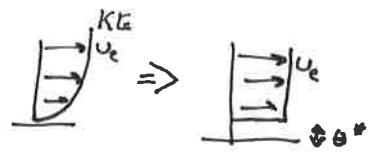


Lesson 26 part 2  
IBL Kinetic Energy  
Crossflow

# Integral Boundary Layer

$$\Theta^* = \int_0^{n_e} \left(1 - \frac{u^2}{U_e^2}\right) \frac{\rho u}{\rho_e U_e} dn \quad \text{Kinetic energy thickness}$$



How does  $\Theta^*$  evolve with  $s$ ?

$$\frac{d}{ds} \left( \frac{1}{2} \rho_e U_e^3 \Theta^* \right) = D - \rho_e U_e^2 \delta^{**} \frac{du_e}{ds} = \frac{dK}{ds}$$

Kinetic energy defect  $K$       Dissipation Integral      density variation in pressure gradient

Compare with vorticity generation when  $\nabla p \times \nabla p \neq 0$  previously in class.

when the density flux thickness is  $\delta^{**} = \int_0^{n_e} \left(1 - \frac{\rho}{\rho_e}\right) \frac{u}{U_e} dn$

$$D = \int_0^{n_e} \tau \frac{du}{dn} dn = \int_0^{n_e} (\nu + \nu_T) \left(\frac{du}{dn}\right)^2 dn \quad \text{when } \sim \text{Newtonian - Boussinesq viscosity flow}$$

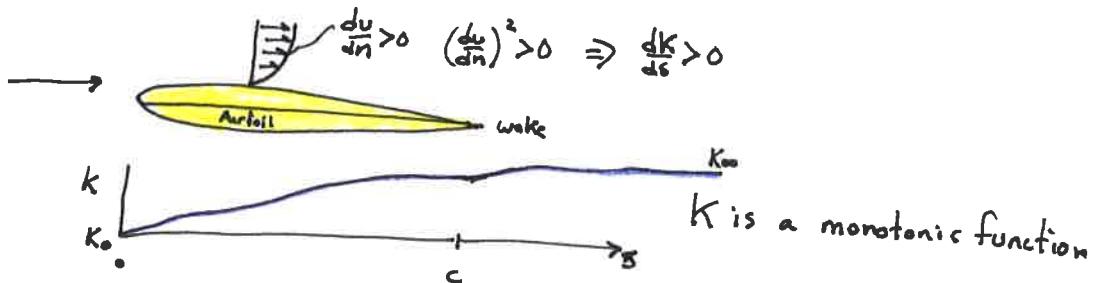
$\geq 0$        $\geq 0$

Dissipation is a positive function of  $\frac{du}{dn}$

$$\tau \approx (\nu + \nu_T) \frac{du}{dn}$$

For incompressible flows (or flows where density in the BL changes only slightly),

$$\frac{dK}{ds} = D \geq 0 \quad \text{on both the airfoil and in the wake.}$$



## Drag Power Balance

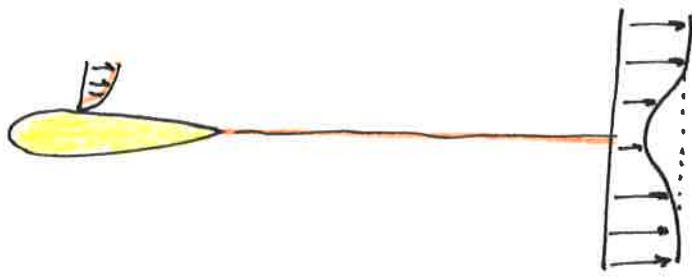
$$D' V_\infty = K_\infty = \int_{-\infty}^{\infty} D ds$$

$$D' = \frac{1}{V_0} \int_0^{\infty} \int_0^{n_e} (\nu + \nu_T) \left(\frac{du}{dn}\right)^2 dn$$

volume integral      boundary layer profile

Remember that this uses the Thin Boundary Layer equation (TSL) as a starting point.

