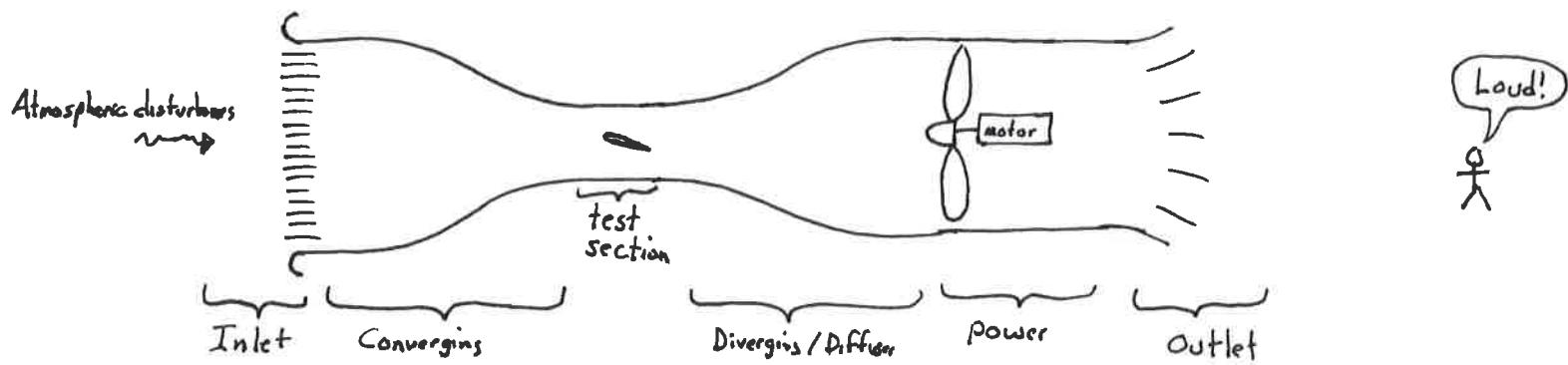
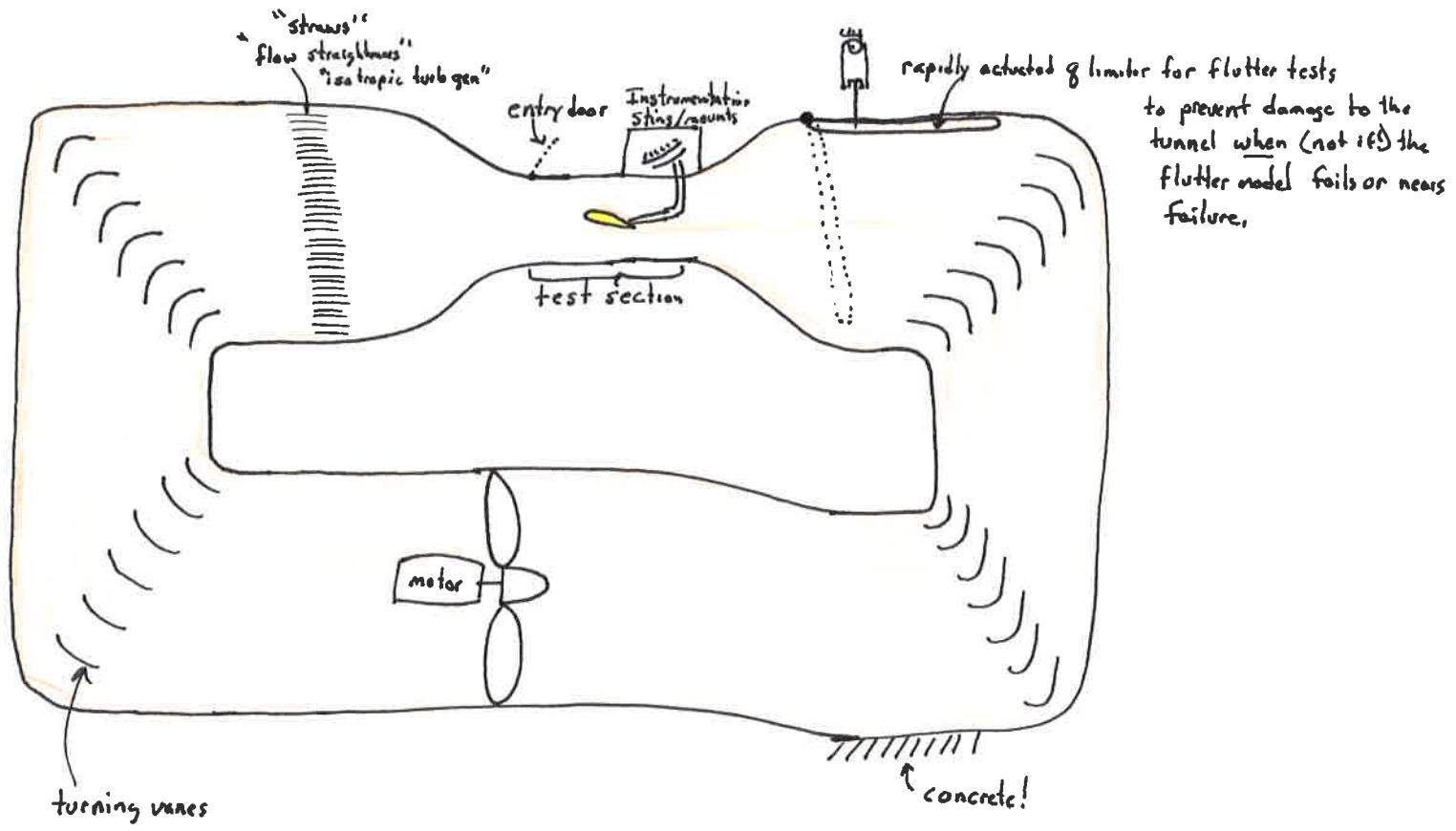


Lesson 31
Wind Tunnel Corrections

Open loop



Closed loop (LSWT)



Q: Why does $Re \#$ change throughout the day?

