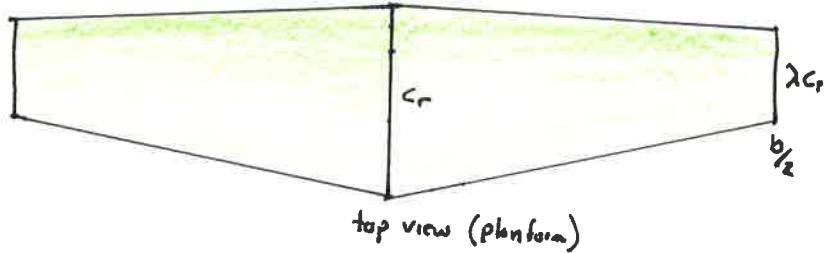


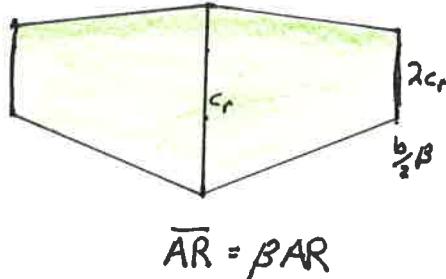
Lesson 24  
Compressible Wings

# Subsonic Compressible Wing



The Prandtl-Glauert equation  $(1 - M_{\infty}^2) \phi_{xx} + \phi_{yy} + \phi_{zz} = 0$  allows a transformation from  $M_{\infty}$  to  $M = 0$  (incompressible).

$$\begin{aligned}\bar{x} &= x \\ \bar{y} &= \beta y \quad \beta = \sqrt{1 - M_{\infty}^2} \\ \bar{z} &= \beta z \quad \text{incomp} \quad \text{comp}\end{aligned}$$



$$\bar{\alpha} = \beta \alpha \quad \bar{n} = \begin{pmatrix} \beta n_x \\ n_y \\ n_z \end{pmatrix} \quad \bar{M} = \frac{M}{\beta}$$

From lifting line theory,

$$\bar{C}_L \approx \frac{C_{ea}}{1 + C_{ea}/\pi \bar{AR}} \cdot \bar{\alpha} \quad \text{and} \quad \bar{C}_{D_L} = \frac{\bar{C}_L^2}{\pi \bar{AR} \bar{\epsilon}}$$

Apply incomp and PG transform, and Götherts rule.

$$C_p = \frac{1}{\beta^2} \bar{C}_p \Rightarrow C_L = \frac{1}{\beta^2} \bar{C}_L \text{ and } C_D = \frac{1}{\beta^2} \bar{C}_D$$

$$C_L = \frac{1}{\beta^2} \cdot \bar{C}_L = \frac{1}{\beta^2} \cdot \frac{C_{L\infty}}{1 + C_{L\infty}/\pi \bar{A}R} \cdot \beta \alpha = \frac{C_{L\infty}}{\beta + \frac{C_{L\infty}}{\pi \bar{A}R}} \alpha$$

$$C_{D_i} = \frac{1}{\beta^3} \bar{C}_{D_i} = \frac{1}{\beta^3} \frac{\bar{C}_L^2}{\pi \bar{A}R \bar{e}} = \frac{1}{\beta^3} \frac{(\beta^2 \bar{C}_L^2)^2}{\pi \bar{A}R \bar{e}} = \frac{\bar{C}_L^2}{\pi \bar{A}R \bar{e}}$$

High AR wings

$$\beta \gg \frac{C_{L\infty}}{\pi \bar{A}R^\infty} \Rightarrow C_{L\alpha} \underset{\bar{A}R \rightarrow \infty}{\approx} \frac{C_{L\infty}}{\beta}$$

Low AR wings

$$\beta \ll \frac{C_{L\infty}^\infty}{\pi \bar{A}R^\infty} \Rightarrow \frac{C_{L\infty}}{\frac{C_{L\infty}}{\pi \bar{A}R}} = \pi \bar{A}R$$

$$C_{L\alpha} \underset{\bar{A}R \rightarrow 0}{\propto} \pi \bar{A}R$$

Not a function of  $\beta$ !

