= \frac{C_L^2}{\pi AR} (1 + \delta) \quad \text{where } \delta = \sum n \left(\frac{A_n}{A_1} \right)^2$$

Roll Moment



Moment about x axis



$$M = \rho V_\infty \int_{-b/2}^{b/2} \Gamma y dy = \rho V_\infty \int_0^\pi 2b V_\infty \sum_{n=1}^{\infty} A_n \sin n\theta \frac{b}{2} \cos \theta \frac{b}{2} \sin \theta d\theta$$

$$= \rho V_\infty 2b V_\infty \frac{b^2}{4} \sum_{n=1}^{\infty} A_n \underbrace{\sin \frac{n\pi}{n^2-4}}_{\frac{\pi}{4} \text{ only when } n=2} = 0$$

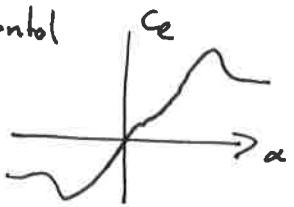
Wrong! Notice $\frac{\sin(n\pi)}{n^2-4}$ = undefined when $n=2$.

$$C_L = \frac{M}{g S b} \cdot \frac{\frac{1}{2} \rho V^2 b \left(b^2 \frac{\pi}{4} A_2 \right)}{\frac{1}{2} \rho V^2 S b} = \frac{\pi R}{4} A_2$$

The wing's roll moment only depends on A_2 .

Arbitrary and Experimental Data

Given experimental data, lifting line theory can incorporate experimental C_e vs α and C_d vs α data.



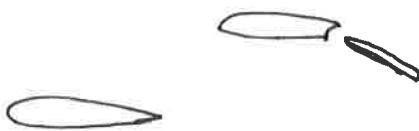
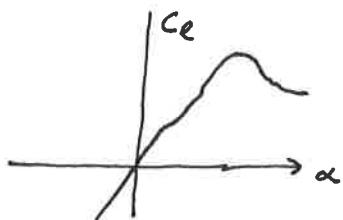
How?

$$\Gamma = 2b V_\infty \sum A_n \sin(n\theta) \quad \text{same as before.}$$

$$\Gamma = \frac{1}{2} V_\infty C(y) \underbrace{C_{e,a}(\alpha + \alpha_{aero} - \alpha_i)}_{}$$

This is just C_e as a function of α , airfoil + wing geometry and the "induced" angle of attack from the wake.

Replace $C_{e,a}(\alpha + \alpha_{aero} - \alpha_i)$ with data



Gov Egu.

$$2b V_\infty \sum A_n \sin(n\theta) = \frac{1}{2} V_\infty C(y) \underbrace{C_e(\alpha)}_{\alpha - \alpha_i}$$

$$\text{where } \alpha_i \approx \frac{\omega_{wake}}{V_\infty} = \sum n A_n \frac{\sin(n\theta)}{\sin \theta}$$

$$4b \sum A_n \sin(n\theta) = C(y) \underbrace{C_e(\alpha - \sum n A_n \frac{\sin(n\theta)}{\sin \theta})}_{}$$

solve for A_n

proceed with regular process