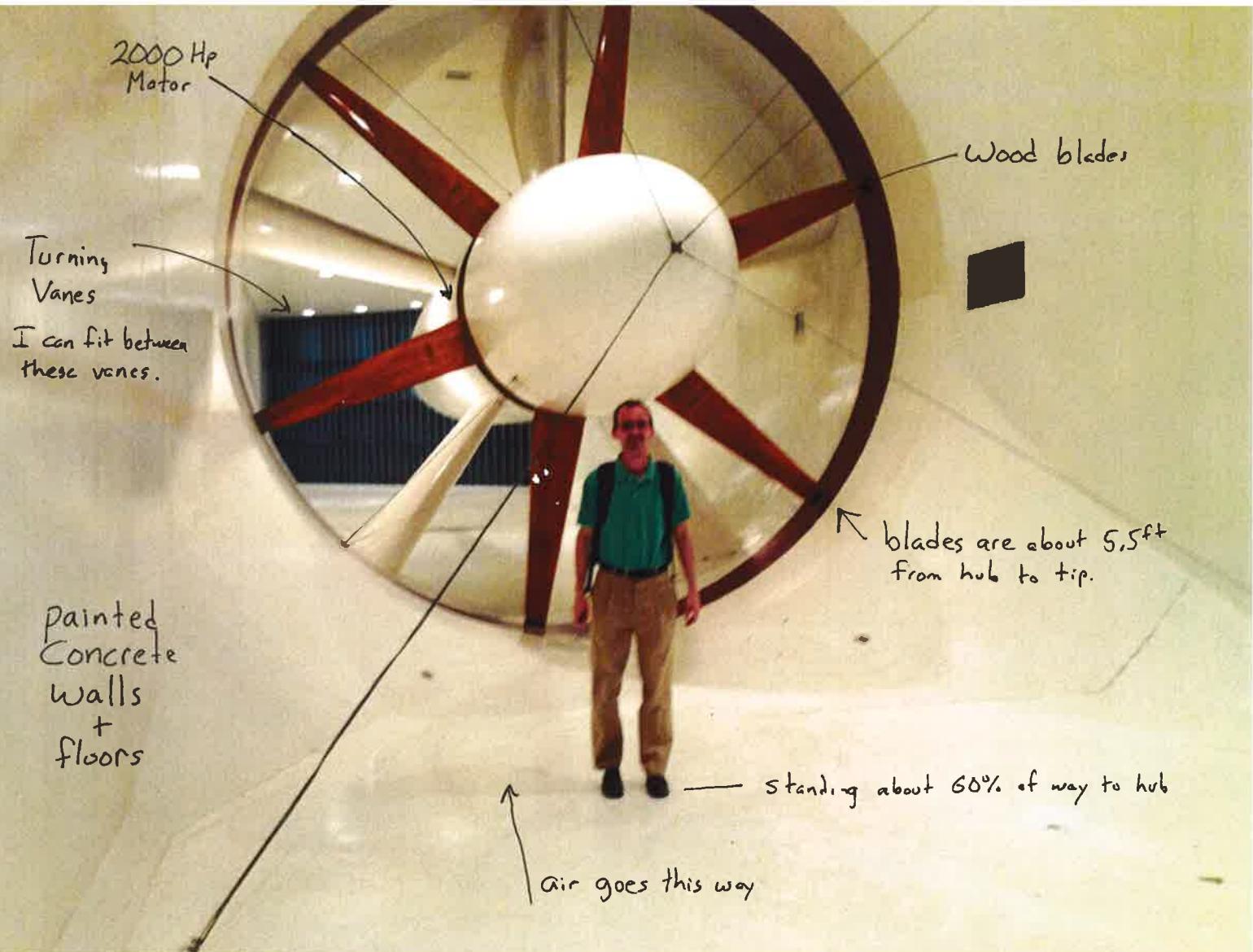
A: Drag + Motor inefficiencies heat the air. Viscosity is a function of temp: $Re = \frac{\rho VL}{\mu}$

$$\mu \approx \mu_0 \frac{T_0 + C}{T + C} \left(\frac{T}{T_0} \right)^{2/3}$$

Sutherland's formula

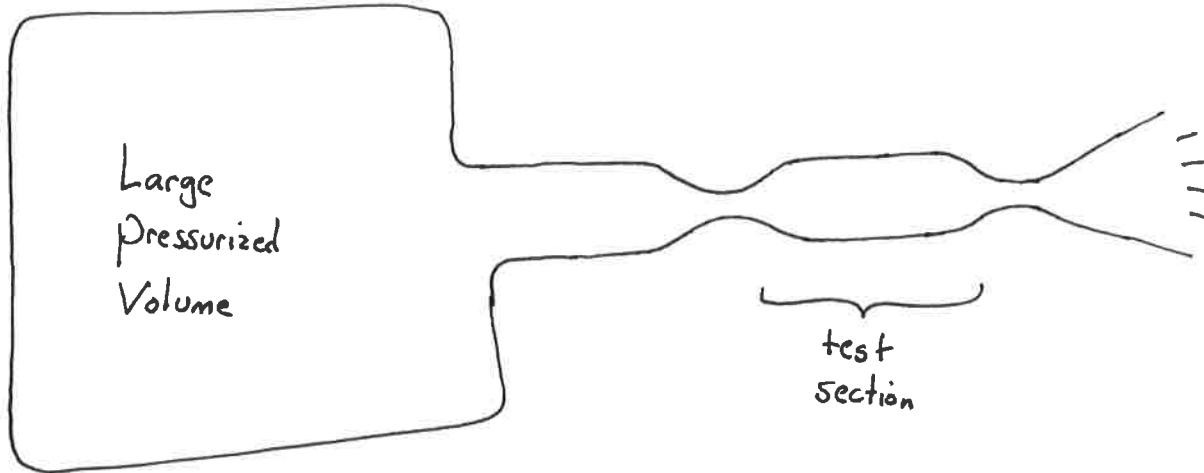
A wind tunnel can be cool/cold in the morning and toasty warm when you leave.