Low aspect ratio wings are less influenced by compressibility.

Compressibility has only a slight negative impact on induced drag for non-elliptical planforms.

$$C_{D_i} \underset{\text{incomp}}{\approx} C_{D_i}$$

# Supersonic Wing

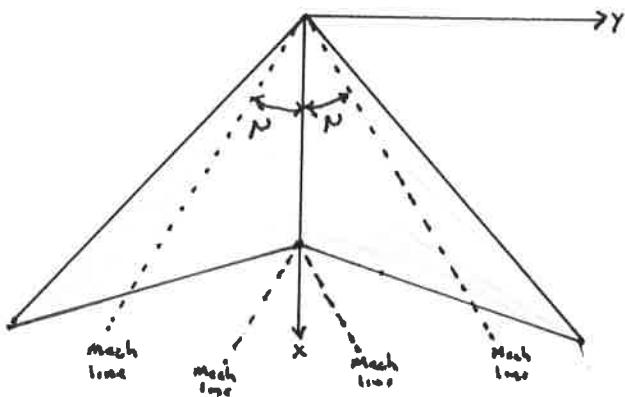
Using the linear hyperbolic PG equation  $-\sqrt{M^2 - 1} \phi_{xx} + \phi_{yy} + \phi_{zz} = 0$

with BCs

$$\phi_z = \frac{\partial z_0}{\partial x} \text{ on upper}$$

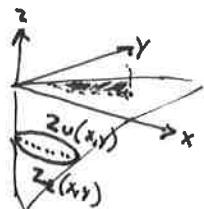
$$\frac{dz_0}{dx} \text{ on lower.}$$

Simple Planform  $\equiv$  supersonic LE and TE



No interaction between upper and lower surfaces since any Mach cone is within planform.

## Non lifting wings



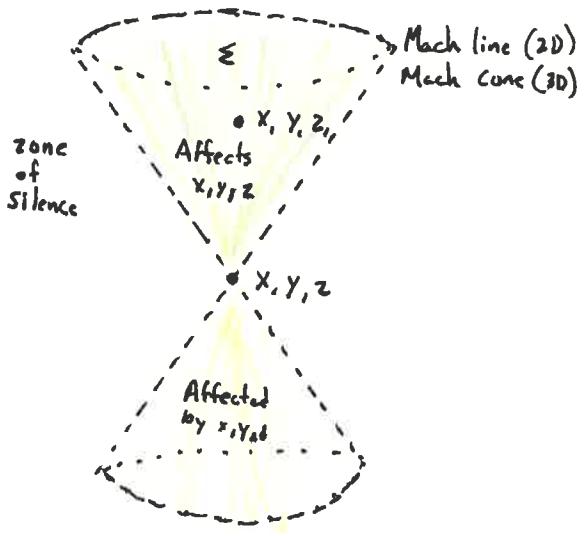
$$z_e = -z_u \quad \text{thus source w velocity is symmetric about } z=0 \\ \text{call it } w_o(x, y)$$

The potential at  $x, y, z$  from an infinitesimal source at  $x_i, y_i, z_i$  is

$$d\phi(x, y, z) = \frac{-1}{2\pi} \frac{C(x_i, y_i) dx_i dy_i}{\sqrt{(x-x_i)^2 - \beta^2((y-y_i)^2 + z^2)}}$$

where only the upstream Mach cone defines the integration.

and the total source strength is double one side ( $C = 2w_o$ )



Thus the potential is

$$\phi(x, y, z) = -\frac{1}{\pi} \iint_R \frac{w_o(x, y, z) dx dy}{(x - x_1)^2 + \beta^2((y - y_1)^2 + z^2)}$$

upstream Mach cone,

The pressure is

$$C_p = -2\phi_x$$

2D Wing: (independent of  $y$ )

$$C_{p+}(x, y, 0^+) = \underset{\text{integral}}{\text{complimented}} = \frac{2}{\beta} \frac{dz_u}{dx}$$

$$C_{p-}(x, y, 0^-) = \dots = \frac{2}{\beta} \frac{dz_c}{dx}$$

$$C_p = C_{p+} + C_{p-} = \frac{4}{\beta} \frac{dz_c}{dx}$$

This is the Ackeret supersonic  $C_{p\alpha}$  result!

