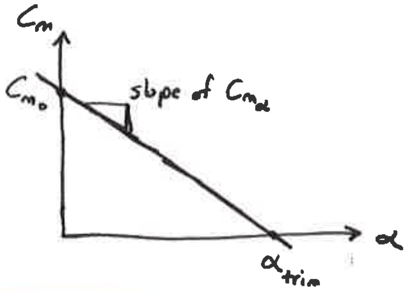
Lesson 14 part 2

Neutral Point + Static Margin

Fuselage + propulsion
Effects on Static
Stability

Wing - Tail Static Stability



$$C_{m_0} = \overbrace{C_{m_{ac_w}} + C_{L_{0_w}}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right)}^{\text{wing}} + \overbrace{\eta V_H C_{L_{\alpha_t}}\left(\epsilon + i_w - i_t\right)}^{\text{tail}}$$

$$C_{m_\alpha} = \underbrace{C_{L_{\alpha_w}}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right)}_{\text{wing}} \underbrace{- \eta V_H C_{L_{\alpha_t}}\left(1 - \frac{d\epsilon}{d\alpha}\right)}_{\text{tail}}$$

with $V_H = \frac{l_t}{\bar{c}} \frac{S_t}{S_w}$

Trim α

$$y = ax + b \Rightarrow C_m = C_{m_\alpha}\alpha + C_{m_0}$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$0 = ax + b \qquad 0 = C_{m_\alpha}\alpha_{trim} + C_{m_0}$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$x = -\frac{b}{a} \qquad \boxed{\alpha_{trim} = -\frac{C_{m_0}}{C_{m_\alpha}}}$$

$$\alpha_{trim} = -\frac{C_{m_{ac_w}} + C_{L_{0_w}}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) + \eta V_H C_{L_{\alpha_t}}\left(\epsilon + i_w - i_t\right)}{C_{L_{\alpha_w}}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) - \eta V_H C_{L_{\alpha_t}}\left(1 - \frac{d\epsilon}{d\alpha}\right)}$$

+ ⇒ TED / LEU

\- ⇒ TEU / LED

Q: How does increasing i_t (the tail incidence angle) affect trim

A: "+" i_t (TED) lowers α_{trim}

Often, we place the CG near or on the A.C. for understanding the concepts and for a $\overset{\text{usually!}}{\vee}$ safe estimate of CG on an unknown aircraft.

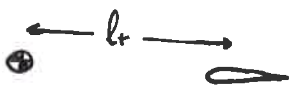
$$\alpha_{trim} = -\frac{C_{m_{ac_w}} + C_{L_{\alpha_w}}\left(\cancelto{0}{\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}}\right) + \eta V_H C_{L_{\alpha_t}}\left(\epsilon_0 + i_w - i_t\right)}{C_{L_{\alpha_w}}\left(\cancelto{0}{\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}}\right) - \eta V_H C_{L_{\alpha_t}}\left(1 - \frac{d\epsilon}{d\alpha}\right)}$$

$$= \frac{C_{m_{ac_w}} + \eta V_H C_{L_{\alpha_t}}\left(\epsilon_0 + i_w - i_t\right)}{+\eta V_H C_{L_{\alpha_t}}\left(1 - \frac{d\epsilon}{d\alpha}\right)}$$

$$= \frac{C_{m_{ac_w}}}{\eta V_H C_{L_{\alpha_t}}\left(1 - \frac{d\epsilon}{d\alpha}\right)} + \frac{\epsilon_0 + i_w - i_t}{1 - \frac{d\epsilon}{d\alpha}}$$

$$= \frac{C_{m_{ac_w}}}{\eta V_H C_{L_{\alpha_t}}\left(1 - \frac{d\epsilon}{d\alpha}\right)} + \frac{\epsilon_0 + i_w}{1 - \frac{d\epsilon}{d\alpha}} \underbrace{- \frac{i_t}{1 - \frac{d\epsilon}{d\alpha}}}_{\alpha \text{ proportional to } i_t}$$

Q: In the absense of a wing, we have



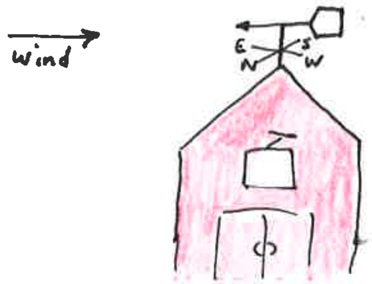
$\frac{d\epsilon}{d\alpha} = 0$ (no wing)

$C_{m_{ac_w}} = 0$

$\epsilon_0 = 0$

$$\alpha_{trim} = -i_t$$

We call this a wind vane

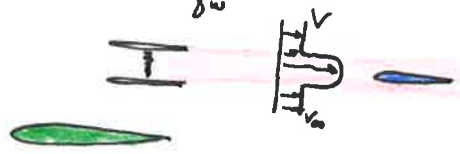


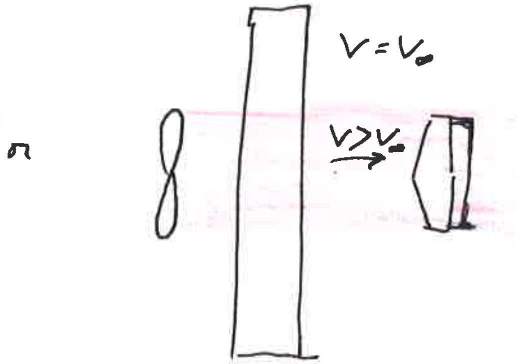
Propulsion Contribution

propulsion greatly affects the stability and trim of aircraft.

$$\uparrow \quad C_{m_\alpha} \qquad\qquad \uparrow \quad \alpha_{trim} = -\frac{C_{m_0}}{C_{m_\alpha}}$$

- Dynamic Pressure $\eta \equiv \frac{q_t}{q_w}$





Stability (cg at ac)

$$C_{m_\alpha} = -\eta V_H C_{L_{\alpha_t}} \left(1 - \frac{d\epsilon}{d\alpha}\right)$$

This is particularly true at low speeds for propeller aircraft.

η is proportional to V^2

Trim (cg at ac)

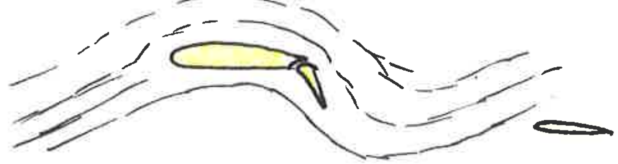
$$\alpha_{trim} = \frac{C_{m_{ac_w}}}{\eta V_H C_{L_{\alpha_t}} \left(1 - \frac{d\epsilon}{d\alpha}\right)} + \text{Constant} - \frac{i_t}{1 - \frac{d\epsilon}{d\alpha}}$$

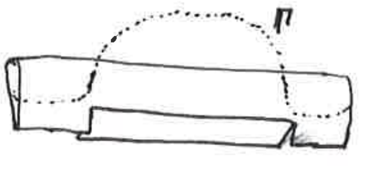
$C_{m_{ac_w}}$ is usually negative.