Bring a jacket or layers!



Closed loop tunnel at San Diego Low Speed Wind Tunnel

Blow Down



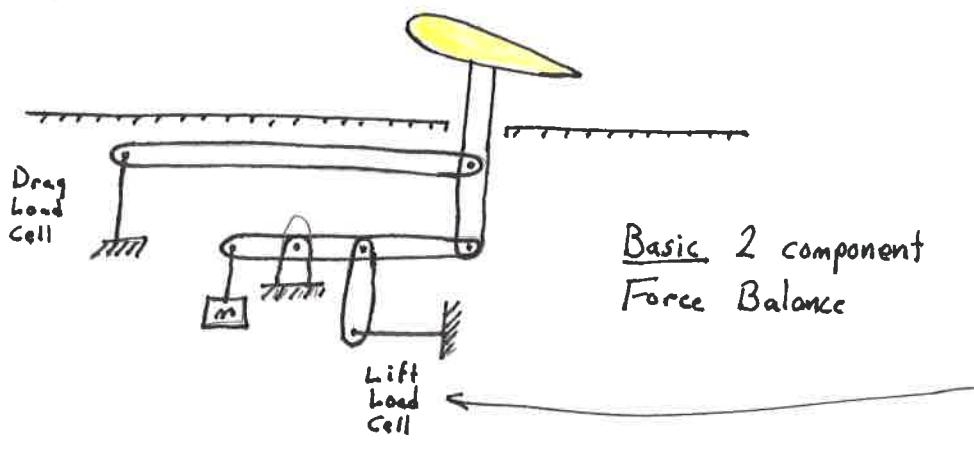
Q: What type of tunnel is drawn above?

A: HSWT (supersonic)

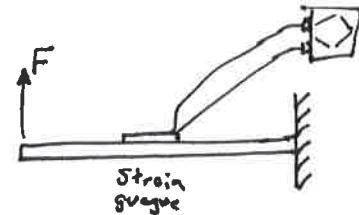
The pressurized tank can be a pressure vessel or in the case of the WwZ German research center at Peenemünde, the large caves were pressurized!

Wind Tunnel Force Measurement

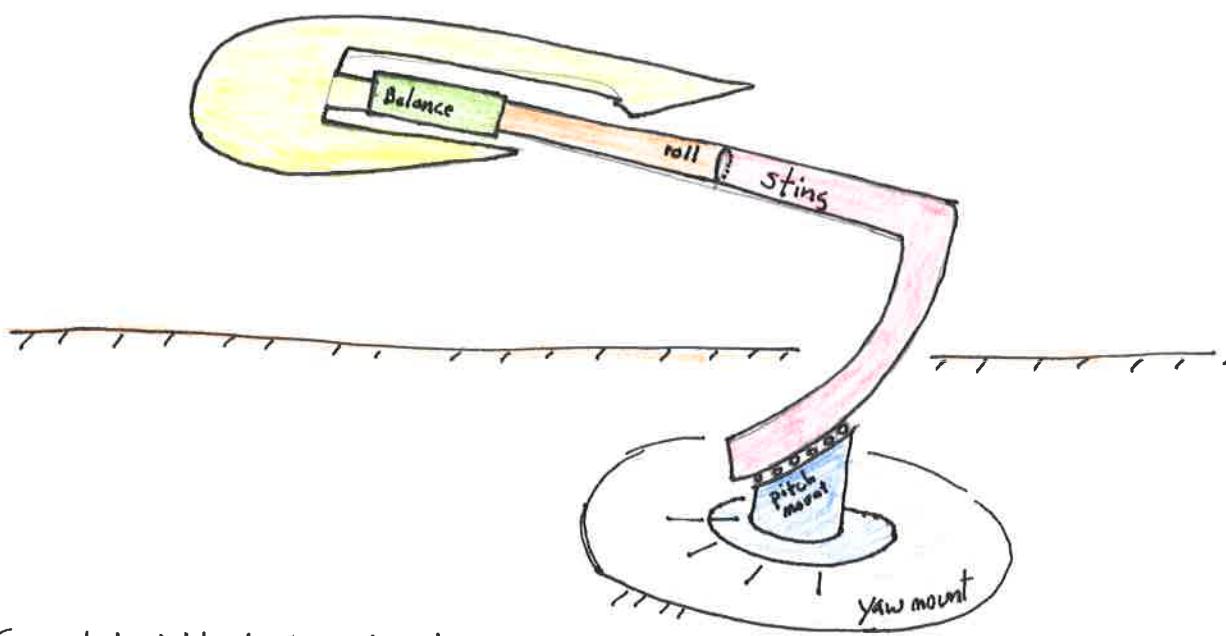
External Force/Moment Balance



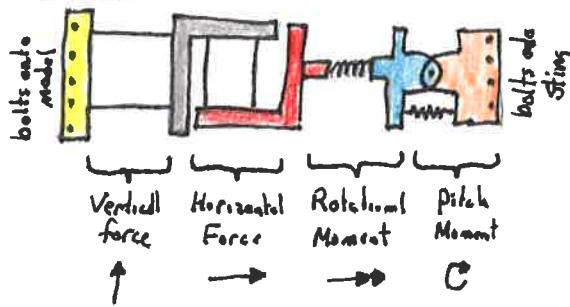
Basic 2 component
Force Balance



Internal Balance



Conceptual Model of Internal Balance



Vertical Force Horizontal Force Rotational Moment Pitch Mount
↑ → → C

Most industry balances are 6 component devices
3 forces and 3 moments

\$\$\$ and \$/year in "calibration"

Refer to Lesson 1 for orientation angles

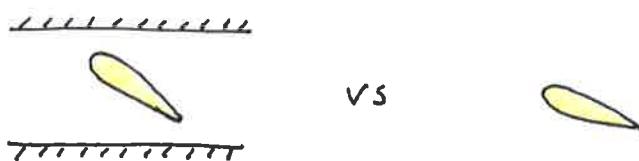
Uncorrected Coefficients

Raw data from measurement devices (w/w₀ nondimensionalization) in wind tunnel

$$C_L = \frac{L}{q_\infty S_{ref}} \quad C_m = \frac{M}{q_\infty S_{ref} c_{ref}} \quad \dots$$

Corrected Coefficients

Data values that would be obtained if the model operated in an unbound environment



The presence of the wind tunnel walls changes the flow!

