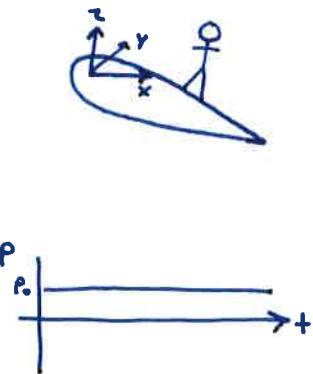
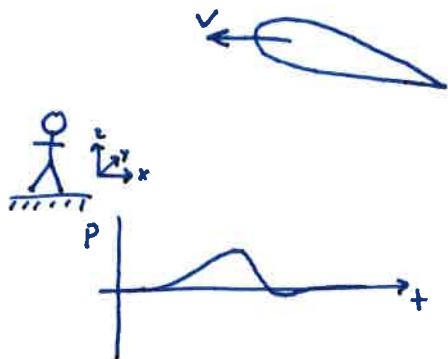


Lesson 33
Unsteady Airfoil Aerodynamics

FVA Chap 7

Reference Frames



A steady aerodynamic case becomes unsteady in a different reference frame.

Some cases are unsteady in every reference frame.

Sources of unsteady aerodynamics:

- Rigid Body Motion
- Atmospheric variations "gusts"
- Deforming body

Further/more details in AEM 626 "Unsteady Flow"

The purpose of this lecture is to show results rather than derivation.

FVA Chap 7. is ok.

Goldstein's "Aeroacoustics" is good if you can find and afford a copy.

We will talk about a few basic unsteady aero flows.

Wagner problem

Recall from Kelvin's theorem that for inviscid, incompressible, irrotational flows, the total system circulation is constant.

Thus, given a bound circulation about an airfoil creating lift, a net ozone circulation exists somewhere....



From the Kutta condition, $\alpha_p = 0$ which implies (Eq 7.22 FVA) that the shed wake potential jump convects.

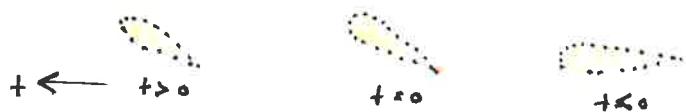
$$\frac{D(\Delta\phi)}{Dt} = 0$$

Thus as the lift changes on an airfoil, a corresponding shed wake is generated.

Notice that $\frac{D(\Delta\phi)}{D} = 0$ indicates convection at the local velocity. The wake can influence itself. (e.g. wake rollup)

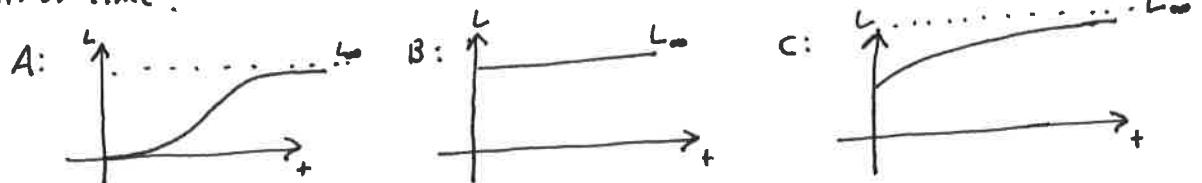
The Wagner problem considers a step change in airfoil attitude.