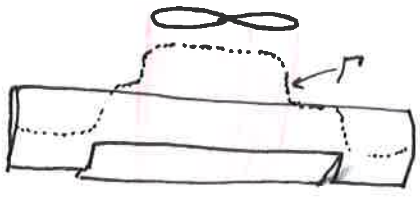
Adding power in a propeller a/c with T in line with a.c. tends to raise the nose from η

- Down wash

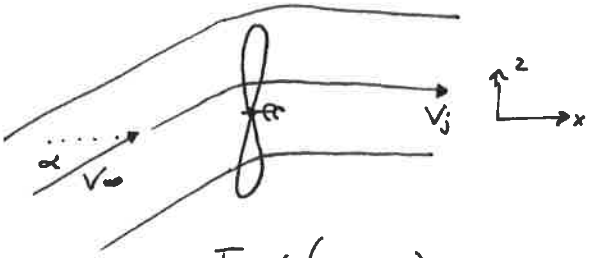
propulsion can change the lift distribution of the wings, which impacts the tail. This is especially notable in a flap condition.

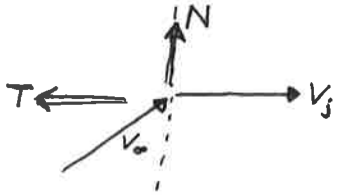






- Normal Forces





$$T = \dot{m}(V_J - V_\infty)$$ vector quantity!

In x-direction,

$$T = \dot{m}(V_{J_x} - V_{\infty_x}) = \dot{m}\left(V_J - V_\infty \cancelto{\approx 1 \text{ for small } \alpha}{\cos\alpha}\right)$$

In z-direction:

$$N = -\dot{m}(V_{J_z} - V_{\infty_z}) = -\dot{m}\left(\cancelto{0}{V_{J_z}} - V_\infty \cancelto{\approx \alpha \text{ for small } \alpha}{\sin\alpha}\right)$$

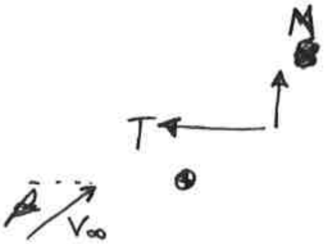
$$= -\dot{m}\left(\cancelto{0}{V_J} - V_\infty \alpha\right)$$

$$= +\dot{m} V_\infty \alpha$$

$$N = \dot{m} V_\infty \alpha \approx \rho V A V \alpha$$

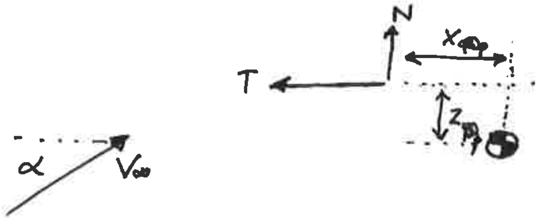
$$\approx \tfrac{1}{2}\rho V^2 \cancel{2} A \alpha 2$$

$$\dot{m} = \frac{T}{V_J - V_\infty} \Rightarrow N = \dot{m} V_\infty \alpha = \frac{T V_\infty}{V_J - V_\infty}\alpha$$



This also occurs with sideslip.

- Thrust Moment



X_p = horizontal fwd distance

Z_p = vertical fwd distance

$$C_{m_{cg_p}} = -\frac{T \cdot Z_p}{\frac{1}{2}\rho V^2 S_w \bar{c}_w} + \frac{N \cdot X_p}{\frac{1}{2}\rho V^2 S_w \bar{c}_w}$$

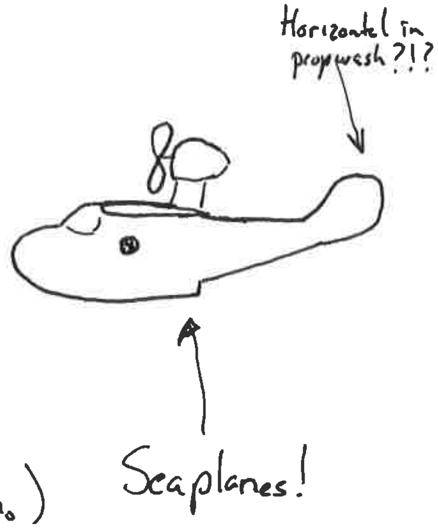
$$= -\frac{T}{q S_w} \cdot \frac{Z_p}{\bar{c}_w} + \frac{N}{q S_w} \cdot \frac{X_p}{\bar{c}_w}$$

for our Normal force model,

$$N \approx q A_p 2\alpha$$

$$C_{m_{cg_p}} = \frac{-T}{q S_w} \cdot \frac{Z_p}{\bar{c}} + \frac{q A_p 2\alpha}{q S_w} \frac{X_p}{\bar{c}_w}$$

$$= -\frac{T}{q S_w} \cdot \frac{Z_p}{\bar{c}} + \frac{A_p}{S_w} 2\alpha \frac{X_p}{\bar{c}_w}$$

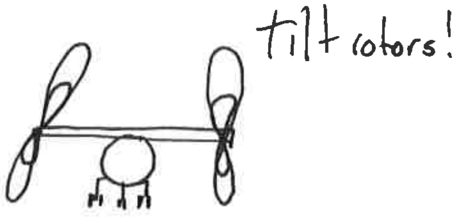


Notice:

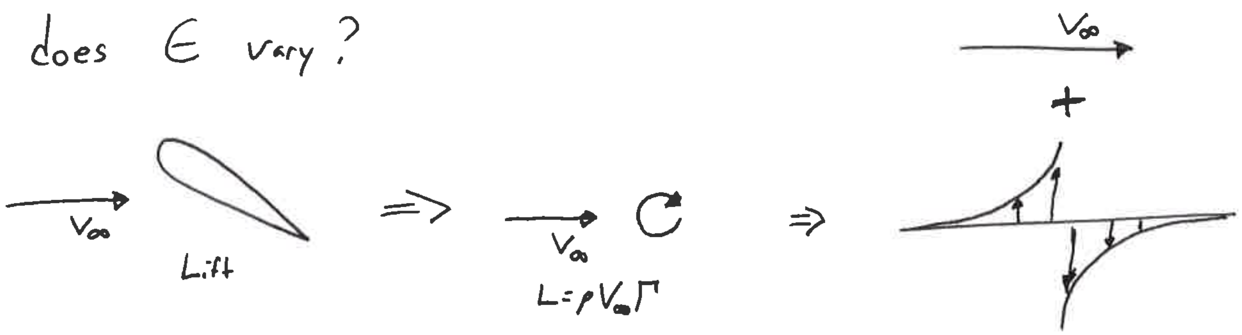
- The Thrust term contributes to trim (C_{m_0})
- The Normal force term <u>decreases</u> stability for forward ($X_p > 0$) locations

$$C_{m_{\alpha_{cg_p}}} = \frac{dC_{m_{cg_p}}}{d\alpha} = \frac{A_p}{S_w} 2 \frac{X_p}{\bar{c}_w}$$

Aircraft with large disk areas $\overset{\text{forward of cg}}{\vee}$ have less stability w/ power



How does ϵ vary?