Why? Flow tangency at the walls.

$$\mathbf{V} \cdot \hat{\mathbf{n}} = 0 \text{ at a wall}$$

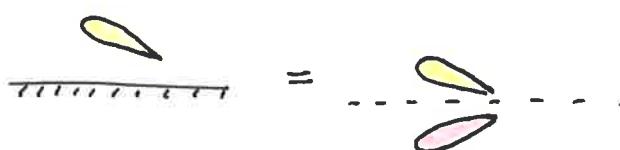
Symmetry in Partial Differential Equations

Given a potential flow ϕ , the velocity is $\mathbf{V} = \nabla \phi$

Thus, $\nabla \phi \cdot \mathbf{n} = 0$ at a wall ($\frac{d\phi}{dx} n_x + \frac{d\phi}{dy} n_y + \frac{d\phi}{dz} n_z = 0$)

Ex:  $\lambda = (0, 0, 1) \Rightarrow \frac{d\phi}{dz} = 0$

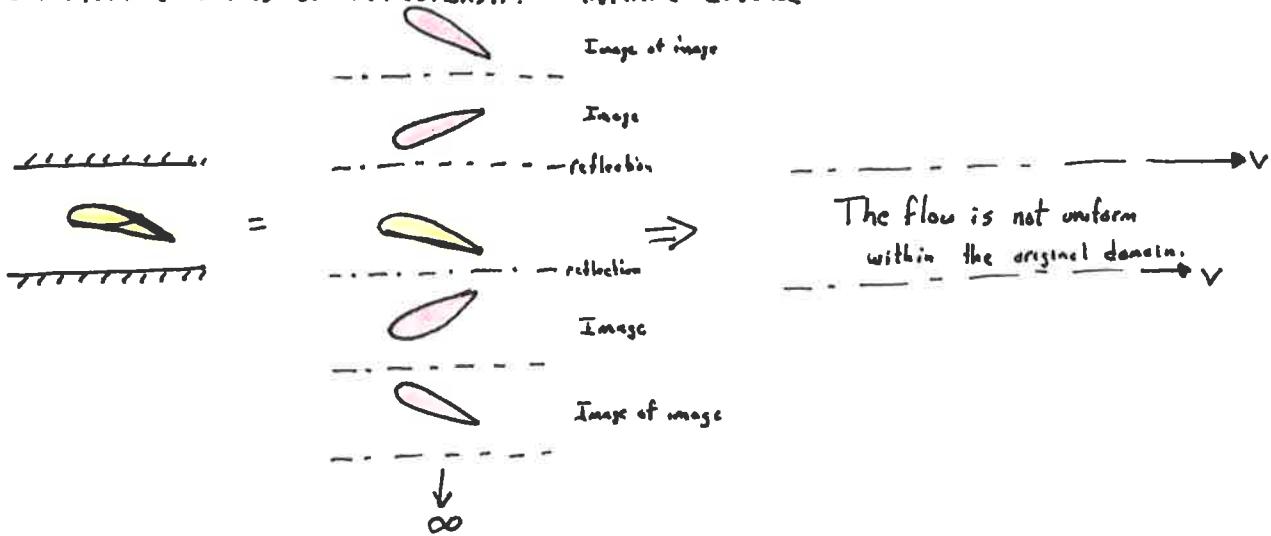
Given an infinite boundary along the x axis with $\frac{d\phi}{dz} = 0$, symmetry can be exploited to give



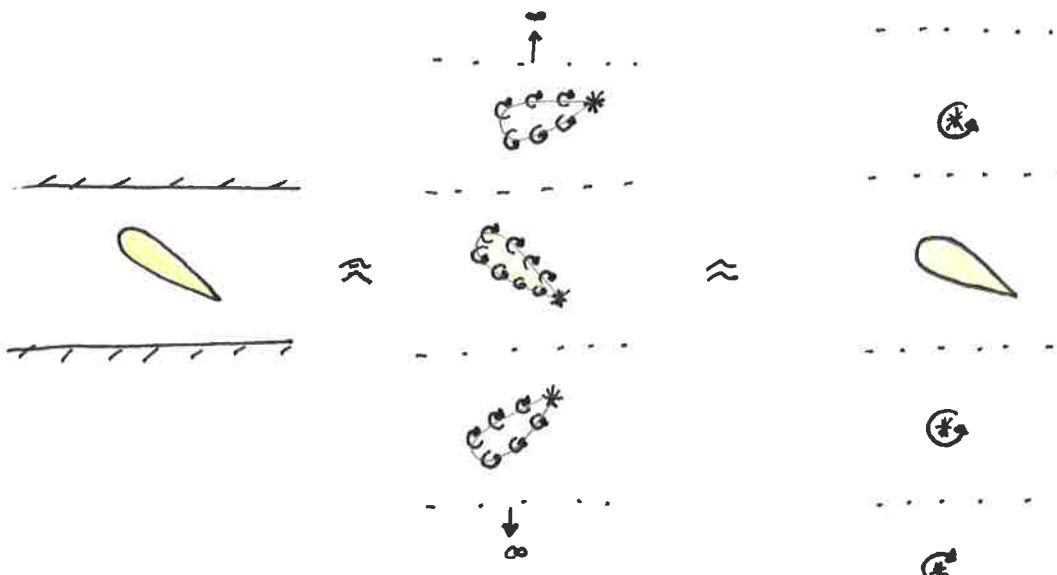
Tunnel walls act as mirrors.