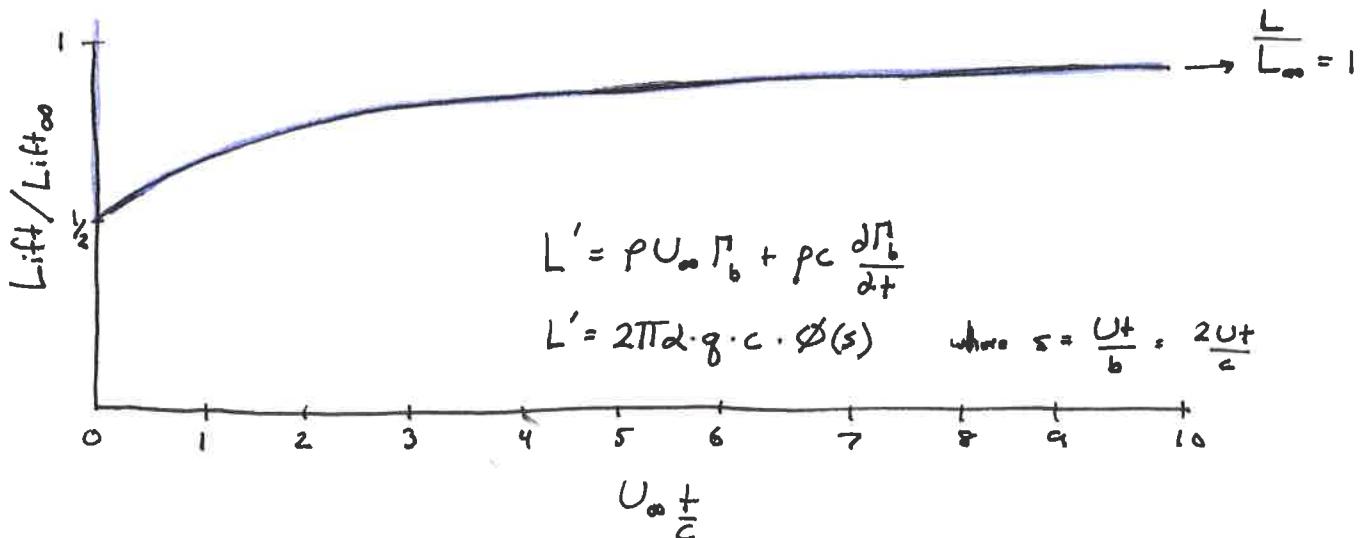
$$\alpha = \begin{cases} 0 & t < 0 \\ \alpha_0 & t \geq 0 \end{cases}$$



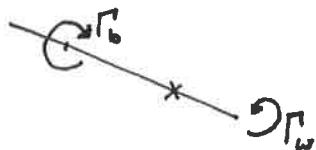
What is the lift and moment and drag?

Q: Lift vs Time?





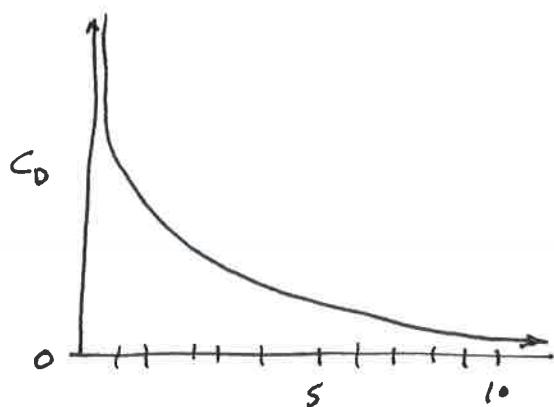
- Lift at $t=0$ is $\frac{1}{2}$ the final value. Why?



- The magnitude of induced velocity due to Γ_b decreases with time

Drag.

$$D' = L' \propto = \underbrace{\rho U_{\infty} \propto \Gamma_b}_{\text{water induced downwash}} + \underbrace{\rho c \frac{d\Gamma_b}{dt}}_{\text{fluid acceleration}}$$