$L = \rho V_\infty \Gamma$

Along x axis, $V = -\frac{\Gamma}{2\pi x}$

For an elliptical wing at the ¼ chord.

$$\epsilon = \frac{2C_L}{\pi AR} \Rightarrow \frac{d\epsilon}{d\alpha} = \frac{2C_{L_\alpha}}{\pi AR} \quad \text{and} \quad \epsilon_0 = \frac{2C_{L_0}}{\pi AR}$$

In the C_m terms, we see ϵ_0 and $1 - \frac{d\epsilon}{d\alpha}$ often.

Vary w̄ Mach

subsonic

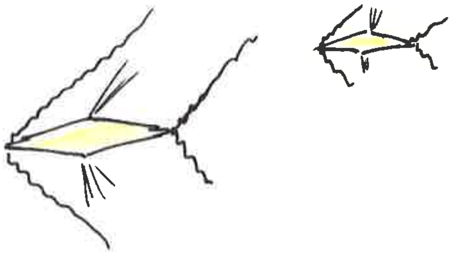
$$\left(\frac{d\epsilon}{d\alpha}\right)_M \approx \left(\frac{d\epsilon}{d\alpha}\right)_{M=0} \cdot \frac{1}{\sqrt{1-M^2}}$$

Prandtl - Glauert.
β

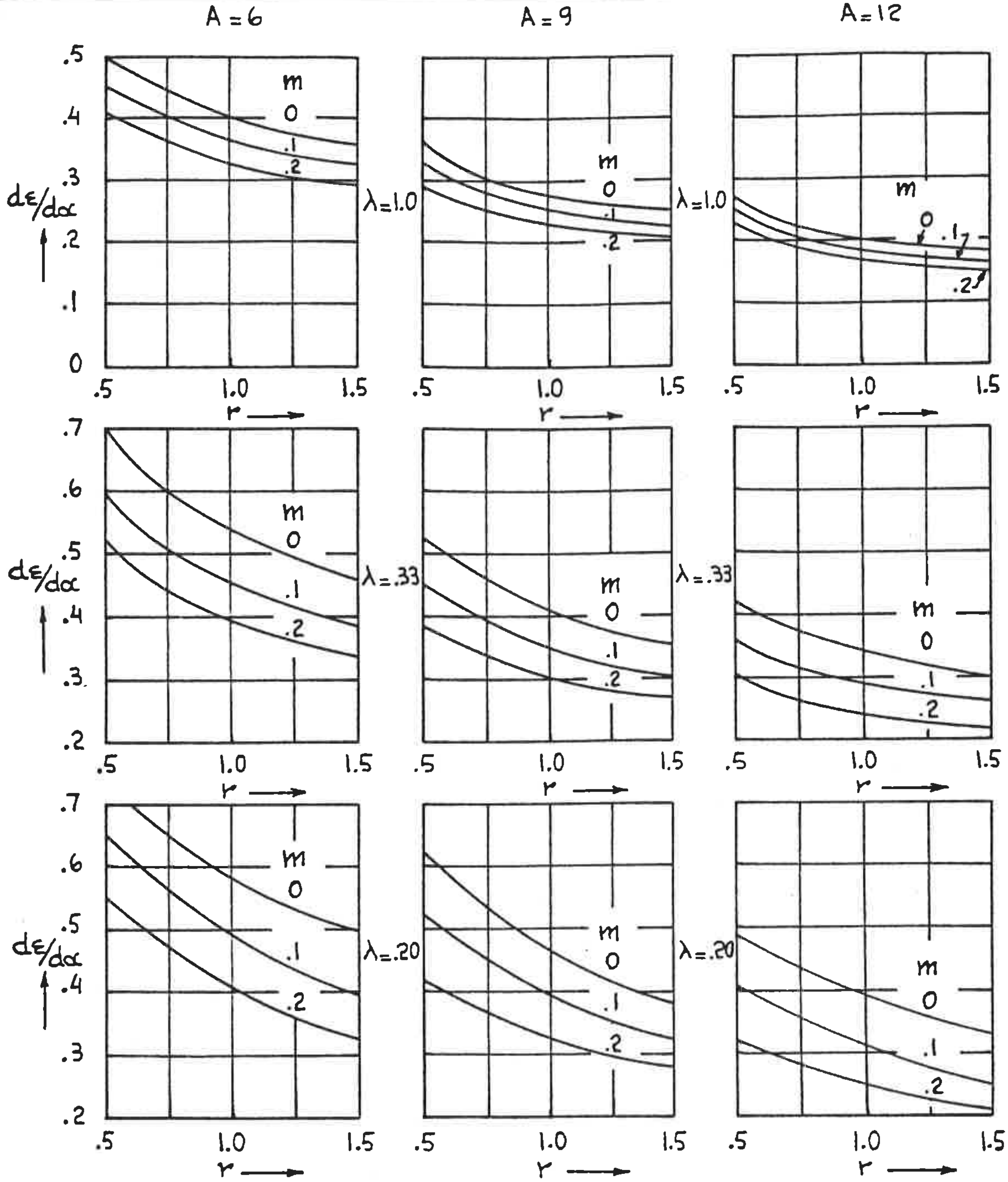
$$\approx \left(\frac{d\epsilon}{d\alpha}\right)_{M=0} \cdot \frac{C_{L_{\alpha_M}}}{C_{L_{\alpha_{M=0}}}}$$

Supersonic

depends on geometry
and
Mach #



Although, in the limit as x→ large, the wake appears similar to subsonic.