Multiple Walls

Each wall mirrors the flow solution in the normal direction. Multiple walls create an infinite series of reflections... "infinite cascade"



- One way to determine the resulting flow would be to create an infinite series of Thin airfoil theory or lifting line solutions.
- Or, the resulting images could be represented by the far field approximations studied in AFVA chapter 2.11
Source, Vortex, and Doublet approximations



Recall the approximations for:

$$\text{Source: } \Delta = \frac{1}{2} V_\infty C C_d \quad (\text{drag})$$

$$\text{Vortex: } \Gamma^2 = \frac{1}{2} V_\infty C C_L \quad (\text{lift})$$

$$\text{Doublet: } K_x = V_\infty A \quad (\text{area after volume})$$

$$\approx V_\infty A \left(1 + \frac{t}{C}\right) \quad \text{for "thicker" airfoils}$$

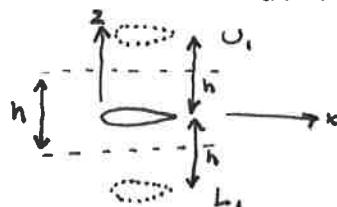
Effective Wind Tunnel Velocity

The far-field expansion of sources, vortices, and doublets is

$$V = \underbrace{V_\infty}_{\text{freestream}} + \underbrace{\frac{\Lambda}{2\pi} \frac{x\hat{x} + z\hat{z}}{r^2}}_{\substack{\text{source} \\ \text{vortex}}} + \underbrace{\frac{\Gamma}{2\pi} \frac{z\hat{x} - x\hat{z}}{r^2}}_{\text{doublets}} \quad \left. \right\} \text{Sources and vortices decay with } \frac{1}{r^2}$$

$$+ \underbrace{\frac{K_x}{2\pi} \frac{(z^2 - x^2)\hat{x} - 2xz\hat{z}}{r^4} + \frac{K_z}{2\pi} \frac{-2xz\hat{x} + (x^2 - z^2)\hat{z}}{r^4}}_{\text{doublets}} \quad \left. \right\} \text{doublets decay with } \frac{1}{r^4}$$

Consider Sources: (assume model is centered in tunnel)



Non-dimensionalize coordinates

$$x = hX$$

$$z = hZ$$

$$r^2 = x^2 + z^2 = h^2(X^2 + Z^2)$$

Consider upper #1 and lower #1

$$V_1 = \frac{\Lambda}{2\pi} \frac{x\hat{x} + z\hat{z}}{r^2} \quad \text{since centered and symmetrical, } \hat{z} \text{ term is zero}$$

$$= \frac{\Lambda}{2\pi} \frac{hX\hat{x}}{h^2(X^2 + Z^2)} \quad Z_{upper} = 1 \quad Z_{lower} = -1$$

$$\Delta V_A = \frac{\Lambda}{h} \frac{1}{2\pi} \frac{X}{X^2 + 1} + \frac{\Lambda}{h} \frac{1}{2\pi} \frac{X}{X^2 + (-1)^2} = \frac{\Lambda}{h} \frac{1}{\pi} \frac{X}{X^2 + 1}$$

Expanding for all mirrored images

$$\Delta V_A = \frac{\Lambda}{h} \frac{1}{\pi} \underbrace{\left(\frac{X}{X^2 + 1} + \frac{X}{X^2 + 4} + \frac{X}{X^2 + 9} + \dots \right)}_{\text{This series converges to } -\frac{1}{2}}$$

This series converges to $-\frac{1}{2}$ when X is far upstream.

So, to keep $\Delta V = 0$ at ∞ , add $\frac{1}{2}$ to above

$$\Delta V_A = \frac{\Lambda}{h} \left[\frac{1}{\pi} \left(\frac{X}{X^2 + h^2} \right) + \frac{1}{2} \right]$$

$$\text{So at } X=0, \Delta V_A = \frac{1}{2} \frac{\Lambda}{h}$$

Remember that Λ is a measure of drag, so drag increases the local velocity at the model. Why?

Apply a similar operation to Γ , and K_x and K_z .

At the model location $X=0$, (and only at $X=0$!)

$$\Delta u = \underbrace{\frac{\Lambda}{h} \frac{1}{2}}_{\text{drag}} + \underbrace{\frac{K_x}{h^2} \frac{\pi}{6}}_{\text{thickness}} \Rightarrow \frac{\Delta u}{V_\infty} = \frac{1}{2} \frac{\Lambda}{V_\infty h} + \frac{\pi}{6} \frac{K_x}{V_\infty h^2} = \text{substitute drag and volume}$$

$$\Delta w = \underbrace{\frac{\pi}{h} \cdot 0}_{\text{lift}} + \underbrace{\frac{K_z}{h^2} \frac{\pi}{12}}_{\text{pitch moment}}$$