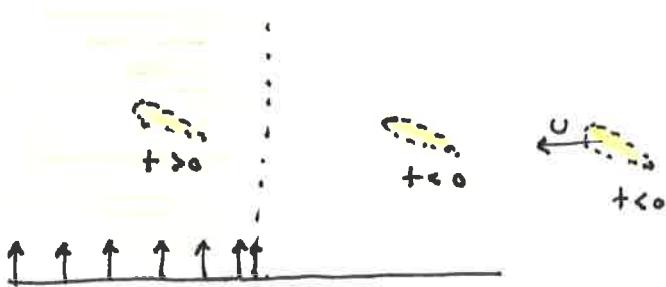


Approximate solution for $\phi(s)$

$$\phi(s) \approx 1 - 0.165 e^{-0.0455s} - 0.335 e^{-0.3s}$$

$$\phi(s) \approx \frac{s+2}{s+4}$$

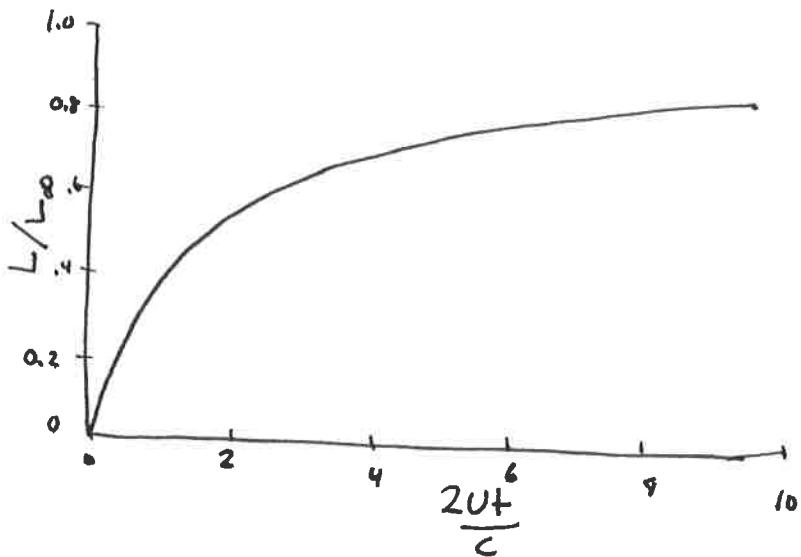
Küssner Gust



Fly into a gust at time = 0 such that $\Delta\alpha$ occurs

$$L' = 2\pi \Delta\alpha \Psi(s)$$

$$\Psi(s) \approx 1 - 0.5 e^{-0.13s} - 0.5 e^{-s}$$

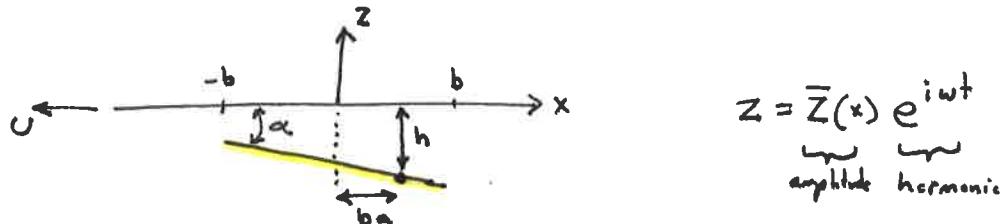


Theodorsen Problem

Source "Unsteady Aerodynamics" Class notes, Spr 2002
MAS 5943, Dr. Falk.

(Harmonic rigid body motion of an airfoil.)

$$\text{Airfoil mean camber line } z(x,t) = -h(t) - \alpha(t)(x - ba)$$



Derivation is involved and requires conformal mapping of unsteady flow.

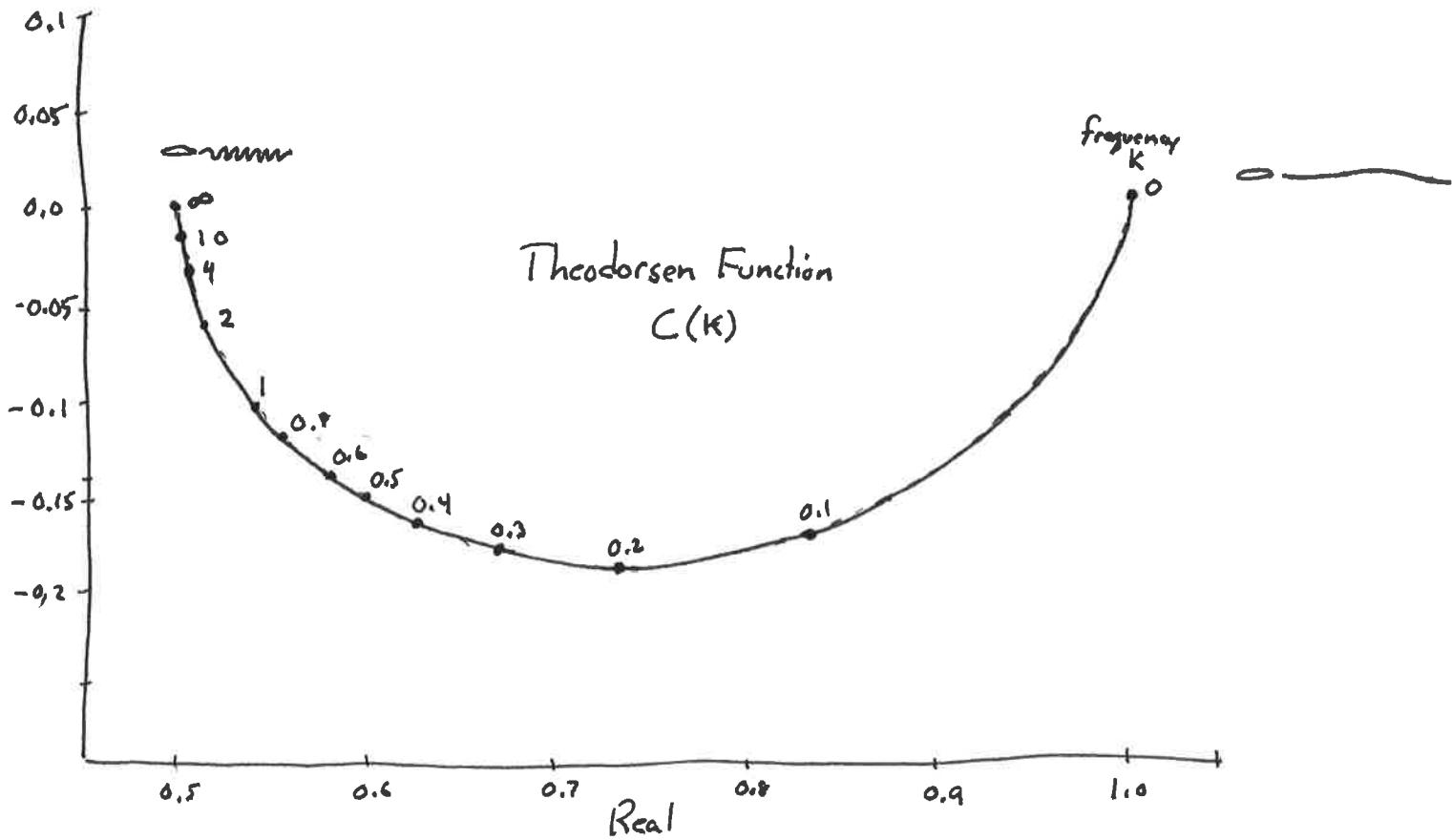
$$\text{Reduced frequency } \equiv k \equiv \frac{\omega b}{U} = \pi \frac{\text{chord}}{\text{distance per period}}$$