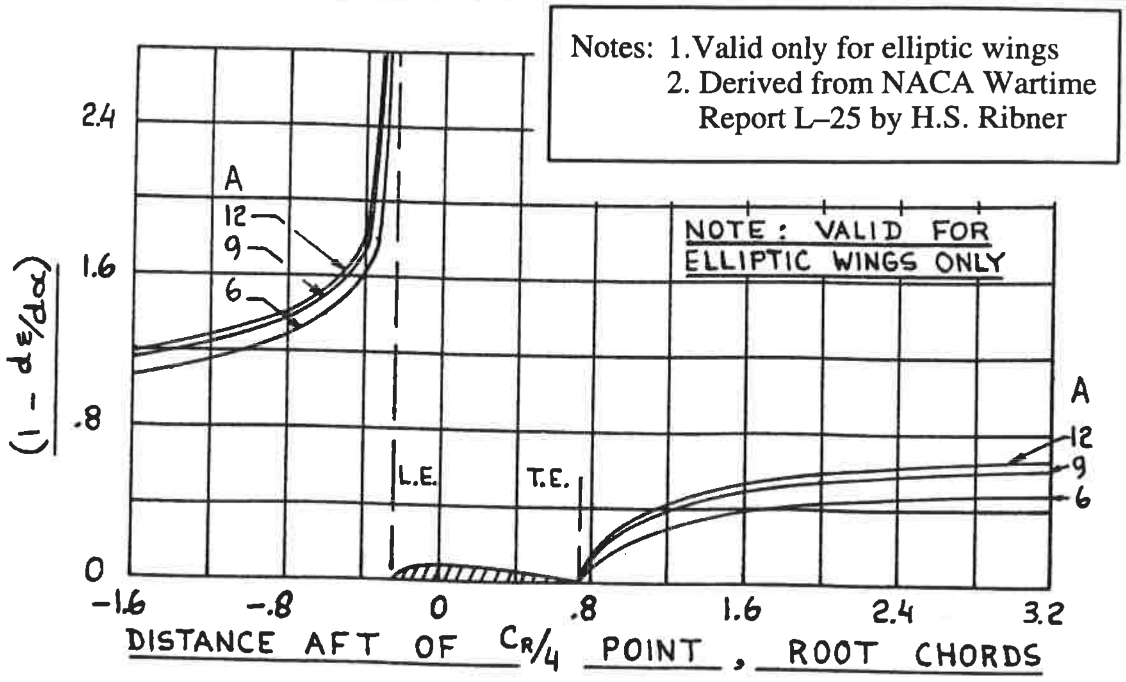
Notes:
1. Valid only for straight, tapered wings at low Mach Numbers
2. For other aspect ratios, interpolate or extrapolate
3. See Part VI of Reference 2.3 for a more general method

$$m = \frac{\text{Vertical distance of horizontal tail } 0.25\bar{c}_h \text{ above/below the wing zero lift line}}{b/2}$$

$$r = \frac{\text{Longitudinal distance of } 0.25c_r \text{ toward the horizontal tail } 0.25\bar{c}_h \text{ location}}{b/2}$$

Figure 2.14 Effect of Wing Aspect Ratio and Horizontal Tail Location on the Downwash Gradient

Roskam



Notes: 1. Valid only for elliptic wings
2. Derived from NACA Wartime Report L–25 by H.S. Ribner

Figure 2.15 Magnitude of $(1 - d\varepsilon/d\alpha)$ on the Longitudinal Axis

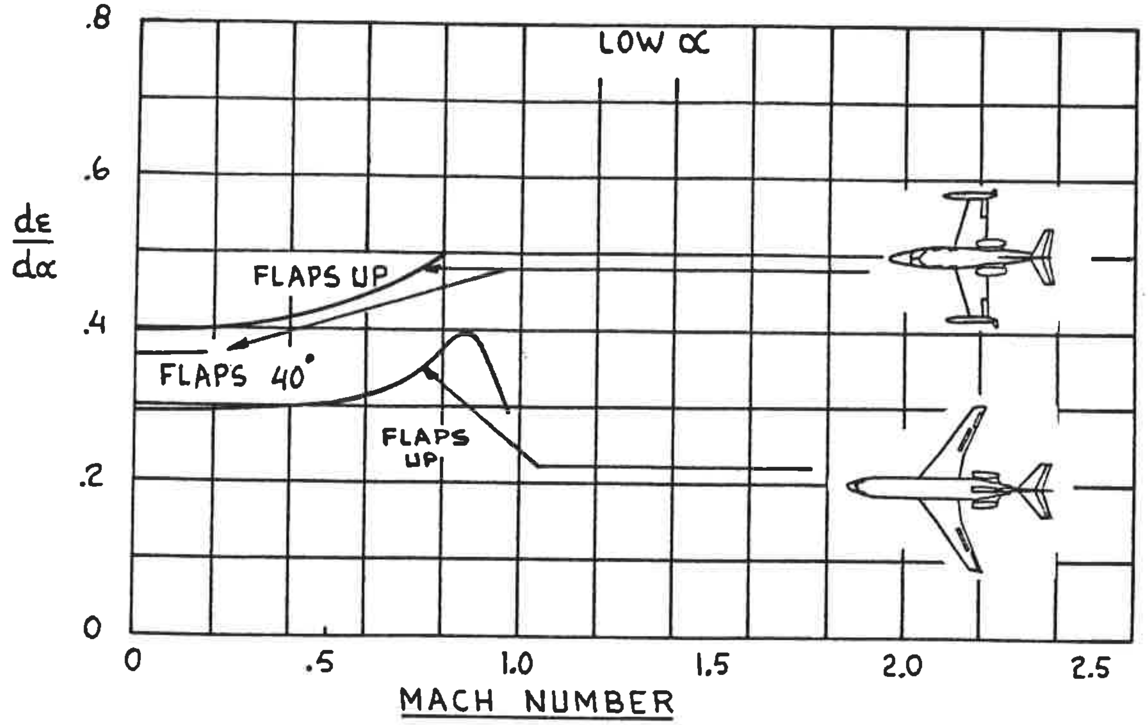
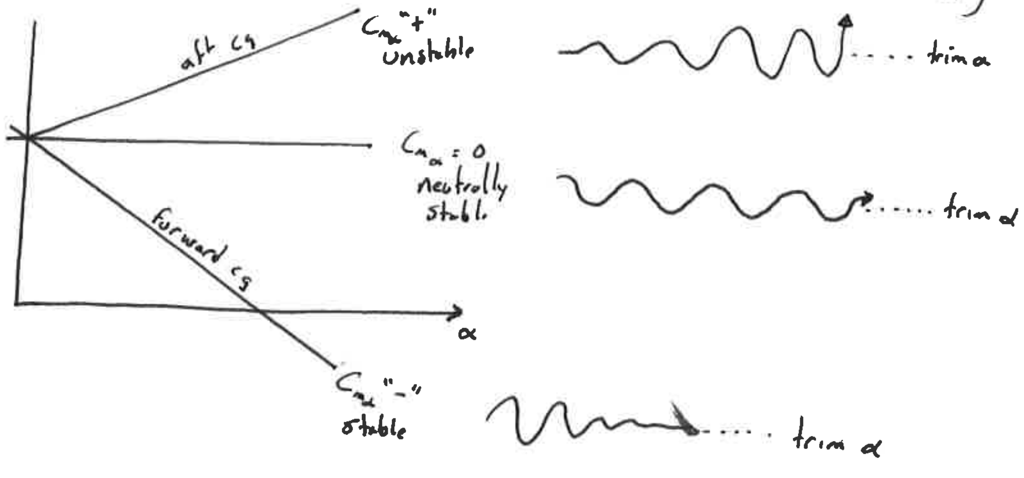


Figure 2.16 Examples of Variation of $d\varepsilon/d\alpha$ with Mach Number

Roskam

Neutral Point (Stick Fixed)

