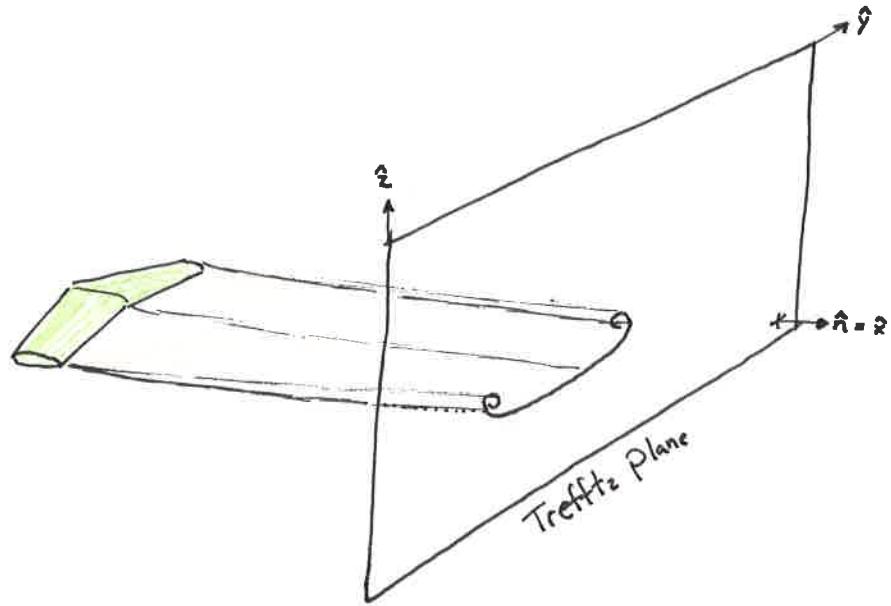


Lesson 27

Aero Forces part 2

Treffitz Plane



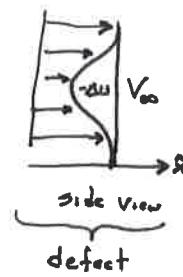
Drag computed on Treffitz plane

$$D = F \cdot \hat{x} = \iint_{\text{outer}} (P_\infty - P) \hat{n} \cdot \hat{x} - \rho (V \cdot \hat{n}) (U - U_\infty) dS$$

$$= \iint_{TP} P_\infty - P - \rho U (U - U_\infty) dy dz$$

Decompose velocity at TP into viscous velocity defect and streamwise vorticity.

$$V = (\underbrace{V_\infty + \Delta U}_{\text{freestream defect}}) \hat{x} + \underbrace{\nabla \phi}_{\text{potential gradient}}$$



CC 99

front view
vorticity in streamwise dir.

Low perturbation velocities (Bernoulli) in y-z plane

$$P = P_\infty + \frac{1}{2} \rho V_\infty^2 = P + \frac{1}{2} \rho_\infty V^2$$

rearrange to

$$P = P_\infty + \frac{1}{2} \rho_\infty V_\infty^2 - \frac{1}{2} \rho_\infty V^2$$

Notice that only the potential portion of the TP velocity impacts pressure. The defect portion does not!

$$P = P_\infty + \frac{1}{2} \rho_\infty V_\infty^2 - \frac{1}{2} \rho_\infty (V_\infty \hat{x} + \nabla \phi)^2$$

Look at the term $(V_\infty \hat{x} + \nabla \phi)^2$

$$V_\infty \hat{x} + \nabla \phi = \begin{pmatrix} V_\infty \hat{x} \\ 0 \hat{y} \\ 0 \hat{z} \end{pmatrix} + \begin{pmatrix} \phi_x \hat{x} \\ \phi_y \hat{y} \\ \phi_z \hat{z} \end{pmatrix} = \begin{pmatrix} (V_\infty + \phi_x) \hat{x} \\ \phi_y \hat{y} \\ \phi_z \hat{z} \end{pmatrix}$$

Square this (Keep vector terms separate!!)