Speed increase from volume blockage

$$\frac{K_x}{Vh^2} = 0.2$$

$$\Delta u > 0$$

Flow curvature from lift

$$\frac{\Gamma}{Vh} = 0.5$$

$$\frac{d\Delta w}{dx} > 0$$

Pressure gradient from wake blockage

$$\frac{\Lambda}{Vh} = 0.25$$

$$\frac{d\Delta u}{dx} > 0$$

Flow angle from pitching moment

$$\frac{K_z}{Vh^2} = 0.125$$

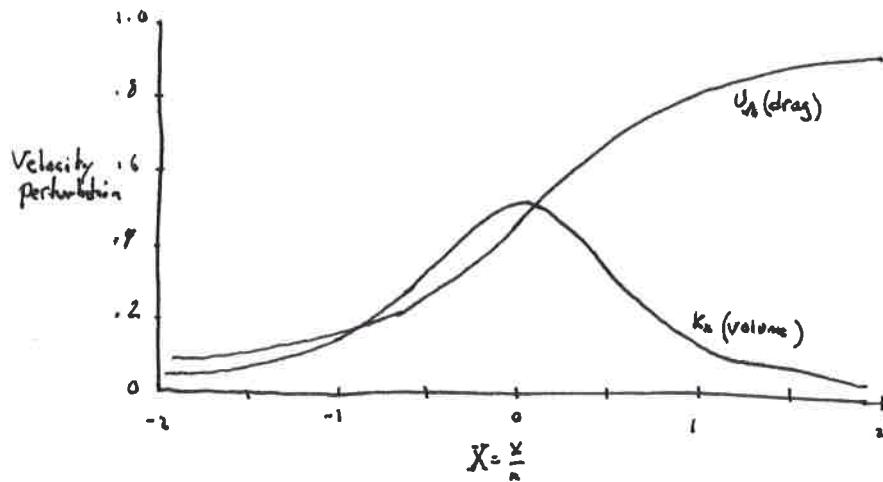
$$\Delta w > 0$$

Figure 10.4: Streamlines of the effective freestream $\mathbf{V}_{\text{eff}}(x, z)$ resulting from model images required to model effects of solid tunnel walls. Shown are models with only volume (left) and only profile drag (right). Images have a positive Δu and $d\Delta u/dx$ along the tunnel centerline at the real model location.

FVA, Drela

Effective Velocity at model

$$V_{\text{eff}} = V_{\infty} + \Delta U$$



Dynamic pressure

$$q_{\text{eff}} = \frac{1}{2} \rho V_{\text{eff}}^2 = \frac{q_{\infty}}{V_{\infty}^2} (V_{\infty} + \Delta U)^2 \approx q_{\infty} \left(1 + 2 \frac{\Delta U}{V_{\infty}} \right)$$

calculated on prev page

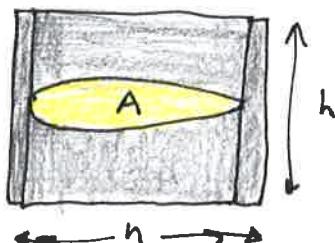
$$q_{\text{eff}} = q_{\infty} \left(1 + \frac{1}{2} \frac{C_d}{h} + \frac{\pi}{3} \frac{A}{h^2} \right)$$

Drag and thickness increase the effective dynamic pressure.

- The drag term scales with $\frac{C_d}{h}$. Reducing the difference in q requires either low drag or a chord smaller than the tunnel height.

The push to increase Re in the tunnel provides the trade off.

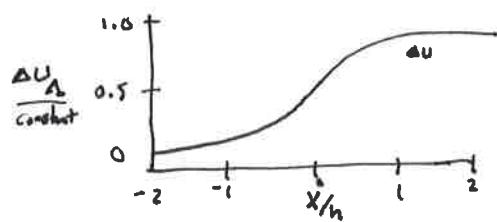
- The airfoil area (volume) term scales with $\frac{A}{h^2}$. This term represents the ratio of airfoil area to tunnel area.



Specifically, by the area ratio of an equivalent circle of radius $\sqrt{\frac{A}{\pi}}$

Wake Induced Buoyancy in 2D ^{Solid wall}_{downstream tunnel}

The wake corresponding to the drag accelerates the flow in the streamwise direction



At $X=0$, a velocity gradient resulting from λ is

$$\frac{dV_{\text{eff}}}{dx} = \frac{\lambda}{h^2} \frac{d\lambda(0)}{dX} = \frac{\pi}{6} \frac{\lambda}{h^2} = \frac{\pi}{12} \frac{V_{\infty} c}{h^2} C_d$$

This velocity gradient implies a pressure gradient

$$\frac{dp_{\text{eff}}}{dx} = - \frac{\pi}{12} \frac{\rho V_{\infty}^2 c}{h^2} C_d$$

Given a pressure gradient and an airfoil cross sectional area, a force results

$$\begin{aligned} \Delta D'_{\text{buoyancy}} &= - \frac{dp}{dx} A \\ &= \frac{\pi}{12} \rho V_{\infty}^2 c C_d \frac{A}{h^2} \end{aligned}$$



$$\Delta C_d^{\text{buoyancy}} = \frac{\Delta D'_{\text{buoyancy}}}{\frac{1}{2} \rho V_{\infty}^2 c} = \frac{\pi}{6} \frac{A}{h^2} C_d$$

$$\Delta C_d^{\text{buoyancy}} = \frac{\pi}{6} \frac{A}{h^2} C_d$$

Buoyancy Drag

When you test a model at a commercial wind tunnel, the staff usually will perform the calculations to remove buoyancy drag (i.e. uncorrected data \rightarrow corrected data)

You will be required to supply model area values.

Also note that this term depends on configuration (e.g. stores-on vs stores-off)

The streamwise curvature resulting from Γ and K_2 (lift and pitch moment) contribute to a correction term.

