Prandtl-Glauert Supersonic Thin Airfoil Theory

$M := 2$

Supersonic version of PG (hyperbolic)

$\beta := \sqrt{M^2 - 1} = 1.732$

Lift Coefficient (FVA 8.137)

$Cl(a) := \frac{4}{\beta} \cdot a$

Drag Coefficient (FVA 8.141)

$Cd(a) := \frac{2}{\beta} \cdot \int_0^1 \left(\left(Z'_u \right)^2 + \left(Z'_l \right)^2 \right) \cdot \frac{1}{c} \, dx$

Forward Section (0<x<0.5)

$FZ'u := \frac{\left(\frac{0.0335}{2} \right)}{0.5 - 0} \qquad FZ'l := -FZ'u$

Aft Section (0.5<x<1.0)

$AZ'u := \frac{\left(\frac{-0.0335}{2} \right)}{1.0 - 0.5} \qquad AZ'l := -AZ'u$

Decompose section into
forward and aft section

$Cd(a) := \frac{2}{\beta} \cdot \int_0^{0.5} \left(\left(FZ'_u \right)^2 + \left(FZ'_l \right)^2 \right) \cdot \frac{1}{c} \, dx + \frac{2}{\beta} \cdot \int_{0.5}^1 \left(\left(AZ'_u \right)^2 + \left(AZ'_l \right)^2 \right) \cdot \frac{1}{c} \, dx$

Squared term is identical for all four facets.

$Z' := FZ'u = 0.034$

$\theta := \operatorname{atan}(Z') \cdot \frac{180}{\pi} = 1.919$

$Cd(a) := \frac{4}{\beta} \cdot a^2 + \frac{2}{\beta} \int_0^1 2 \cdot Z'^2 \, dx \qquad Cd(0) = 0.00259 \qquad \theta 2 := 2 \cdot \theta = 3.837$

$\cos\left(\theta \cdot \frac{\pi}{180} \right) = 0.999$

Plotting

$$L := \sqrt{0.5^2 + 0.0335^2} = 0.501$$

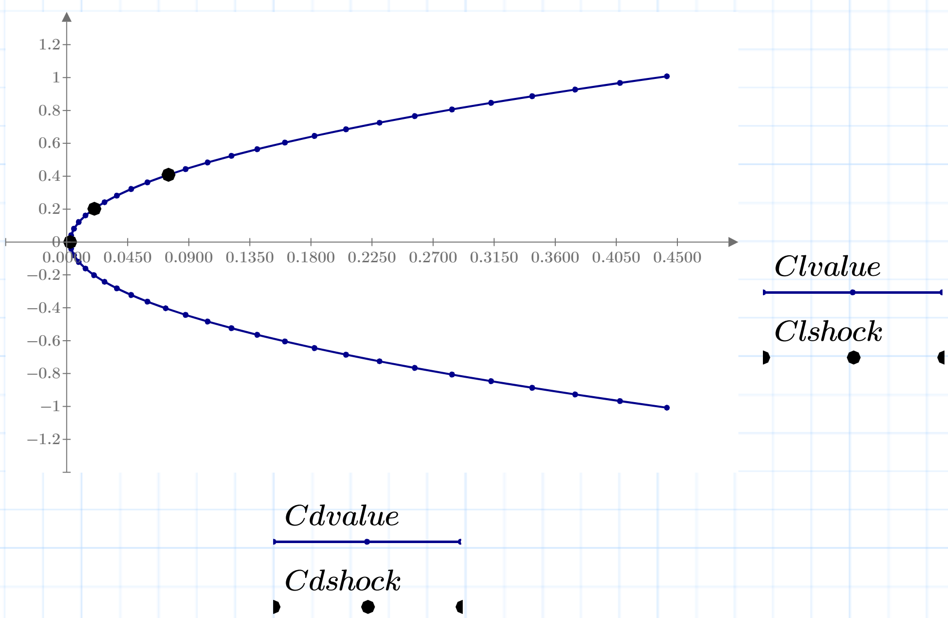
$$i := 0, 1 .. 50 \qquad aoa_i := (i - 25)$$

$$Cdvalue_i := Cd\left(aoa_i \cdot \frac{\pi}{180}\right) \qquad Clvalue_i := Cl\left(aoa_i \cdot \frac{\pi}{180}\right)$$

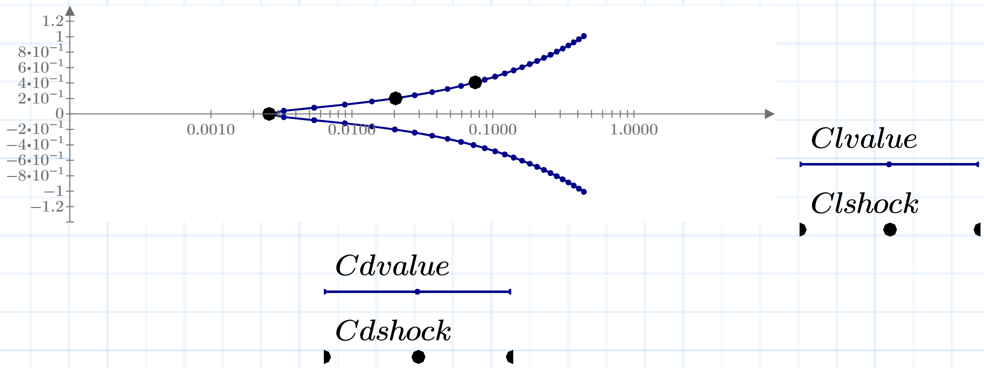
$$ashock := \begin{bmatrix} 0 \\ 5 \\ 10 \end{bmatrix} \quad Clshock := \begin{bmatrix} 0 \\ 0.202 \\ 0.409 \end{bmatrix} \quad Cdshock := \begin{bmatrix} 0.002586506 \\ 0.0204 \\ 0.0750 \end{bmatrix}$$

$$CdError := \frac{Cdshock - \begin{bmatrix} Cd\left(0 \cdot \frac{\pi}{180}\right) \\ Cd\left(5 \cdot \frac{\pi}{180}\right) \\ Cd\left(10 \cdot \frac{\pi}{180}\right) \end{bmatrix}}{Cdshock} \qquad CdError = \begin{bmatrix} -0.002 \\ 0.011 \\ 0.027 \end{bmatrix}$$

Drag Polar



Cdo should always be reported $Cdo := Cd(0) = 0.002592$



Lift to drag ratio: A primary driver from the Breuget Range Equation