$$\begin{aligned} \text{Theodorsen function } C(k) &= \frac{\int_1^{\infty} \frac{s^x}{\sqrt{s^{2x}-1}} e^{-ik s^x} ds^x}{\int_1^{\infty} \frac{\sqrt{s^x+1}}{s^x-1} e^{-ik s^x} ds^x} \\ &= \frac{H_1^{(2)}(k)}{H_1^{(2)}(k) + i H_0^{(2)}(k)} = F(k) + i G(k) \end{aligned}$$

Theodorsen showed that this equals this

$$\begin{aligned} H_n^{(2)} &\equiv n^{\text{th}} \text{ order Hankel function of 2nd kind} \\ &\equiv J_n - i Y_n \\ &\quad \begin{matrix} \uparrow & \nwarrow \\ n^{\text{th}} \text{ Bessel} & n^{\text{th}} \text{ Bessel} \\ \text{of 1st} & \text{of 2nd} \\ \text{kind.} & \text{kind.} \end{matrix} \end{aligned}$$

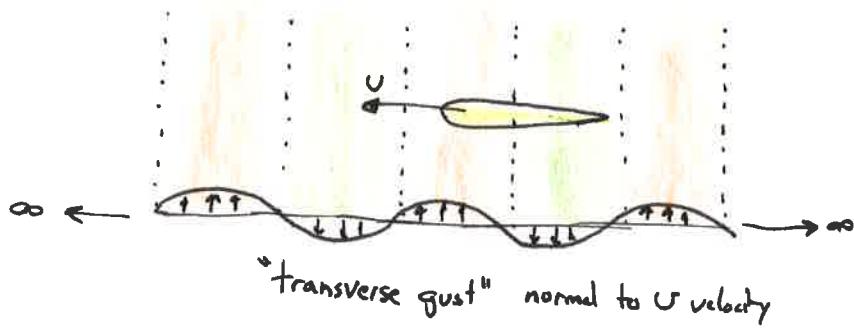
$$\text{Lift} = \underbrace{\pi \rho b^2 [h' + U\dot{\alpha} - ba\ddot{\alpha}]}_{\text{Non circulatory lift} \\ (\text{aka generalized}) \\ \text{impulsive.}} + \underbrace{2\pi b \rho U [h + U\alpha + ba(\frac{1}{2} - \alpha)] C(k)}_{\text{circulatory lift}}$$



The Theodorsen function represents the lift's amplitude and phase of an airfoil harmonically moving in a uniform flow. At steady state, $k=0$, the $C(k)$ term has magnitude 1 and phase 0° . This corresponds to steady state lift. As frequency increases from 0, the magnitude decreases from 1 and the phase decreases from 0. A low frequency oscillation results in a quasi-steady lift term where the lift always "catches up" with the motion thru shed vortices. As the frequency increases, each cycle of shed vorticity has more effect causing a further reduction in lift and less time dependency. For high frequencies, the convection of vorticity is so slow compared to the motion that there is no phase difference.