$$(V_\infty \hat{x} + \nabla \phi)^2 = \begin{pmatrix} (V_\infty + \phi_x)^2 \\ \phi_y^2 \\ \phi_z^2 \end{pmatrix} = V_\infty^2 + 2V_\infty \phi_x + \phi_x^2 + \phi_y^2 + \phi_z^2$$

Substitute into p

$$\begin{aligned} p &= P_\infty + \underbrace{\frac{1}{2} \rho_\infty V_\infty^2}_{\textcircled{1}} - \frac{1}{2} \rho_\infty \left(V_\infty^2 + 2V_\infty \phi_x + \phi_x^2 + \phi_y^2 + \phi_z^2 \right) \\ &= P_\infty - \rho_\infty V_\infty \phi_x - \frac{1}{2} \rho_\infty (\phi_x^2 + \phi_y^2 + \phi_z^2) \end{aligned}$$

Substitute into TP Drag eqn

$$\begin{aligned} D &= \iint_{TP} P_\infty - p - \rho u (u - V_\infty) dy dz \quad U = V \cdot \hat{x} = V_\infty + \phi_x + \Delta u \\ &= \iint_{TP} \underbrace{P_\infty - P_\infty}_{=0} + \rho_\infty V_\infty \phi_x + \frac{1}{2} \rho_\infty (\phi_x^2 + \phi_y^2 + \phi_z^2) - \rho (V_\infty + \Delta u + \phi_x) \underbrace{(V_\infty + \Delta u + \phi_x - V_\infty)}_{\Delta u + \phi_x} dy dz \\ &= \iint_{TP} \underbrace{\rho_\infty V_\infty \phi_x + \frac{1}{2} \rho_\infty (\phi_x^2 + \phi_y^2 + \phi_z^2)}_{\text{Potential only term "vorticity"}, \text{ "viscous!}} - \rho V_\infty \Delta u - \rho V_\infty \phi_x - \rho \Delta u^2 - \rho \Delta u \phi_x - \rho \phi_x \Delta u - \rho \phi_x^2 - 2\rho \Delta u \phi_x dy dz \\ &= \iint_{TP} \underbrace{\frac{1}{2} \rho_\infty (\phi_y^2 + \phi_z^2 - \phi_x^2)}_{\text{Potential only term "vorticity"}, \text{ "viscous!}} + \rho (-\Delta u) \underbrace{(V_\infty + 2\phi_x + \Delta u)}_{\text{Velocity defect term "viscous!}} dy dz \end{aligned}$$

Separate into induced drag and profile drag components

$$\text{Drag} = D_i + D_p$$

$$D_i = \iint_{TP} \frac{1}{2} \rho_\infty (\phi_y^2 + \phi_z^2 - \phi_x^2) dTP$$

$$\frac{\phi}{\phi_x} = \frac{d\phi}{dx} =$$

$$D_p = \iint_{TP} \rho (V_\infty + 2\phi_x + \Delta u) (-\Delta v) dTP$$

↑ when $\phi_x \ll V_\infty + \Delta u$

$$D_p \approx \iint \rho u (V_\infty - u) dS$$

seen this before? Momentum defect term P .

$$= \int P dS$$

$$= \iint \rho u (V_\infty - u) dS$$



$$D_p = \rho_\infty V_\infty \theta_\infty$$

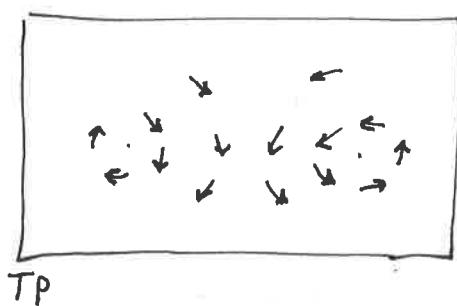
Induced drag components

$$\text{What is } \phi_x? \quad \phi_y = ? \quad \phi_z = ?$$

$$\text{perturbation velocities.} \quad \phi_x = u \quad \phi_y = v \quad \phi_z = w$$

$$D_i = \iint_{TP} \frac{1}{2} \rho_\infty (v^2 + w^2 - u^2) dTP \approx \iint_{TP} \frac{1}{2} \rho_\infty (v^2 + w^2) dTP$$