Summarizing FVA Chapter 10.3.1

$$\Delta C_d = \frac{\pi^2}{48} \frac{c^2}{h^2} C_{d_u}$$

$$\Delta C_m = -\frac{\pi^2}{192} \frac{c^2}{h^2} C_{d_u}$$

$$\Delta \alpha = \frac{\pi}{3} \frac{c^2}{h^2} \left(C_{m_u} + \frac{1}{4} C_{d_u} \right)$$

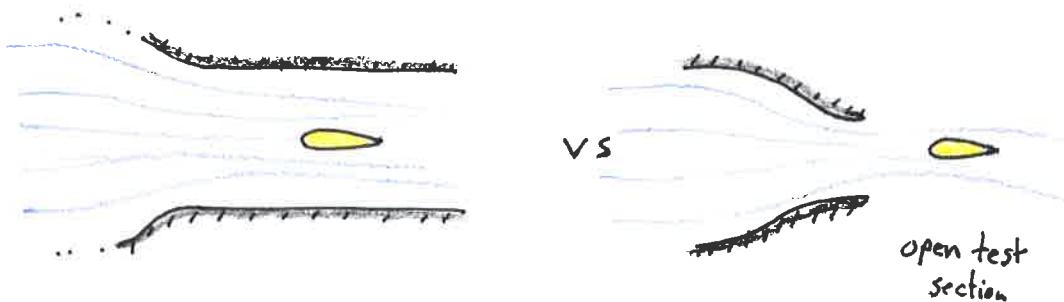
The uncorrected lift contributes to streamwise curvature which creates a change in both lift and moment generated and the flow angle.

Final process for processing uncorrected data to yield corrected data

← Iterative?

$C_d = \left(1 - \frac{1}{2} \frac{c}{h} C_d - \frac{\pi}{2} \frac{A}{h^2} \right) C_{d_u}$
 $C_d = \left(1 - \frac{1}{2} \frac{c}{h} C_d - \frac{\pi}{3} \frac{A}{h^2} - \frac{\pi^2}{48} \frac{c^2}{h^2} \right) C_{d_u}$
 $C_m = \left(1 - \frac{1}{2} \frac{c}{h} C_d - \frac{\pi}{3} \frac{A}{h^2} \right) C_{m_u} + \frac{\pi^2}{192} \frac{c^2}{h^2} C_{d_u}$
 $\alpha = \alpha_u + \frac{\pi}{24} \frac{c^2}{h^2} \left(C_{m_u} + \frac{1}{4} C_{d_u} \right)$
 $Re = \left(1 + \frac{1}{4} \frac{c}{h} C_d + \frac{\pi}{6} \frac{A}{h^2} \right) Re_u$

Each far field term corresponds to a correction term. The correction depends on the type of tunnel. A 2D tunnel has different correction terms than a 2D open jet tunnel.



See FVA Chapter 10 for details.

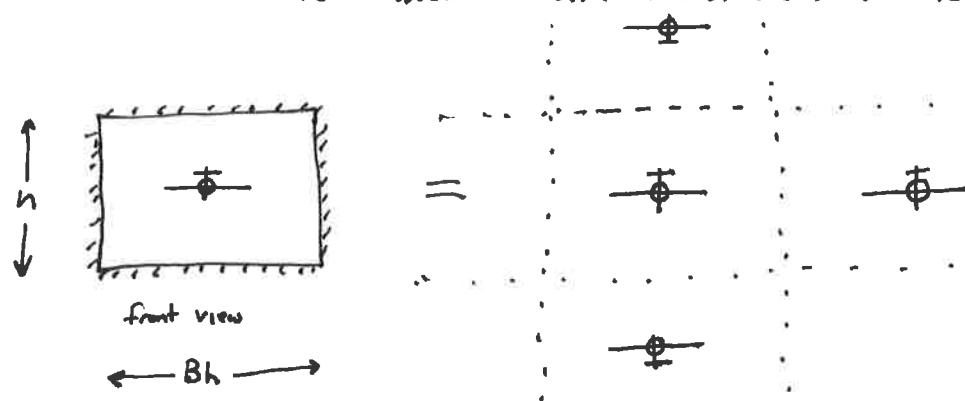
In particular, the open test section has an opposite sign for buoyancy drag!

Warning: All analysis given here assumes that the model is centered.

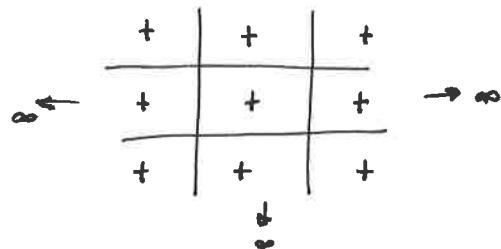
The non centered version is more complicated with terms that don't cancel or aren't equal.

3D Tunnel

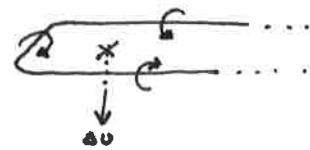
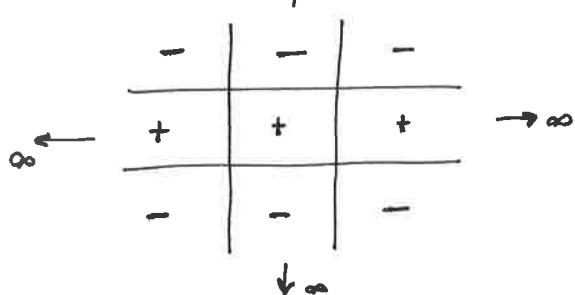
The 3D correction terms are associated with tunnel walls of size h by Bh .