The phase difference in motion and lift and moment can drive aeroelasticity ("flutter")

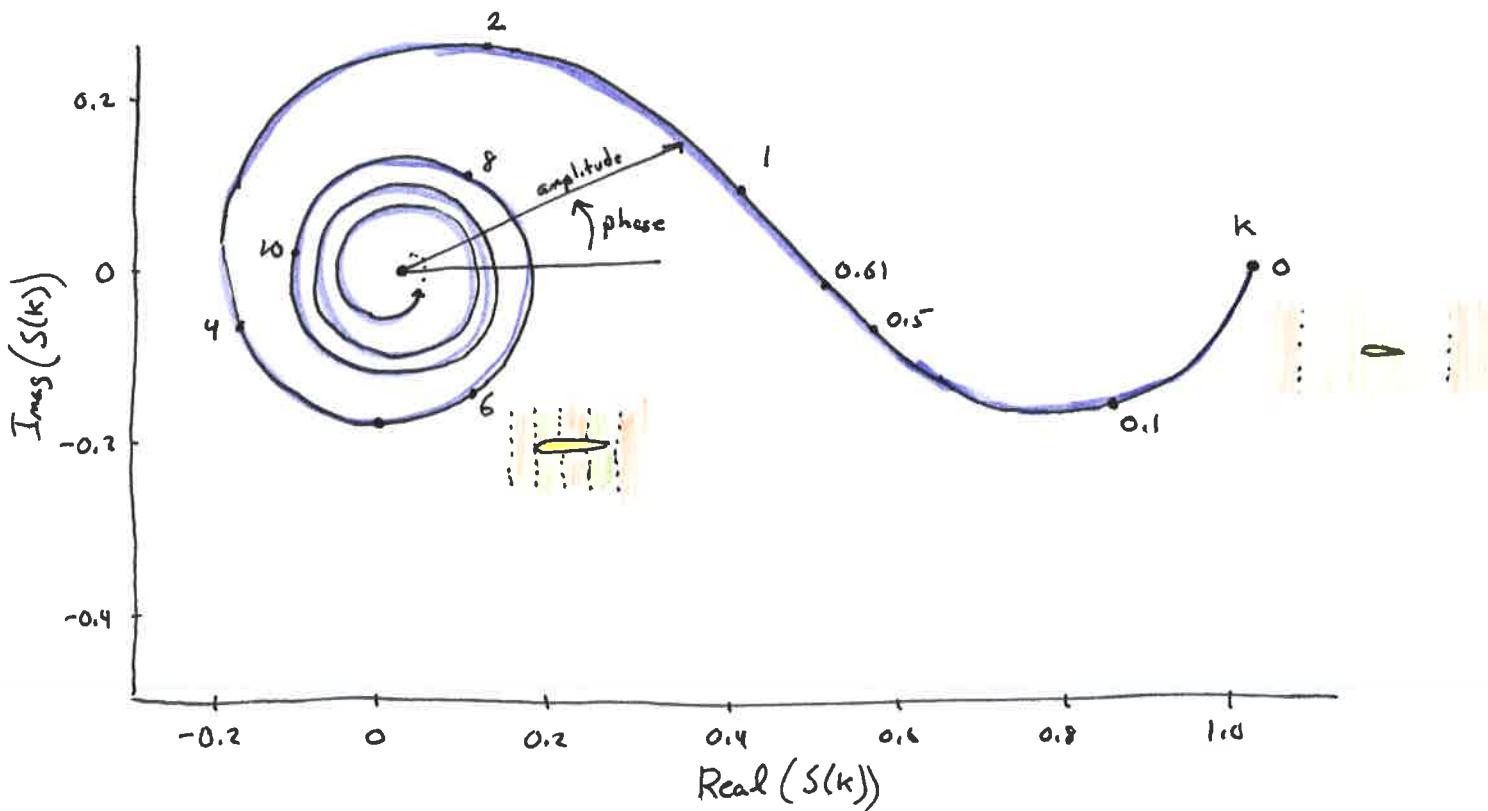
Sears Problem (Harmonic Gust)



Assume that the airfoil does not distort gust.

$$\text{Sears function} = S(k) \equiv \frac{1}{\pi \frac{k}{2} \left[H_0^{(2)}(k) - i H_1^{(2)}(k) \right]}$$

$$L' = 2\pi f_U \bar{w} b e^{i\omega t} S(k)$$



The Sears function expresses the amplitude and phase of an airfoil moving through a harmonic disturbance. As the frequency increases, each disturbance cycle covers an increasingly smaller chord percentage. For very high frequencies, the average disturbance is small (i.e. each disturbance covers only a small portion of the chord and only creates a small wake).