The CG location where $C_{m_\alpha} = 0$ (neutrally stable aircraft)



$$C_{m_\alpha} = C_{L_{\alpha_w}}\left(\frac{X_{cg}}{\bar{c}} - \frac{X_{ac}}{\bar{c}}\right) + C_{m_{\alpha_f}} + C_{m_{\alpha_{stores}}} + \eta V_H C_{L_{\alpha_t}}\left(1 - \frac{d\epsilon}{d\alpha}\right)$$

$0 =$ ↗ ↑ find this

Rearrange

$$\frac{X_{cg}}{\bar{c}} = \underbrace{\frac{X_{ac}}{\bar{c}}}_{\approx 1/4 c \text{ for subsonic}} - \underbrace{\frac{C_{m_{\alpha_f}} + C_{m_{\alpha_{stores}}}}{C_{L_{\alpha_w}}}}_{\text{small?}} + \underbrace{\frac{\eta V_H C_{L_{\alpha_t}}\left(1 - \frac{d\epsilon}{d\alpha}\right)}{C_{L_{\alpha_w}}}}_{\text{tail}}$$

$$= \frac{X_{NP}}{\bar{c}} \approx \frac{X_{ac}}{\bar{c}} + \eta V_H \frac{C_{L_{\alpha_t}}}{C_{L_{\alpha_w}}}\left(1 - \frac{d\epsilon}{d\alpha}\right)$$

CG test before flying

Static Margin

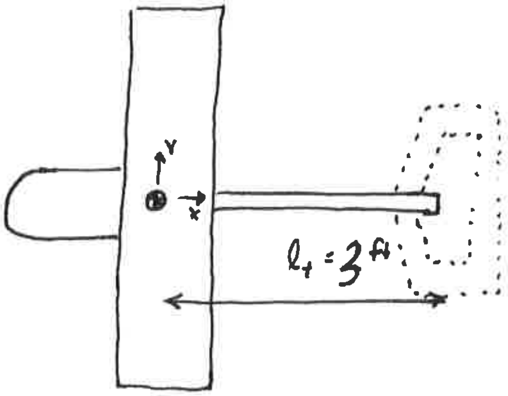
The distance between the $\frac{\text{neutral point}}{\bar{c}}$ and the actual $\frac{CG.}{\bar{c}}$

SM ≈ 10%-15% is ok

SM ≈ 2-5% is not pleasant to fly (usually)

SM > 25% is often too stable and not trimmable at low speeds.

Ex: Size the horizontal tail for a static margin of 10% with the CG at $1/4 c$



$AR_w \approx AR_t$

Lookup $\frac{d\epsilon}{d\alpha} \approx 0.35$

$b = 4$ ft
$S = 4$ sq ft $\Rightarrow AR \approx 4$

$$SM = +\frac{X_{NP}}{\bar{c}} - \frac{X_{CG}}{\bar{c}} = 10\% \quad \Rightarrow \quad \frac{X_{NP}}{\bar{c}} = +0.10 + 0.25 = 0.35$$

Assuming no power or fuselage effects.

$$\cancelto{0.35}{\frac{X_{np}}{\bar{c}}} = \cancelto{0.25}{\frac{X_{ac}}{\bar{c}}} + \cancelto{1}{\eta}\, V_H \cancelto{AR_w \approx AR_t \;\to\; 1}{\frac{C_{L\alpha t}}{C_{L\alpha w}}} \cancelto{1 - \cancel{0}.35 \;\to\; 0.65}{\left(1 - \frac{d\epsilon}{d\alpha}\right)}$$

Solve for $V_H = \dfrac{0.35 - 0.25}{1 \cdot 1 \cdot 0.65} = \dfrac{0.10}{0.65} = 0.15$

$$V_H = \frac{l_t}{\bar{c}} \frac{S_t}{S_w} = 0.15 \quad \Rightarrow \quad S_t = 0.15 \frac{S_w \bar{c}}{l_t}$$

$$= \frac{0.15 \cdot 4\,\text{ft}^2 \cdot 1\,\text{ft}}{3\,\text{ft}} \approx 0.2\ \text{ft}^2$$

Ex: Same Aircraft but CG at 75% $\bar{c}$

$$SM = \frac{X_{NP}}{\bar{c}} - \cancelto{0.75}{\frac{X_{cg}}{\bar{c}}} = 10\% \qquad \Rightarrow \frac{X_{NP}}{\bar{c}} = 85\%$$

$$\cancelto{0.8}{\frac{X_{np}}{\bar{c}}} = \cancelto{0.25}{\frac{X_{ac}}{\bar{c}}} + \cancelto{1}{\eta} V_H \cancelto{1}{\frac{C_{L_{\alpha_t}}}{C_{L_{\alpha_w}}}} \cancelto{0.65}{\left(1 - \frac{d\varepsilon}{d\alpha}\right)} \qquad V_H \approx 0.85$$

$$0.8 = 0.25 + \cancelto{3/4}{\frac{l_t}{\bar{c}}} \frac{S_t}{\cancelto{4}{S_w}} 0.65 \qquad \Rightarrow S_t = (0.8 - 0.25) \cdot \frac{4}{3} \frac{1}{0.65}$$

$$S_t = 1.13 \text{ ft}^2$$

We would also need to check C_{m_0} to ensure that the horizontal is operating at a reasonable C_{L_t} for a given α trim range.

$$C_{m_0} = 0 = \cancelto{\approx 0.1}{C_{m_{ac_w}}} + \cancelto{\approx 0.1}{C_{L_{0_w}}} \cancelto{0.75 - 0.25}{\left(\frac{X_{cg}}{\bar{c}} - \frac{X_{ac}}{\bar{c}}\right)} + \cancelto{1}{\eta} V_H \cancelto{\frac{2\pi}{1+\frac{2\pi}{\pi 4}} \approx 4.2}{C_{L_{\alpha_t}}} \left(\cancelto{0}{\varepsilon_0} + \cancelto{0}{i_w} - i_t\right)$$

$$0.85 \cdot 4.2(-i_t) + 0.1 + 0.1(0.5) = 0$$

$$i_t \approx -0.04 \text{ rad} \approx -2.4° \quad \text{reasonable}$$

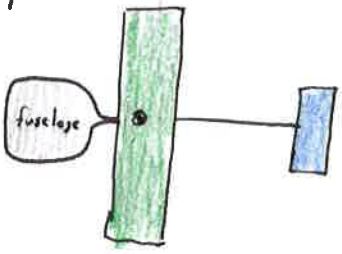
$$\alpha_{trim} = -\frac{C_{m_0}}{C_{m_\alpha}} = \frac{0}{\cdots} = 0$$

Obviously since we specified that $C_{m_0} = 0$!

