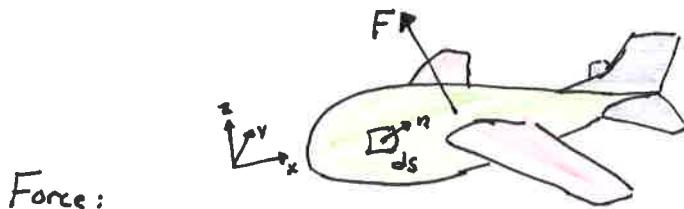


Lesson 27  
Aerodynamic Force Analysis

FVA Chapter 5

# Aerodynamic Force Analysis

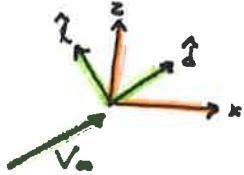
Near Field = On the body's surface



Force:

$$\begin{aligned}
 F &= F_{\text{pressure}} + F_{\text{friction}} \\
 &= \iint_S -P \hat{n} dS + \iint_S \bar{\tau}_w dS \\
 &= \underbrace{\iint_S (P_\infty - P) \hat{n} dS}_{\text{pressure}} + \underbrace{\iint_S \bar{\tau}_w \cdot \hat{n} dS}_{\text{viscous}} \quad \text{since } \iint_S P_\infty dS = 0
 \end{aligned}$$

Decompose into  $\hat{x}$ ,  $\hat{y}$ , and  $\hat{z}$  forces or lift  $\hat{l}$  and drag  $\hat{d}$  forces  
where  $\hat{l}$  is perpendicular to  $V_\infty$  and  $\hat{d}$  is parallel to  $V_\infty$



$$D = F \cdot \hat{d} = \iint_S (P_\infty - P) \hat{n} \cdot \hat{d} dS + \iint_S (\bar{\tau}_w \cdot \hat{n}) \cdot \hat{d} dS$$

and similarly for Lift.

Warning: Since drag is usually small for airfoils and wings, the near-field drag calculation is often inaccurate.

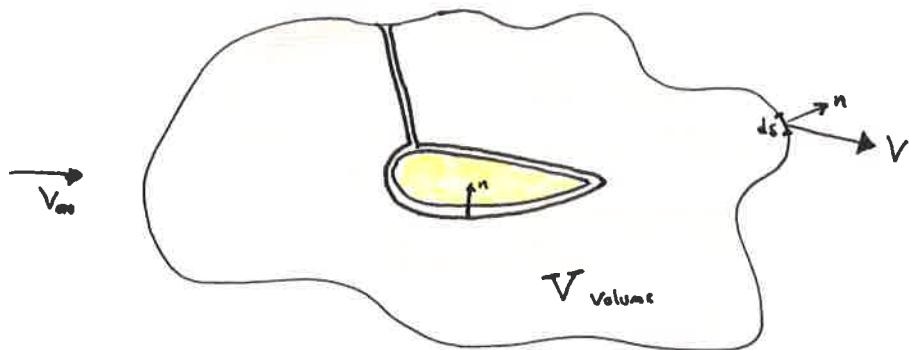
Warning:

Also, calculations / measurements of  $P$ ,  $\bar{\tau}$  and  $\hat{n}$  may be inaccurate in experimental environments.

See FVA 5.1.2

## Far Field Forces

Consider a control volume around/enclosing the vehicle (FVA Fig 5.3)



The general integral momentum equation is

$$\iiint \frac{\partial \rho \mathbf{V}}{\partial t} dV + \iint \rho (\mathbf{V} \cdot \hat{n}) \mathbf{V} dS = \iiint \rho f dV + \iint -p \hat{n} dS + \iint \bar{\tau} \cdot \hat{n} dS$$

Steady state, no body forces

$$\iint \rho (\mathbf{V} \cdot \hat{n}) \mathbf{V} dS + \iint p \hat{n} dS - \iint \bar{\tau} \cdot \hat{n} dS = 0$$

$$\underbrace{\iint (\rho (\mathbf{V} \cdot \hat{n}) \mathbf{V} + p \hat{n} - \bar{\tau} \cdot \hat{n}) dS}_{I} = 0$$

The CV above has 3 parts/boundaries: outer, cut, and body

$$\iint_{\text{outer}} I dS + \iint_{\text{cut}} I dS + \iint_{\text{body}} I dS = 0$$

equal + opposite = 0

On the body,  $\mathbf{V} \cdot \hat{n} = 0$  (even valid for Euler flows)

$$\iint_{\text{body}} I dS = \iint_{\text{body}} (\rho (\mathbf{V} \cdot \hat{n}) \mathbf{V} + p \hat{n} - \bar{\tau} \cdot \hat{n}) dS = \iint_{\text{body}} (p \hat{n} - \bar{\tau} \cdot \hat{n}) dS$$

This term is exactly the Near Field Force (see prev slide)

$$\iint_{\text{body}} I dS = F$$

## Integral Momentum Theorem

$$\iint_{\text{outer}} I dS + \iint_{\text{body}} I dS = 0$$

Thus, if  $\bar{T}$  on the outer boundary is zero,

$$F = \iint_{\text{outer}} (P_\infty - P) \hat{n} - \rho (V \cdot \hat{n}) (V - V_\infty) dS$$