# Drag Comparison of Momentum Deficit/Detect and Dissipation / Kinetic Energy Deficit



Momentum:

$$\frac{\text{Drag}}{\text{span}} = D' = P_\infty \quad \text{where} \quad \frac{dp}{ds} \approx \gamma_w + \delta^* \frac{du}{ds}$$

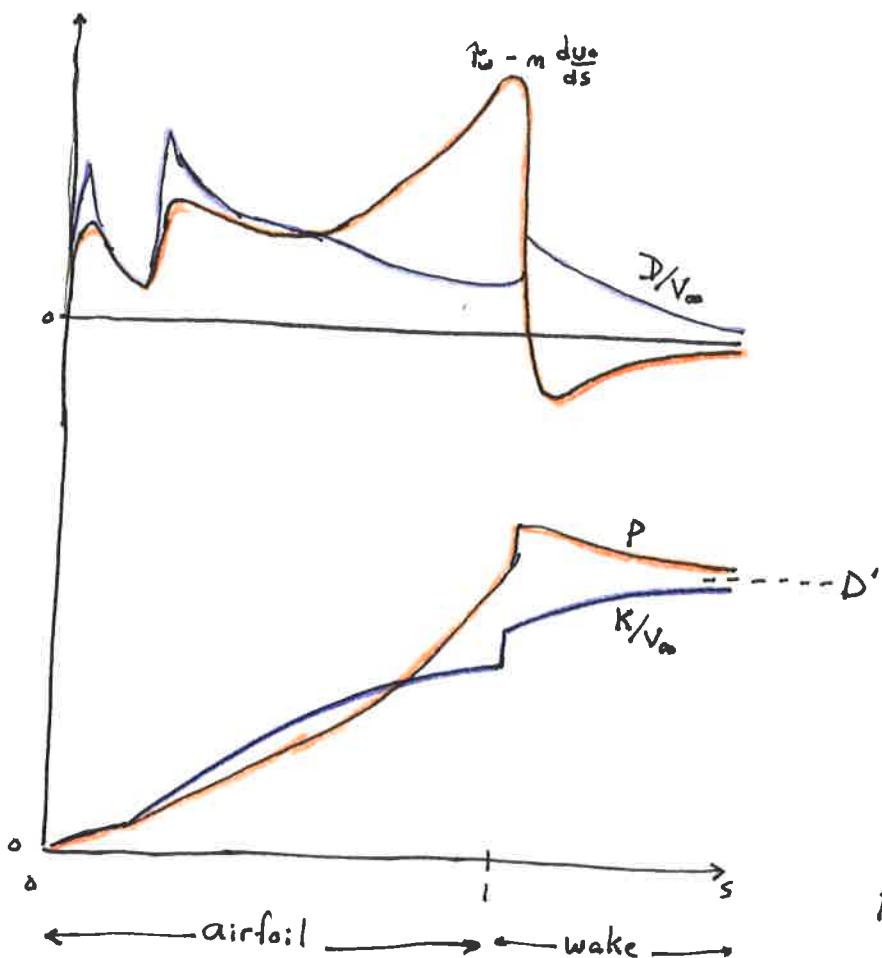
$$= \gamma_w + f_e U_e \delta^* \frac{du_e}{ds} = \gamma_w + m \frac{du_e}{ds}$$

Energy:

$$D' = \frac{1}{V_\infty} \int_0^\infty \int_{\infty}^{\infty} (\nu + \nu_r) \left( \frac{du}{dn} \right)^2 dn$$

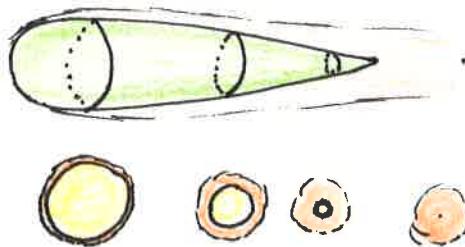
This is obviously strictly positive!

strictly positive?  
Separation gives  $\gamma_w < 0$   
Magnitude of  $\gamma_w$  vs.  $m \frac{du_e}{ds}$ ?