Kinetic energy of non streamwise flow

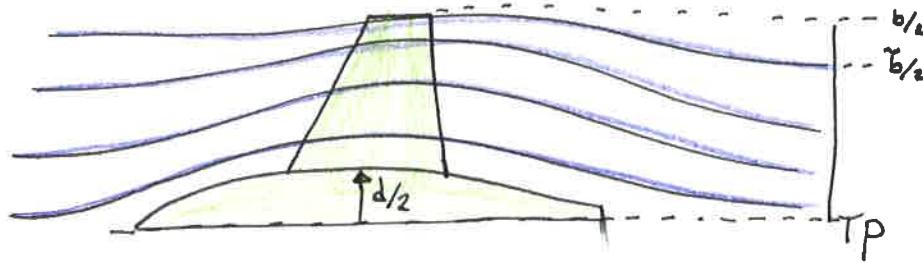


Induced drag is what we call the kinetic energy contained in the non-streamwise flow.

"Rotational Flow Drag"

Fuselage Wake Contraction

Fact: Wings are usually mounted on a fuselage at a certain width.



The aft fuselage contraction reduces the effective span at the TP.

Geometric span $b \Rightarrow$ Trefftz span \tilde{b}

From mass conservation at the wing and at the TP: $m_{\text{wing}} = m_{\text{TP}}$

$$y = \sqrt{\tilde{y}^2 + (\frac{d}{2})^2}$$

for a ratio

$$\left(\frac{\tilde{b}}{b}\right)^2 = 1 - \left(\frac{d}{b}\right)^2 \Rightarrow AR = \frac{b^2}{S} \Rightarrow \tilde{AR} = \frac{\left(\frac{\tilde{b}}{b}\right)^2 b^2}{S} = AR \left(1 - \left(\frac{d}{b}\right)^2\right)$$

Replace all occurrences of span b^2 in previous theory by \tilde{b}^2

For an elliptical wing

~~$$C_{L\alpha} \approx \frac{2\pi}{1 + \frac{2}{AR}}$$~~
$$\Rightarrow C_{L\alpha_{\text{fuselage}}} \approx \frac{2\pi}{1 + \frac{2}{AR} \left(1 - \left(\frac{d}{b}\right)^2\right)^{-1}}$$

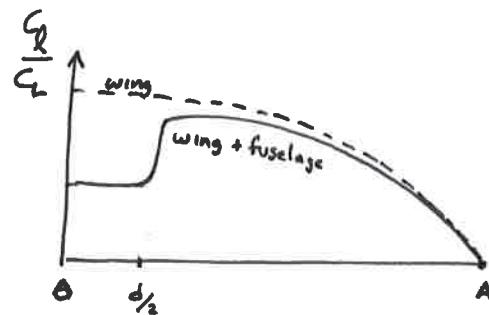
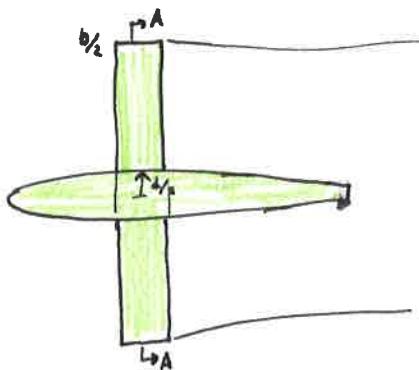
$$C_{D_i} = \frac{C_L^2}{\pi AR e} \Rightarrow \frac{C_L^2}{\pi AR \left(1 - \left(\frac{d}{b}\right)^2\right) e}$$

A fuselage slightly decreases aero performance of a raw wing.

$$\left(\frac{\tilde{b}}{b}\right)^2 = 1 - \left(\frac{d}{b}\right)^2 \quad \text{for } \frac{d}{b} \approx 10\% \Rightarrow \left(\frac{\tilde{b}}{b}\right)^2 = 99\%$$

Worse, the fuselage is less efficient at lift production.

The spanwise loading ~~max~~ will decrease efficiency.



Note:

We have now entered the portion of the class where material will come from papers, other books, and the Aerodynamic Design of Transport Aircraft.

We have almost finished the FVA book.