The thickness and drag terms (K_x and Σ) are symmetric



Wake from lift is not (K_z) since the wake doublet has a direction associated with it



Correction terms for 3D tunnels

$$C_D = \left(1 - \frac{S_{\text{ref}}}{h^2} C_{D_p} \hat{u}_{\Sigma}(0) - 2 \frac{\mathcal{V}}{h^3} \hat{u}_{K_x}(0) \right) C_{D_u} - \frac{1}{2} \frac{S_{\text{ref}} c_{\text{ref}}}{h^3} C_{L_q} \frac{C_{L_u}^2}{\pi A Re} \frac{d\hat{w}_{K_z}(0)}{dX} \\ - \frac{\mathcal{V}}{h^3} C_{D_p} \hat{u}_{K_x}(0) + \frac{1}{2} \frac{S_{\text{ref}}}{h^2} C_{L_u}^2 \hat{w}_{K_z}(0) \quad (10.92)$$

$$C_L = \left(1 - \frac{S_{\text{ref}}}{h^2} C_{D_p} \hat{u}_{\Sigma}(0) - 2 \frac{\mathcal{V}}{h^3} \hat{u}_{K_x}(0) \right) C_{L_u} - \frac{1}{4} \frac{S_{\text{ref}} c_{\text{ref}}}{h^3} C_{L_q} C_{L_u} \frac{d\hat{w}_{K_z}(0)}{dX} \quad (10.93)$$

$$C_m = \left(1 - \frac{S_{\text{ref}}}{h^2} C_{D_p} \hat{u}_{\Sigma}(0) - 2 \frac{\mathcal{V}}{h^3} \hat{u}_{K_x}(0) \right) C_{m_u} - \frac{1}{4} \frac{S_{\text{ref}} c_{\text{ref}}}{h^3} C_{m_q} C_{L_u} \frac{d\hat{w}_{K_z}(0)}{dX} \quad (10.94)$$

$$\alpha = \alpha_u + \frac{1}{2} \frac{S_{\text{ref}}}{h^2} C_{L_u} \hat{w}_{K_z}(0) \quad (10.95)$$

$$Re = \left(1 + \frac{1}{2} \frac{S_{\text{ref}}}{h^2} C_{D_p} \hat{u}_{\Sigma}(0) + \frac{\mathcal{V}}{h^3} \hat{u}_{K_x}(0) \right) Re_u \quad (10.96)$$

FVA, Drela, Chap 10

For a solid wall tunnel, the coefficients are:

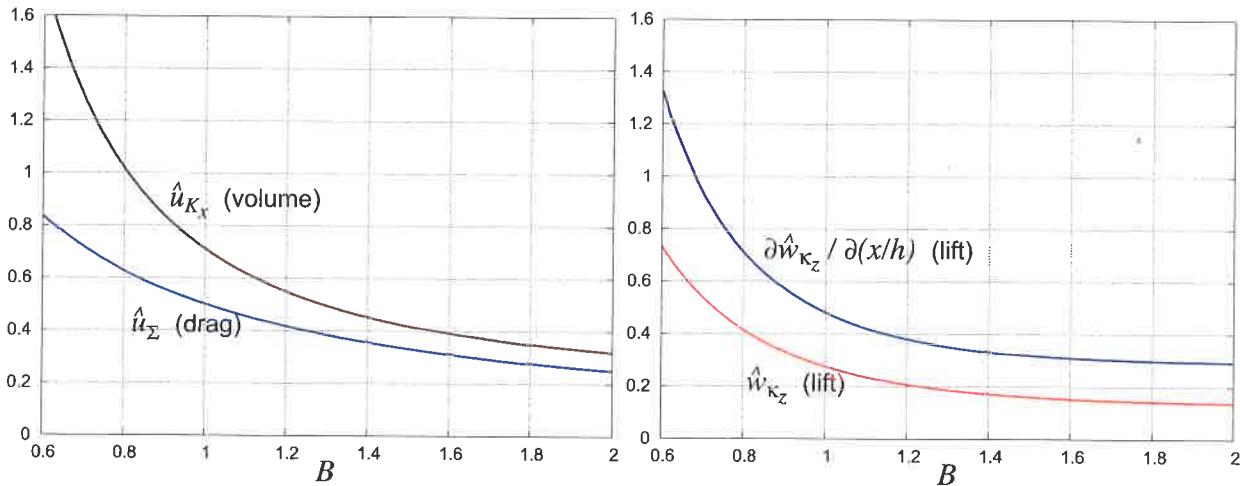
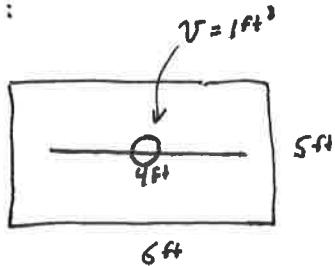


Figure 10.15: 3D velocity disturbances at the model location $x = 0$ versus tunnel section aspect ratio, for a solid-wall tunnel.

FVA, Drela, Chap
10

Ex: In a 5×6 foot tunnel, the zero lift drag is 200 counts. The aircraft has a reference wing area of 4 ft^2 and a volume of 1 ft^3 . Determine the open-air corrected C_D .

Geometry:



3D solid wall coefficients.

$$C_D = \left(1 - \frac{S_{ref}}{h^2} C_{D_p} \hat{U}_\xi(0) - 2 \frac{V}{h^3} \hat{U}_{K_x}(0) \right) C_{D_u} - \frac{V}{h^3} C_{D_p} \hat{U}_{K_x}(0) + \underbrace{\dots}_{\text{terms not needed}}$$

$$S_{ref} = 4 \text{ ft}^2$$

$$h = 5 \text{ ft} \Rightarrow B = \frac{6}{5} = 1.2 \Rightarrow \hat{U}_\xi(0) = 0.42$$

$$C_{D_u} = C_{D_p} = 0.0200$$

$$V = 1 \text{ ft}^3$$

$$C_D = \underbrace{\left(1 - \frac{4 \text{ ft}^2}{25 \text{ ft}^2} \cdot 0.0200 \cdot 0.42 - 2 \cdot \frac{1 \text{ ft}^3}{5^3 \text{ ft}^3} \cdot 0.56 \right) 0.0200 - \frac{1 \text{ ft}^3}{5^3 \text{ ft}^3} \cdot 0.0200 \cdot 0.56}_{0.9897}$$

$$C_D = 0.0197$$

The difference is about 3 counts. (within noise of measurement in tunnel or flight test)

Additional Resources:

"Low-Speed Wind Tunnel Testing", Rae and Pope, Wiley.

AEM 514 "Experimental Aerodynamics" Dr Hubner, Fall 2016