FVA Fig 4.10

# Axi-symmetrical Bodies



$b$  = perimeter at a section

- BL behavior for 3D shapes differs from a 2D shape because of flow divergence resulting from the area change

"Over the rear of the body, the momentum thickness  $\theta(s)$  increases faster than in 2D, due to the viscous fluid flowing onto a progressively smaller perimeter" (FVA Fig 4.17)

3D:

$$\frac{d\theta}{ds} = \frac{C_f}{2} - \left( H + 2 - M_e^2 \right) \frac{\theta}{U_e} \frac{du_e}{ds} - \frac{\theta}{b} \frac{db}{ds}$$

2D:

$$\frac{d\theta}{ds} = \frac{C_f}{2} - \left( H + 2 - M_e^2 \right) \frac{\theta}{U_e} \frac{du_e}{ds}$$

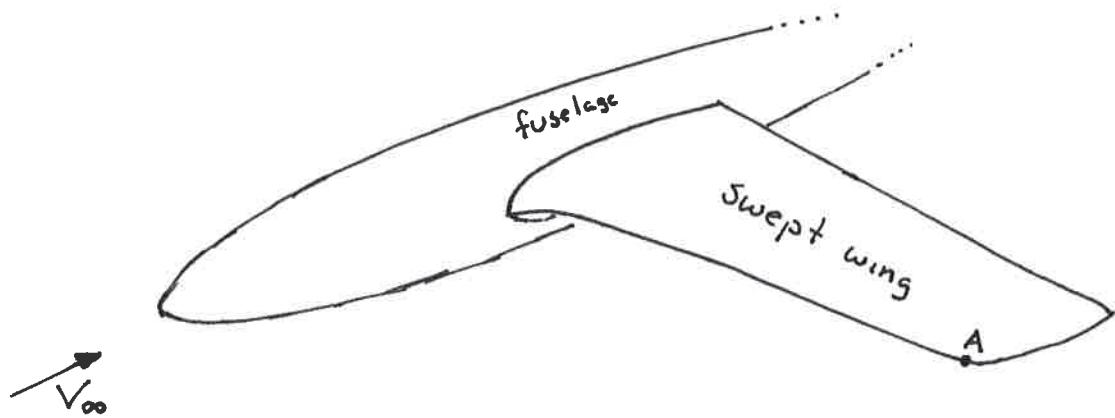
- When the perimeter is growing (i.e. forward portion)  $\frac{db}{ds} > 0$  and  $\frac{d\theta}{ds}$  grows more slowly than 2D.
- When the perimeter is shrinking (aft portion),  $\frac{db}{ds} < 0$  and  $\frac{d\theta}{ds}$  grows faster than 2D.

Aside:

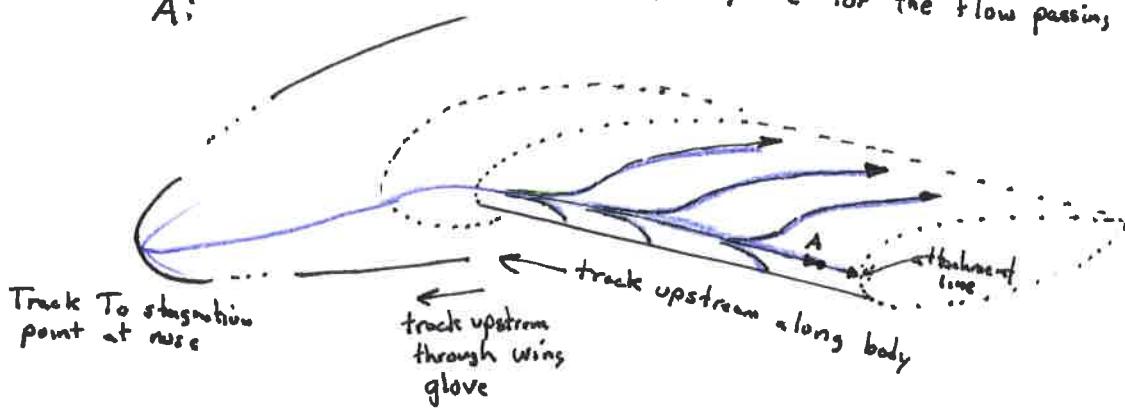
Have you ever wondered why the BL in a wind tunnel is relatively ~~thin~~ thin?

