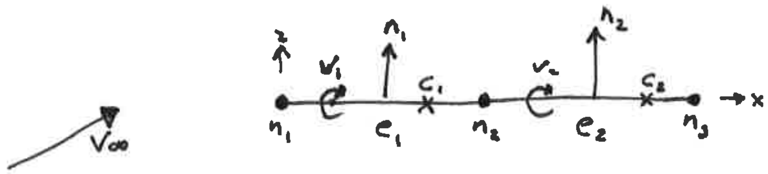
Discrete Vortex Implementation

Misc

5 Oct 2015

Notes on HW3 with 2 element case



$$X = \begin{array}{cc} x & z \\ \left[\begin{array}{c} 0, \; 0 \\ 1, \; 0 \\ 2, \; 0 \end{array}\right] & \begin{array}{l} \text{node}_1 \\ \text{node}_2 \\ \text{node}_3 \end{array} \end{array} \qquad Ele = \begin{array}{cc} \begin{array}{cc} n_1 & n_2 \end{array} & \\ \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} & \begin{array}{l} e_1 \\ e_2 \end{array} \end{array} \qquad Normal = \begin{array}{cc} \begin{array}{cc} n_x & n_z \end{array} & \\ \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} & \begin{array}{l} e_1 \\ e_2 \end{array} \end{array}$$

$$V = \begin{bmatrix} 0.25 & 0 \\ 1.25 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 0.75 & 0 \\ 1.75 & 0 \end{bmatrix}$$

$$V_\Gamma = \frac{\Gamma}{2\pi} \frac{(z-z')\hat{x} - (x-x')\hat{z}}{(x-x')^2 + (z-z')^2}$$

r' is Γ location
r is velocity location

BCs
at C_1:

$$V \cdot n = 0 \text{ at } C_1$$

$$V = \left(V_\infty \cos\alpha \, \hat{x} + V_\infty \sin\alpha \, \hat{z} + \sum V_\Gamma\right) \cdot \hat{n}_1 \quad \text{Generic}$$

$$= V_\infty \sin\alpha + \frac{-\Gamma_1}{2\pi} \frac{1}{X_{c_1} - X_{V_1}} + \frac{-\Gamma_2}{2\pi} \frac{1}{X_{c_1} - X_{V_2}}$$

Particular V_Γ for flat a/f
Same for C_2

$$AIC = \frac{1}{2\pi} \begin{bmatrix} \frac{1}{X_{c_1} - X_{v_1}} & \frac{1}{X_{c_1} - X_{v_2}} \\ \frac{1}{X_{c_2} - X_{v_1}} & \frac{1}{X_{c_2} - X_{v_2}} \end{bmatrix} \begin{pmatrix} \Gamma_1 \\ \Gamma_2 \end{pmatrix} = V_\infty \begin{pmatrix} \sin\alpha \\ \sin\alpha \end{pmatrix}$$

$$AIC = \frac{1}{2\pi} \begin{bmatrix} \frac{1}{.5} & \frac{1}{-.5} \\ \frac{1}{1.5} & \frac{1}{.5} \end{bmatrix} = \frac{1}{2\pi} \begin{bmatrix} 2 & -2 \\ \frac{2}{3} & 2 \end{bmatrix} \Rightarrow AIC^{-1} = 2\pi \frac{\begin{pmatrix} 2 & 2 \\ -\frac{2}{3} & 2 \end{pmatrix}}{4 + 4/3} = 2\pi \begin{bmatrix} 3/8 & 3/8 \\ -1/8 & 3/8 \end{bmatrix}$$

$$\begin{pmatrix} \Gamma_1 \\ \Gamma_2 \end{pmatrix} = AIC^{-1} V_\infty \begin{pmatrix} \sin\alpha \\ \sin\alpha \end{pmatrix} 2\pi = V_\infty \sin\alpha \begin{pmatrix} 6/8 \\ 2/8 \end{pmatrix} 2\pi \Rightarrow \Gamma_{total} = V_\infty \sin\alpha$$

$$L = \rho V \Gamma = \rho V_\infty^2 \sin\alpha \, 2\pi$$

$$C_\ell = \frac{L}{\frac{1}{2}\rho V^2 c} = \frac{\rho V_\infty^2 \sin\alpha}{\frac{1}{2}\rho V^2\, 2} = \sin\alpha \; 2\pi$$

Also:

$$C_p = \frac{p - p_\infty}{q} \quad \text{with B'} \quad p = p_\infty + \frac{1}{2}\rho V_\infty^2 - \frac{1}{2}\rho V^2$$

$$= \frac{\frac{1}{2}\rho V_\infty^2 - \frac{1}{2}\rho V^2}{\frac{1}{2}\rho V_\infty^2} = 1 - \left(\frac{V}{V_\infty}\right)^2$$

What is V at collocation points?

$$V_1 = V_\infty \cos\alpha\, \hat{x} + V_\infty \sin\alpha\, \hat{z} + V_\Gamma$$

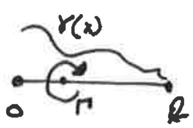
well, we know $V \cdot n = 0$, thus we could eliminate those terms. or, just plug into the above.

$$= V_\infty \sin\alpha + \frac{\Gamma_1}{2\pi}\left(\cancelto{0}{\frac{z - z'}{(x - x')^2}}\right)$$

No change in Velocity!?!
No C_p variation?!?
This is a problem.