# Lifting Supersonic Wings (Simple Planform)

The pressure difference is (~~Ashley and Landahl~~)

$$\Delta C_p = \frac{4\alpha}{\pi\beta} \frac{m}{\sqrt{m^2-1}} \operatorname{Re} \left( \cos \frac{1-m\tau}{m-\gamma} + \cos \frac{1+m\tau}{m+\gamma} \right)$$

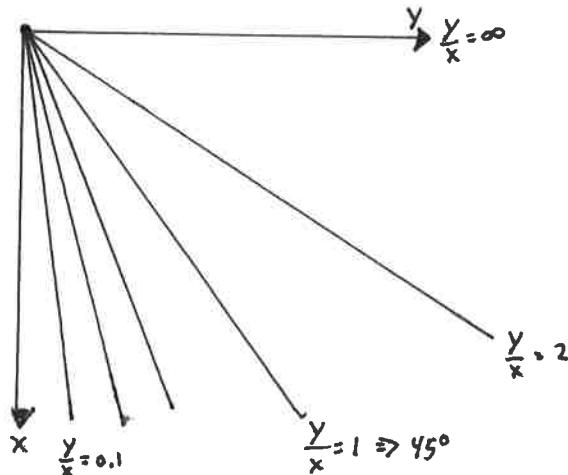
where  $m = \frac{\beta}{\tan \Lambda}$

$$\tau = \frac{By}{x}$$

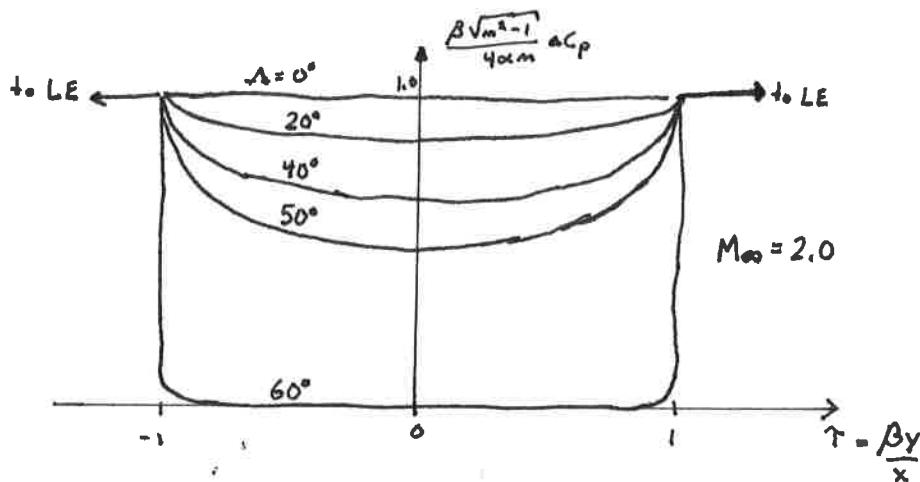
What does this represent?

