Energy

$$\underbrace{E}_{\text{total}} = \underbrace{PE}_{\text{potential}} + \underbrace{KE}_{\text{Kinetic}} = W\cdot h + \frac{1}{2}mV^2 = Wh + \frac{1}{2}\frac{W}{g}V^2 \quad \text{units of energy}$$

Ex: what is the total energy of a 12500 lb Twin Otter at 12000 ft and 120 kts?

$$E = \underbrace{\frac{12500\ \text{lbf} \mid 12000\ \text{ft}}{\ }}_{150\times10^6\ \text{ft lbf}} + \underbrace{\frac{1}{2}\,\frac{12500\ \text{lbf} \mid s^2 \mid 120^2\ \text{kt}^2 \mid 1.688^2\ \text{ft}^2}{32.174\ \text{ft} \mid \quad \mid \text{kt}^2\,s^2}}_{8\times10^6\ \text{ft lbf}} = \cancel{\ \ } \ 158\times10^6\,[\text{ft lbf}]$$

$\gg$ mostly altitude for this slow a/c

Specific Energy

$$E_s = \frac{E}{W} = \frac{Wh + \frac{1}{2}\frac{W}{g}V^2}{W} = h + \frac{1}{2}\frac{V^2}{g} \quad \text{units of length! "energy height"}$$

Derivative wrt time.

$$\frac{dE_s}{dt} \equiv P_s = \frac{dh}{dt} + \frac{1}{2}\frac{1}{g}2V\frac{dV}{dt} = \dot{h} + \frac{V}{g}\frac{dV}{dt} = P_s$$

"specific excess" power

units $[ft/s]$

where have you seen this before? Accelerated Climb $\frac{dh}{dt} = V\frac{T-D}{W} - \frac{V}{g}\frac{dV}{dt}$

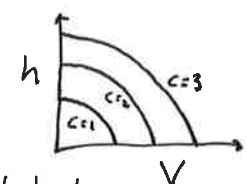
$$\boxed{P_s = V\frac{T-D}{W} = \dot{h} + \frac{V}{g}\frac{dV}{dt}}$$

Now you see why we multiplied by V/W, as this gives specific power.

Lines of constant E_s

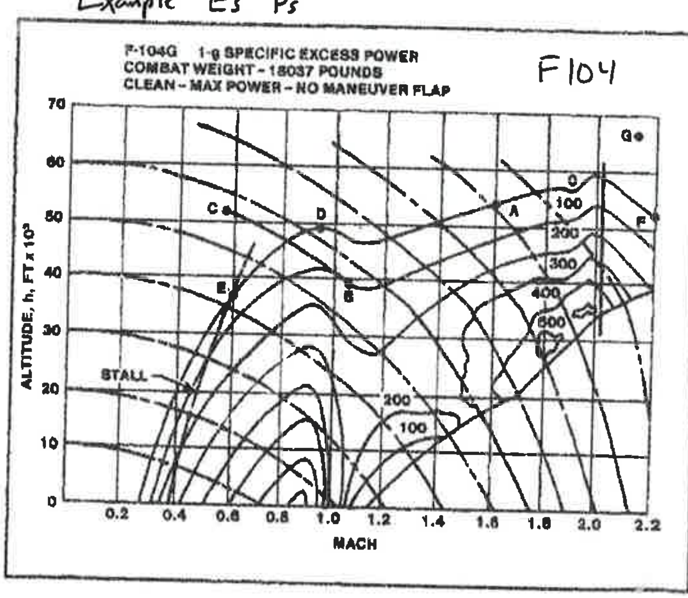
$$E_s = C = h + \frac{1}{2}\frac{V^2}{g} \Rightarrow h = C - \frac{1}{2}\frac{V^2}{g}$$

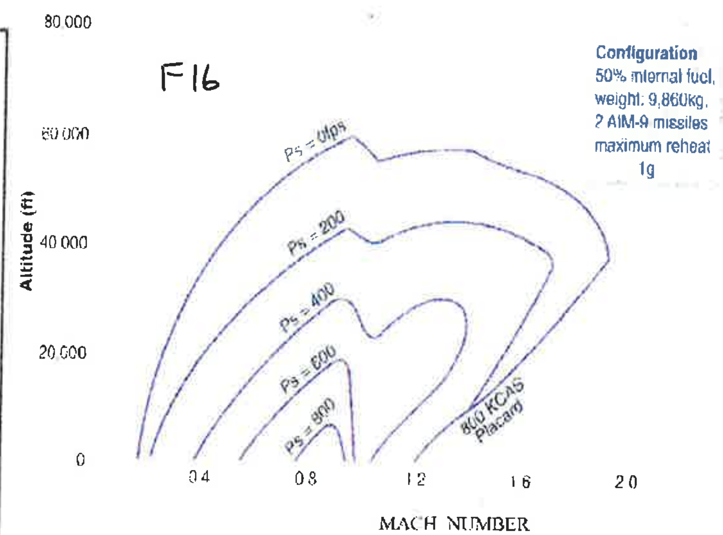
parabolas



In a frictionless environment, objects would travel along a line of constant E trading h and V.