Trimmed for $C_L = 0$

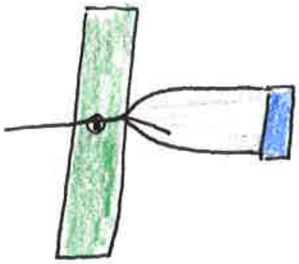
Fuselage Contribution

The fuselage generates lift and drag and moments. Neglecting the impact is a serious error.



less stable

vs

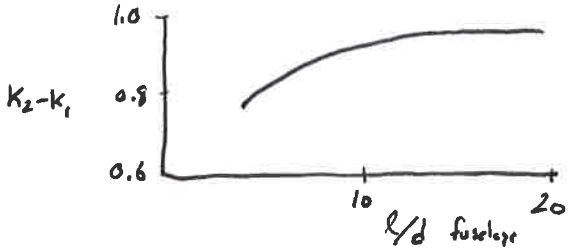


more stable

Multhopp's method for computing $C_{m_{0_f}}$ and $C_{m_{\alpha_f}}$ is: NACA TM-1036, 1942

$$C_{m_{0_f}} = \frac{k_2 - k_1}{36.5 \cdot S \cdot \bar{c}} \int_0^{l_f} w_f^2 (\alpha_{0_w} + i_f)\, dx$$

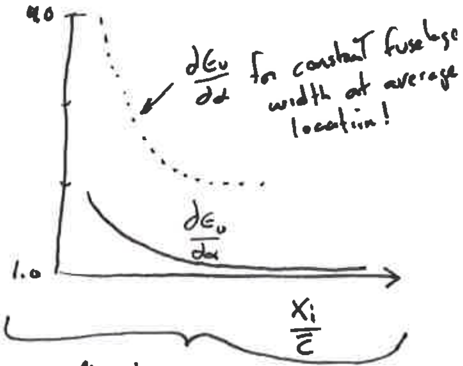
$$\approx \frac{k_2 - k_1}{36.5 \cdot S \cdot \bar{c}} \sum w_f^2 (\alpha_{0_w} + i_f)\, \Delta x$$



w_f = fuselage width

i_f = incidence fuselage $\left({+ \; / \atop - \; \backslash} \right)$

$$C_{m_{\alpha_f}} = \frac{1}{36.5 \cdot S \cdot \bar{c}} \int_0^{l_f} w_f^2 \frac{\partial \epsilon}{\partial \alpha}\, dx$$



Ahead of wing only

$\bar{X}$ is an average/integrated term

$$\underbrace{\frac{d\epsilon_u}{d\alpha} = \frac{X_i}{l_h}\left(1 - \frac{d\epsilon}{d\alpha}\right)}_{\text{behind wing}}$$

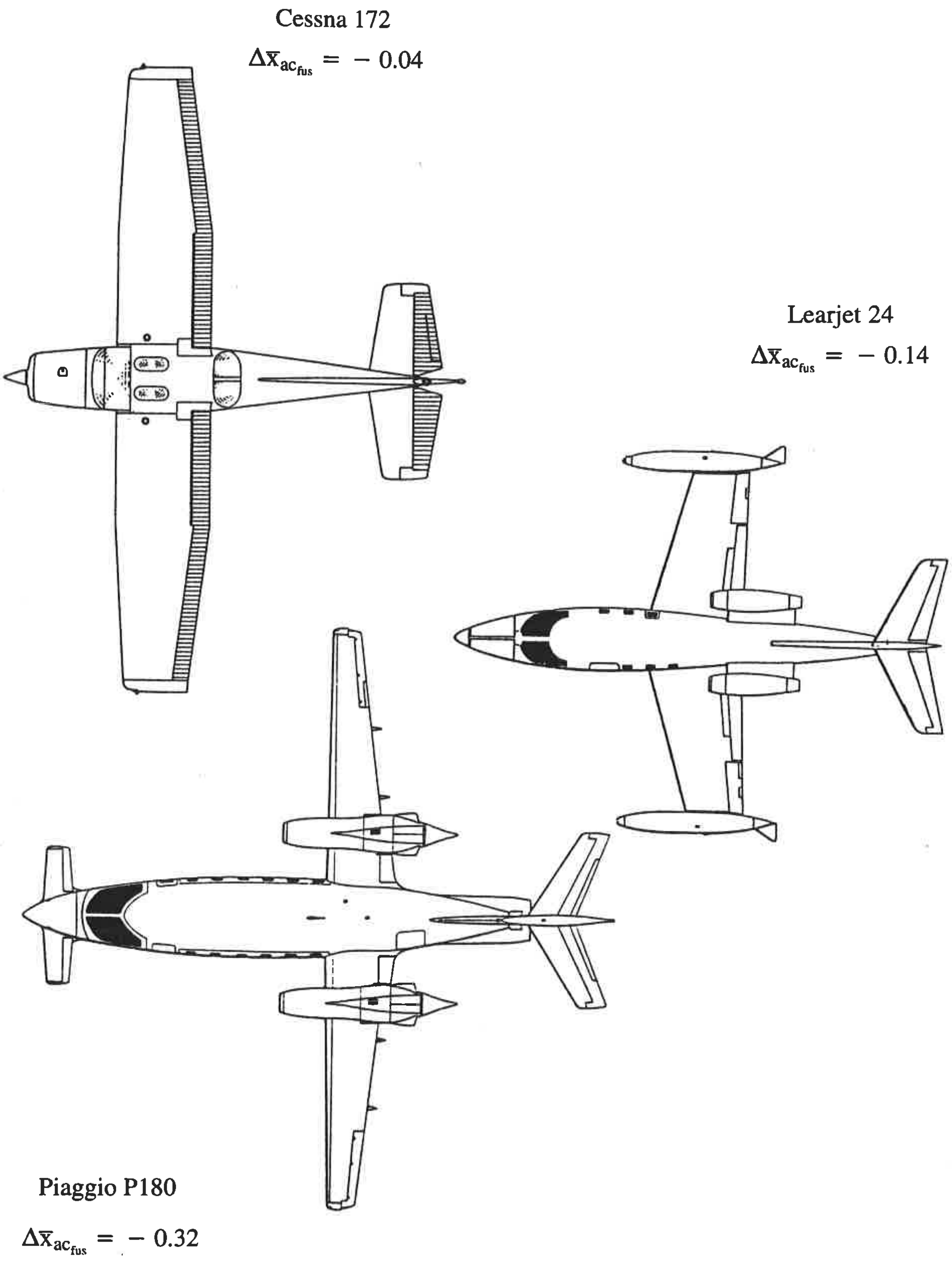
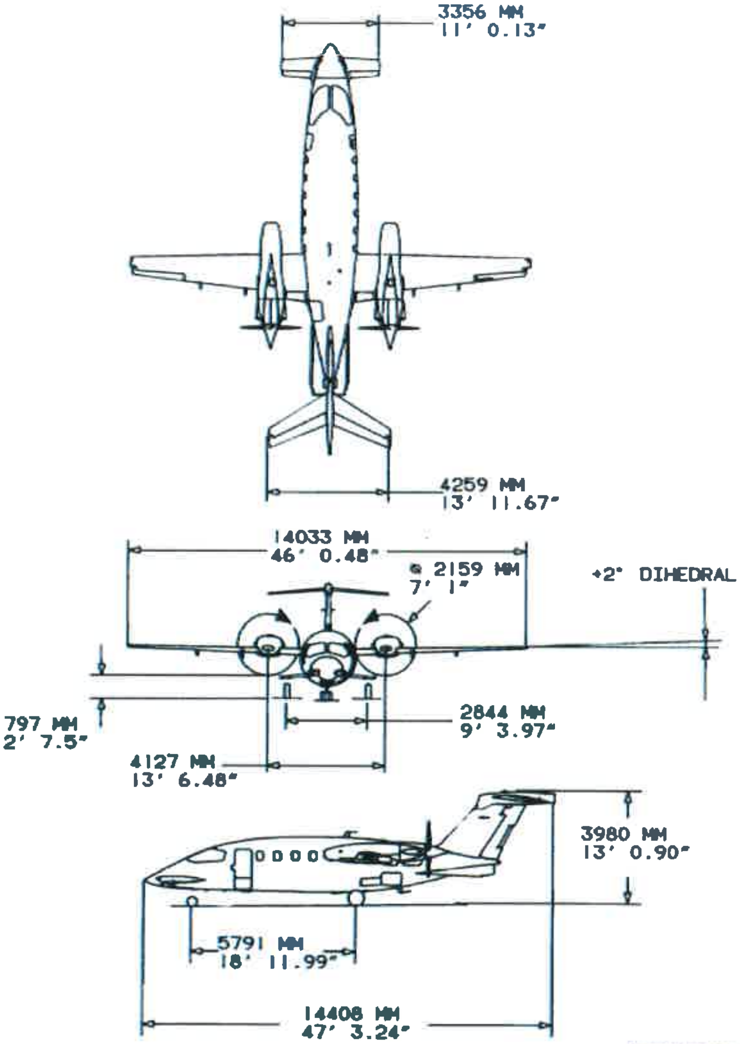


