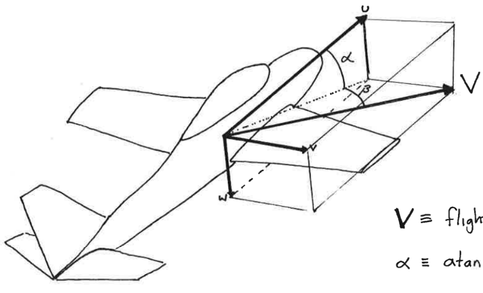
TuskaUAV: Aerodynamics and Propulsion

Angle of Attack and Sideslip

Dr. O'Neill



$V \equiv$ flight velocity vector

$\alpha \equiv \operatorname{atan}\left(\frac{w}{u}\right)$ Angle of Attack

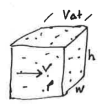
$\beta \equiv \operatorname{asin}\left(\frac{v}{V}\right)$ Sideslip

α is defined wrt the projection of V onto the body frame (i.e. U)

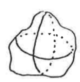
β is defined wrt the v projection and V.

- Physics

Impulse is Δmv , Force is $\frac{\text{Impulse}}{\Delta t}$



Blob of air moving at V velocity with density ρ. Cross sectional area is $h \cdot w = A$



Interacts with a solid surface of exactly $h \cdot w$ cross section

Ignore this and just put a constant

$$J = (\text{mass} \cdot \text{velocity})_{\text{start}} - (\text{mass velocity})_{\text{end}} = C \cdot \text{mass} \cdot \text{velocity}$$

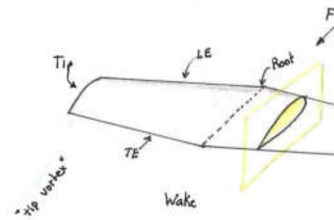
$$= \underbrace{\Delta t \rho V h w}_{\text{mass}} \cdot V \cdot C \quad \Rightarrow \quad \text{Force} = \frac{J}{\Delta t} = \rho V^2 A \cdot C$$

Example: Lift $= \frac{1}{2}\rho V^2 S_{\text{wing}} \cdot C_L$

$$\boxed{\text{Dynamic Pressure} = q = \tfrac{1}{2}\rho V^2}$$

Wing:

- Three dimensional closed surface
- Generates aerodynamic force
- Cross sections are airfoils

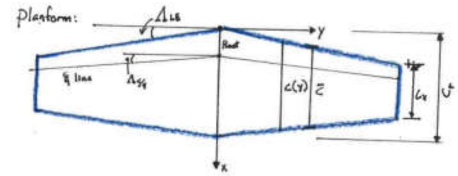


Etymology:

wing — English
↓
wenge — middle English
↓
vent (english) — Vaenger — Norse
↓
vent (Latin) — we-ingyaz — Old German
↓
we (to blow) — Indo-European

Interestingly, not from Indo-European word for "fly" which is "peth" from which we get "feather" and "pen".

Planform:



$S \equiv$ Wing Area $[L^2]$

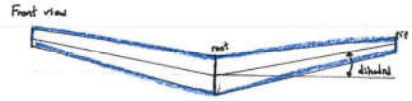
$b \equiv$ Span $[L]$

$c \equiv$ chord

$AR \equiv$ Aspect Ratio $\frac{b^2}{S}$

$\lambda \equiv$ taper ratio $\equiv \frac{c_t}{c_r}$, $\left(\frac{\text{tip}}{\text{root}}\right)$

Front view

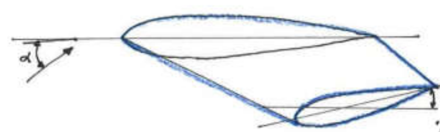


MAC $\equiv$ Mean Aerodynamic Chord

$$\frac{1}{S}\int_{-b/2}^{b/2} c(y)^2\, dy$$

$\bar{c} \equiv$ Average Chord

Side view



wash out $\equiv$ decrease in incidence angle at tip

"wash in" is a positive increase ... usually a bad idea

Lift

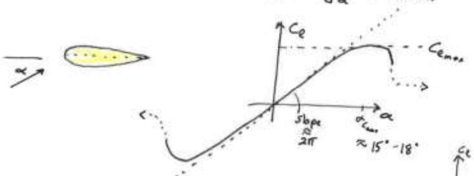
$\boxed{2\pi \text{ corrected for aspect ratio}}$

$$\text{Lift} = \underbrace{C_L}_{\text{lift coeffic}}\ \underbrace{q}_{\text{dynamic pressure}}\ \underbrace{S}_{\text{area}}$$

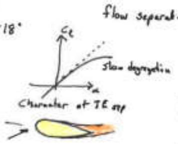
2-Dimensional airfoil

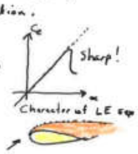
$C_{\ell_\alpha} \approx 2\pi$

$C_{\ell_\alpha} \equiv \frac{dC_\ell}{d\alpha}$ in $\frac{1}{\text{radians}}$



Lower C_ℓ deviation from $2\pi\alpha$ as α increases is due to flow separation.