$$\frac{dp}{ds} \text{ and } \frac{db}{ds}$$

# Crossflow Convergence and Divergence.

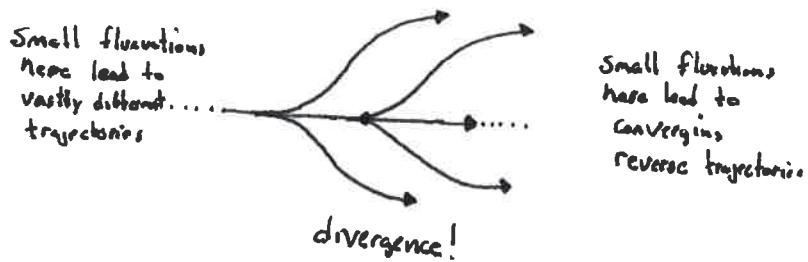


Q: Where is the stagnation point/line for the flow passing through point A?



Q: Boundary Layer thickness depends on distance "S" from start/stagnation pt. So, how thick is the BL at A? It should be very thick after traveling that distance, right?

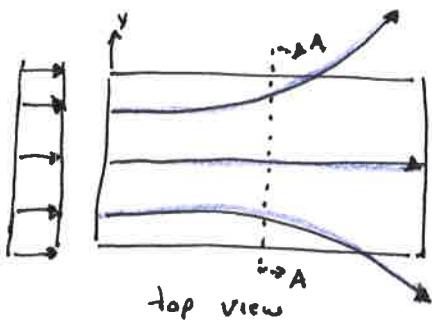
A: Very thin! Divergence along the attachment line keeps the BL thin.



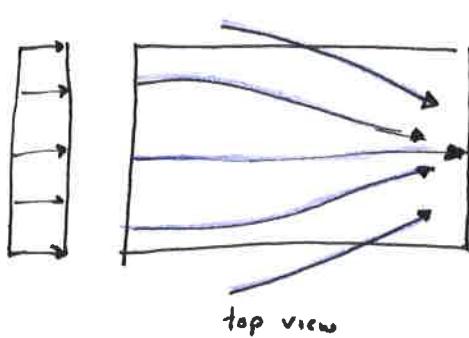
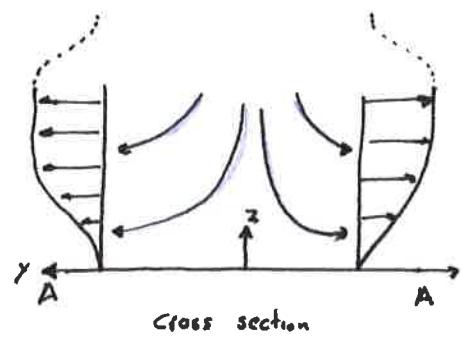
Recall the 3D axisymmetric term  $-\frac{\partial}{\partial s} \frac{db}{ds}$  and reform into a velocity divergence term,

Q: What about the TE?

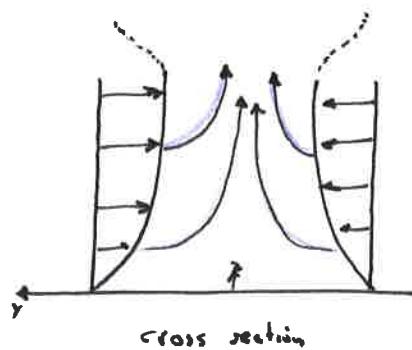
## Cross flow Convergence and Divergence



Divergence.



Convergence



Convergence tends to increase the growth rate of BL.

or

Convergence tends to "pump" BLs upward. Divergence "pulls" freestream properties down.

Please review the ONERA 3D Separation pdfs.

Interpretation of surface flows allows for 3D flow understanding.