Figure 2.19 Numerical Examples of Fuselage Induced Shift in Aerodynamic Center



blueprintbox.com

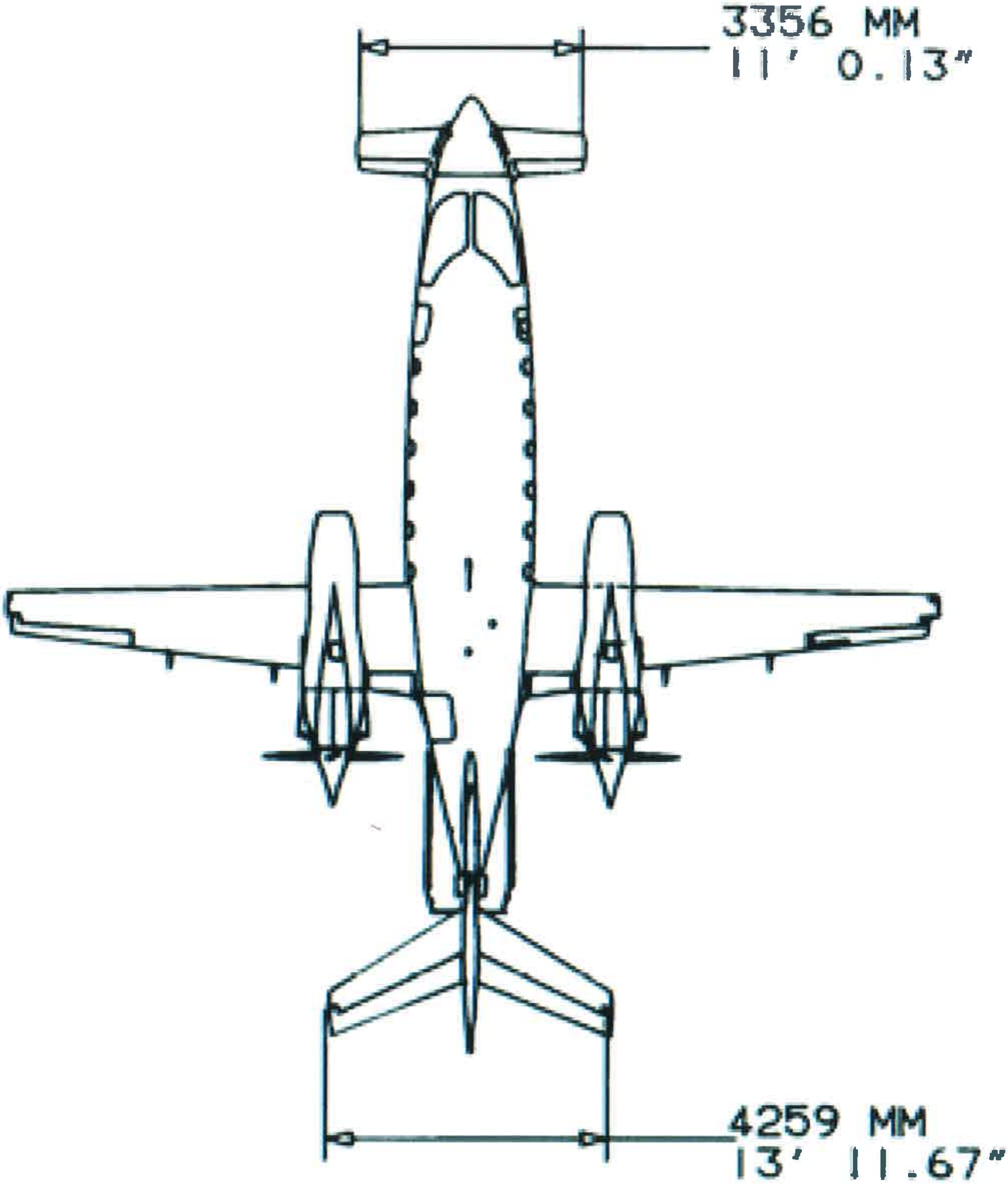
Piaggio 180

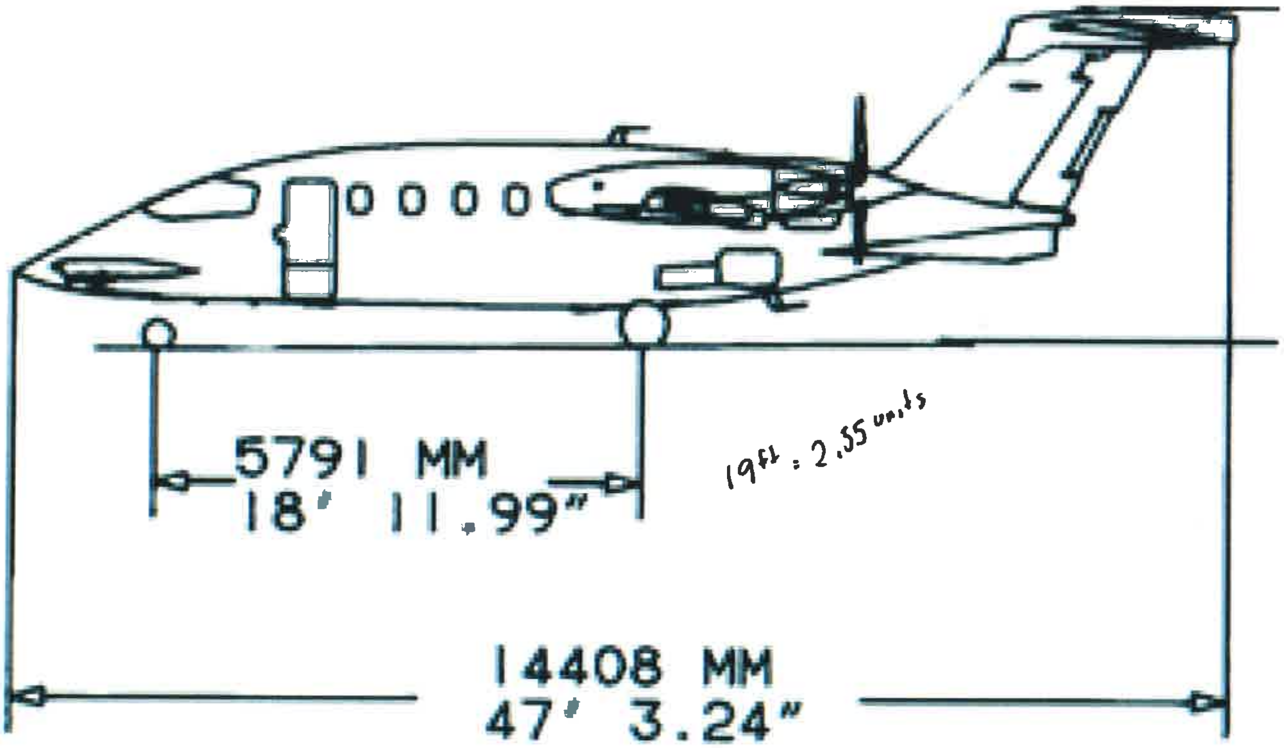
Avanti = "Forward"

"Advance"

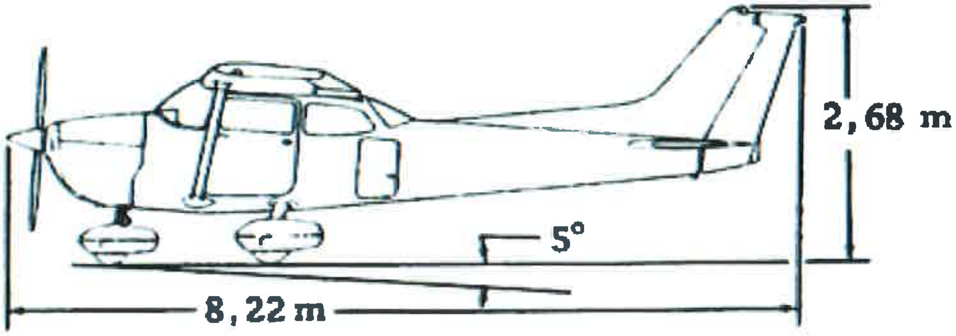