Distribute Γ over element

$$\Gamma = \int_0^\ell \gamma \, d\ell$$



$$\Rightarrow \quad \gamma \approx \frac{\Gamma}{\ell}$$

From TAT derivation, $C_p = 2\frac{\gamma}{V_\infty}$

Given an average pressure, compute total force

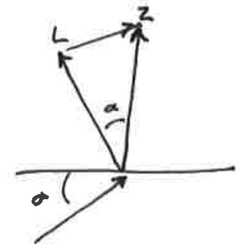
$$F = q \cdot C_p \cdot \ell\, \hat{n} = q \cdot 2\frac{\gamma}{V_\infty} \cdot L\, \hat{n} = q \cdot 2\frac{\Gamma_i}{\ell}\frac{1}{V_\infty}\, \ell\, \hat{n}$$

Force in z direction

$$Z_i = F \cdot \hat{z} \qquad C_z = \frac{Z}{qc} = \frac{2\Gamma_i\, n_{z_i}}{V_\infty\, q c}$$

For our HW example case

$$C_\ell = \frac{2\,\frac{6}{8}\,2\pi V_\infty \sin\alpha}{V_\infty\, g\, 2} + \frac{2\,\frac{2}{8}\,2\pi V_\infty \sin\alpha}{V_\infty\, g\, 2}$$



$$= 2\pi \sin\alpha$$

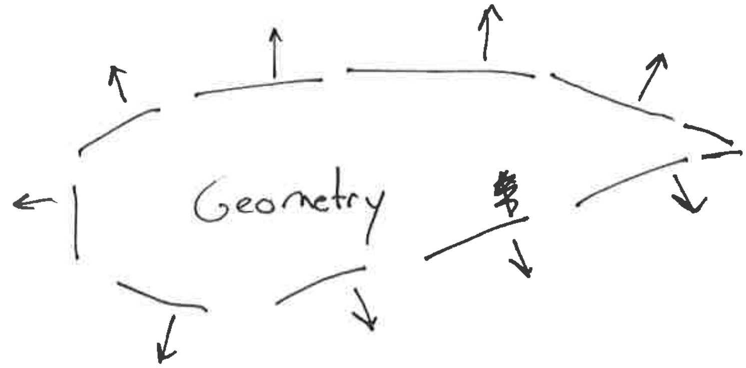
1 when $\alpha \approx 0$

$$C_\ell = C_z \cancel{\cos\alpha}$$

$$\boxed{C_\ell = 2\pi \sin\alpha}$$

$$C_m = F \cdot \text{distance}$$

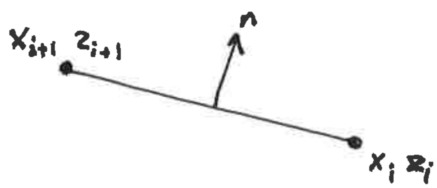
Challenges

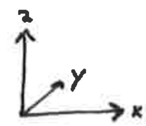
1) 

2) $AIC \rightarrow AIC^{-1}$

3) $C_p \rightarrow \int C_p$

Find the normal





$$\Delta X = (x_{i+1} - x_i)\,\hat{x} + (z_{i+1} - z_i)\,\hat{z}$$

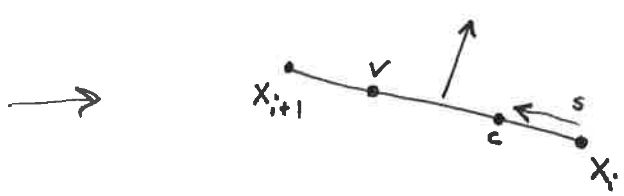
$$n = -\Delta X \times \hat{y}$$

$$n = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ x_i - x_{i+1} & 0 & z_i - z_{i+1} \\ 0 & 1 & 0 \end{vmatrix}$$

$$= -(z_i - z_{i+1})\,\hat{x} + (x_i - x_{i+1})\,\hat{z}$$

$$= (z_{i+1} - z_i)\,\hat{x} + (x_i - x_{i+1})\,\hat{z}$$

Find V and C



$$V = X_i + \frac{X_{i+1} - X_i}{1} \cdot 0.75$$

$$C = \underbrace{\qquad\qquad\qquad} \cdot 0.25$$