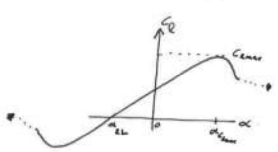




Camber

Mean chord line is through the forward LE and rear TE pts.

Adding camber offsets the C_ℓ vs α curve such that zero lift occurs at α_{ZL}

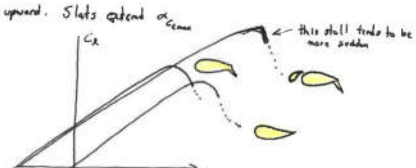


Knowing $C_{\ell_\alpha} \approx 2\pi$ and α_{ZL}, what is a good estimate for $C_\ell(\alpha=0)$?

$C_\ell(\alpha=0) \approx 2\pi \cdot (-\alpha_{ZL})$

Slats and Flaps

Flaps shift the C_ℓ vs α curve upward. Slats extend $\alpha_{C_{\ell max}}$



Analysis of Prandtl Lifting Line Theory:

The elliptical lift distribution is optimal for minimizing induced drag.

$$C_{L_\alpha} = \frac{C_{\ell_\alpha}}{1 + \frac{C_{\ell_\alpha}}{\pi AR}}$$

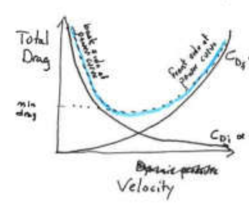
Reducing AR reduces lift slope to away from 2D value.

$$C_D = \frac{C_L^2}{\pi AR e}$$

where $e=1$ for elliptic only

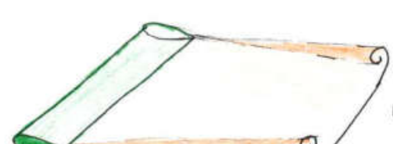
Induced drag depends on Lift squared and the inverse of AR.

$$= \frac{\left(\frac{L}{b}\right)^2}{q^2 \pi e} = C\,\frac{(\text{Span loading})^2}{(\text{dynamic pressure})^2}$$



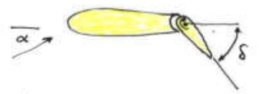
Non-Elliptical distributions have an "e" value are essentially a ratio of actual to elliptical performance

$e_{\text{non-elliptical}} < e_{\text{elliptical}} = 1$



Wake rollup

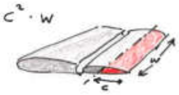
Hinge Moments (Necessary for the upcoming flight control system portion of the class)



A moment is necessary to maintain the control surface at δ.

$$C_h = f(\alpha, \delta, Re, gap, etc)$$ and possibly time

$$H = C_h \cdot q \cdot \underset{\text{area aft of hinge}}{S} \cdot \underset{\text{chord aft of hinge}}{c} = C_h \cdot q \cdot c^2 \cdot w$$



The pilot is connected to the surface in some way

$$\delta = f(\underset{\delta_s}{\text{stick angle}})$$

$$F l_s \delta_s = H \delta_e \Rightarrow F = \left(\frac{\delta_e}{l_s \delta_s}\right) H_e$$ Gearing Ratio

How much force can a pilot exert? How long? $H \propto V^2$; pilot is limited human!

Constraints? Stick position, structural stresses, . . .

Trim Tab

$$C_h = C_{h_0} + \underbrace{C_{h_\alpha}}_{\frac{dC_h}{d\alpha}} \alpha + \underbrace{C_{h_{\delta_e}}}_{\frac{dC_h}{d\delta_e}} \delta_e + \underbrace{C_{h_{\delta_t}}}_{\frac{dC_h}{d\delta_t}} \delta_t$$

For the pilot to have zero stick force, $C_h = 0$

We (as pilots) can adjust δ_t to ensure $C_h = 0$ for a particular δ_e required at a particular α.