$$AR = \frac{b^2}{S} = \frac{(46^{ft})^2}{172^{ft^2}} \approx$$

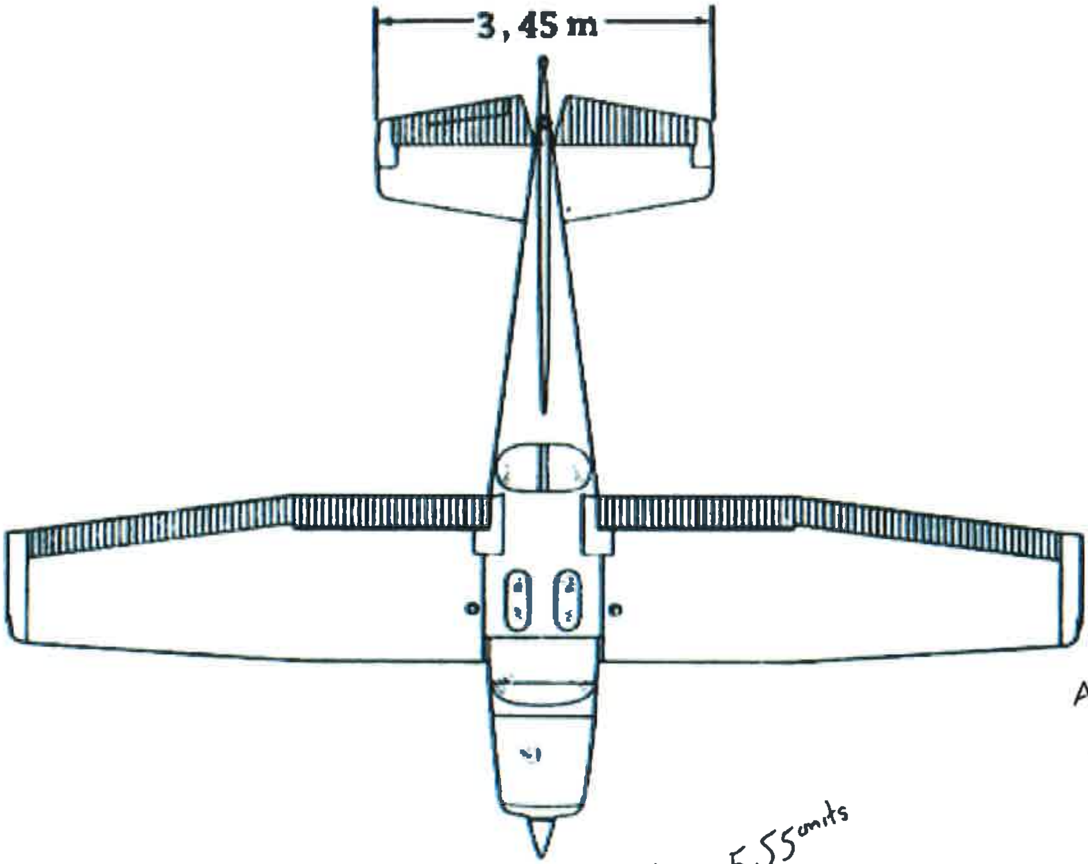
$$C_{L_\alpha} \approx \frac{C_{\ell_\alpha}}{1 + \frac{C_{\ell_\alpha}}{\pi AR}}$$


3356 MM
11' 0.13"
4259 MM
13' 11.67"



19ft : 2.55 units





AR = 7.32

S = 174 ft²

c̄ =

36 ft = 5.55 units