Integrating over the wing gives  
 $C_L = \frac{4\alpha}{\beta}$

Conical flow

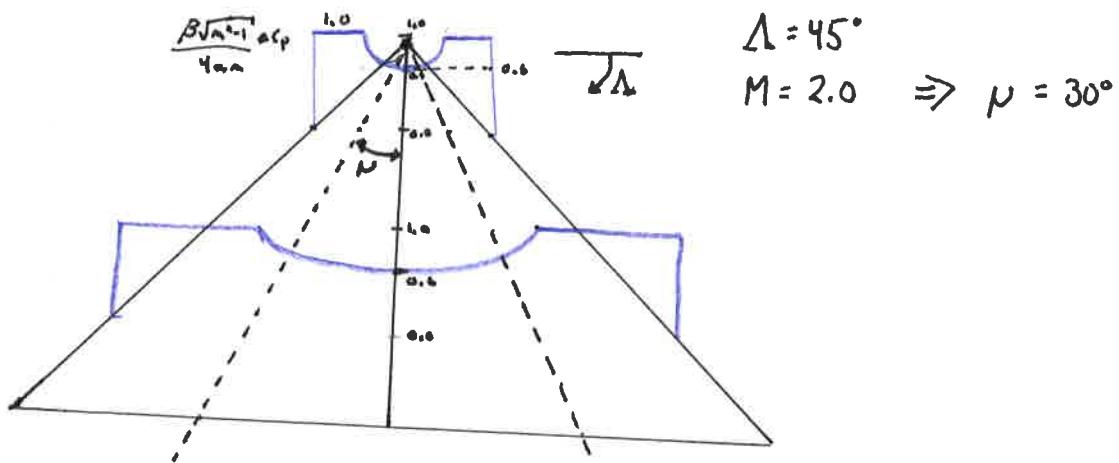


$\Delta C_p$  is constant along  $\tau$



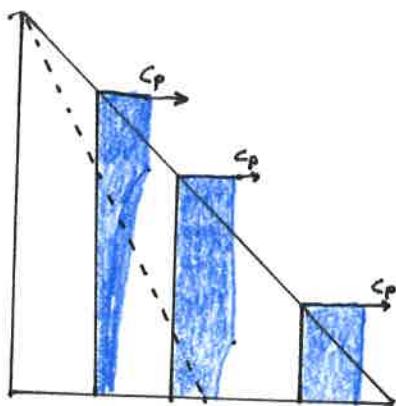
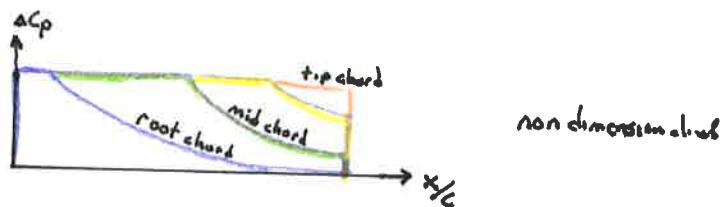
[ Increasing sweep tends to load the LE more ]

Ex.



Note that when  $\Lambda$  is behind  $p$  line, the upper surface communicates with the lower through the LE.

Chordwise  $\Delta C_p$



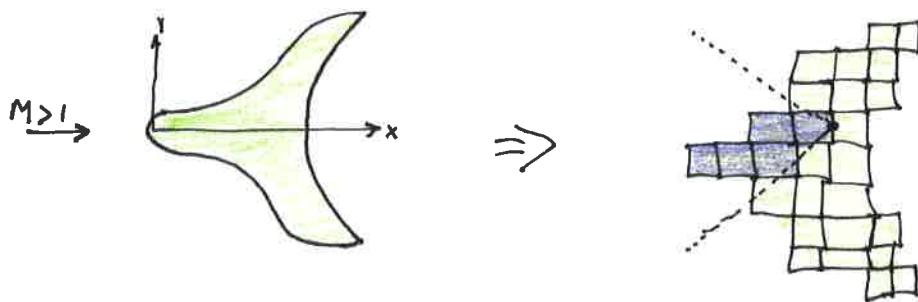
# Supersonic Pressure Distribution (Carlson + Miller Numerical Approach)

$$\Delta C_p(x, y) = -\frac{4}{\beta} \frac{\partial Z_c(x, y)}{\partial x} + \frac{1}{\pi} \iint_S R(x-x_i, y-y_i) \Delta C_p(x_i, y_i) d\beta y_i dx_i$$

with  $R(x-x_i, y-y_i) = \frac{x-x_i}{\beta^2(y-y_i)^2 \left[ (x-x_i)^2 - \beta^2(y-y_i)^2 \right]^{1/2}}$

where  $S$  is the zone of action (upstream Mach cone)

This method is applicable to a finite patch approximation.



This method work well for supersonic wings

See Bertin + Smith, Section 11.7 for implementation details.