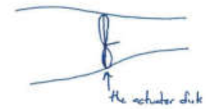
Stick free:

$$C_h = 0 = \left(C_{h_0} + \cancel{C_{h_{\delta_t}} \delta_t}^{\text{trimmed}}\right) + \underbrace{C_{h_\alpha} \alpha}_{\text{usually negative}} + \underbrace{C_{h_{\delta_e}} \delta_e}_{\text{usually negative}} \Rightarrow \delta_e = -\frac{C_{h_\alpha}}{C_{h_{\delta_e}}} \alpha$$

The elevator "floats". TEU $\leftrightarrows$ α increase

Propeller

Converts rotational power into linear power



Efficiency

$$\eta = \frac{P_{available}}{P_{input}}$$

Advance Ratio

$$J = \frac{V_\infty}{ND} = \frac{\text{Forward velocity}}{\text{revolutions per second} \cdot \text{diameter}}$$

Ex: A 60 inch propeller is operating at 60 mph at 2500 rpm. What is J?

$$J = \frac{V_\infty}{ND} = \frac{60\,\text{miles}}{\text{hr}} \left| \frac{\text{min}}{2500\,\text{rev}} \right| \frac{}{60\,\text{in}} \left| \frac{\text{hr}}{60\,\text{min}} \right| \frac{5280\,\text{ft}}{1\,\text{mile}} \left| \frac{12\,\text{in}}{1\,\text{ft}} \right.$$

$$= \boxed{0.42}$$

Thrust Coefficient

$$C_T = \frac{T}{\rho n^2 D^4} \qquad C_T = f(J)$$

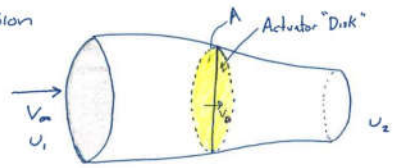
Power Coefficient

$$C_P = \frac{P}{\rho n^3 D^5} \qquad C_P = f(J)$$

Efficiency

$$\eta = \frac{TV}{P} = \frac{C_T \rho n^2 D^4 V}{C_P \rho n^3 D^5} = \frac{C_T}{C_P} \underbrace{\frac{V}{nD}}_{J} = \boxed{\frac{C_T}{C_P} J}$$

Propulsion



From the CV,

$$T = -D = -\iint_S \underbrace{\rho_2 U_2}_{\dot{m}/A \text{ mass flow per area}} \underbrace{(U_1 - U_2)}_{V_\infty - U_2} \, dS$$

$$\boxed{T = \dot{m}(U_2 - U_\infty)}$$

The thrust is also the pressure difference across the disk $\cdot$ disk area

$$T = \Delta P_{disk} \cdot A_{disk}$$

Applying Bernoulli's eqn, gives

$$V_0 = \frac{1}{2}(U_1 + U_2)$$ the average of upstream and downstream

rearranged, this is

$$V_0 = U_1 + w$$ w: the increment in velocity across the disk

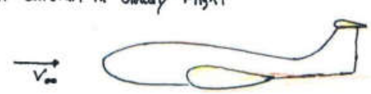
$$T = 2\rho A(U_1 + w) w = \underbrace{\rho A(U_1 + w)}_{\text{mass flux through disk}} \cdot \underbrace{2w}_{\text{Velocity increment through disk}}$$

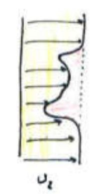
Power.

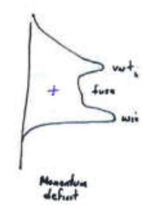
$$P = 2w \cdot \rho A (U_1 + w)^2 = \underbrace{2w}_{V \text{ incr' disk}} \cdot \underbrace{\rho A(U_1 + w)}_{\text{mass flux}} \cdot \underbrace{U_1 + w}_{\text{velocity disk}}$$

$$= 2w\rho A(U_1 + w)^2$$

Full aircraft in steady level flight





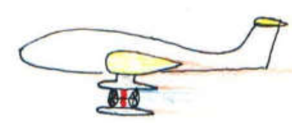


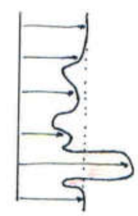
$C_D \geq 0$

D > 0

$\vec{F} = m\vec{a} \Rightarrow$ the aircraft decelerates

Propulsion!



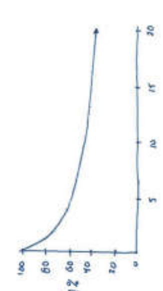


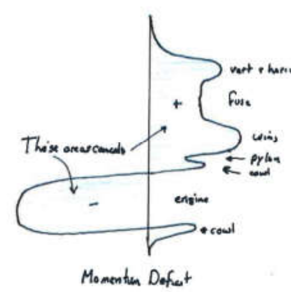
$$\eta = \frac{2}{1 + \sqrt{1 + T_c}}$$

T_c = thrust coefficient $= \frac{1}{q} \underbrace{\frac{T}{A}}_{\text{disk loading}}$

Efficient engines have a low $\frac{T}{qA}$ — Freight engines

High thrust engines have low η — military





$$C_D = \frac{\int \rho_2 U_2 (U_1 - U_2)\, dl}{qS} = 0$$