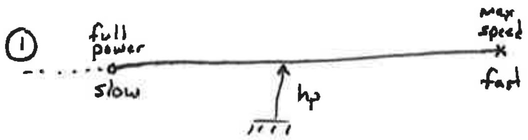
Example E_s P_s

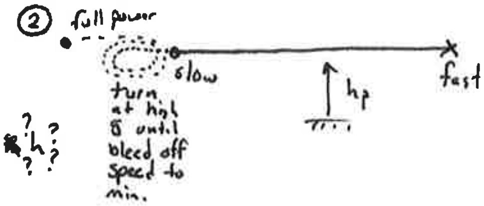




F-16C SEP datas, from air international.

Level Acceleration

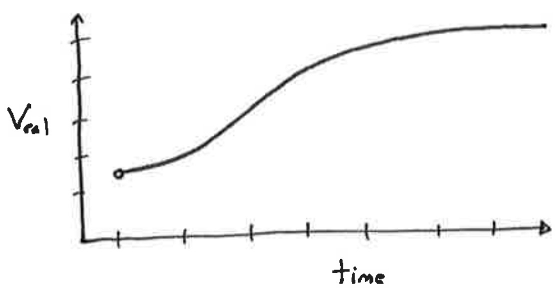




measure $t, V, \dot{h}$???

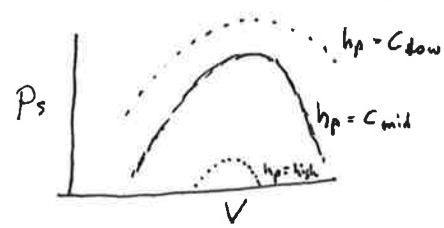
$$P_s = \dot{h} + \frac{V}{g}\frac{dV}{dt}$$

Track time, velocity while maintaining h



⇒

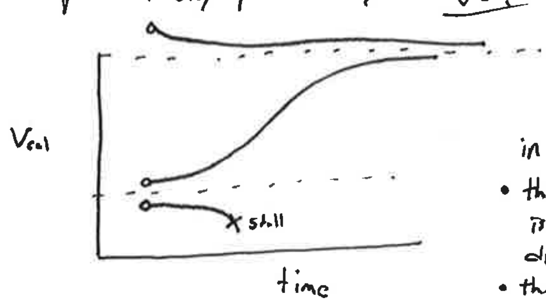
time	V_{cal}	V_{true}	$\frac{dV}{dt} \approx \frac{\Delta V}{\Delta t}$	P_s
0	⋮	⋮	⋮	⋮
Δt				
⋮				



Ex:
An aircraft accelerated from 80-85 ^{KTAS} in 2 seconds, what is the equivalent climb rate?

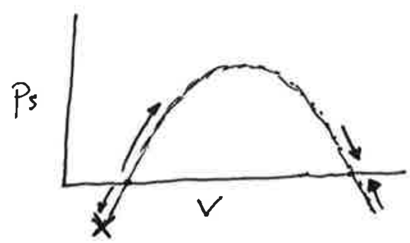
$$P_s = \cancel{\dot{h}}^{\,0} + \frac{V}{g}\frac{dV}{dt} = \frac{82.5\ \text{KTAS}}{} \left|\frac{1.688\ \text{ft}}{\text{kt}\ \text{s}}\right| \frac{\text{s}^2}{32.174\ \text{ft}} \left|\frac{5\ \text{kt}}{\text{s}}\right| \frac{1.688\ \text{ft}}{\text{kt}\ \text{s}} = 36.5\ \text{ft/s}$$

Q: Is P_s strictly positive? No!



in other words
- the low speed $P_s = 0$ is a speed divergence.
- the high speed $P_s = 0$ is a speed convergence

Q: what happens when $P_s < 0$? slow or descent



P_s is extremely useful for determining optimal flight profiles for:
time to climb, fuel to climb, glide path, etc.

Calculus of variations: minimize $J(y) = \int_a^b F(x, x', y, y')\, dx$ in other words, what $\underbrace{y(x)}_{\text{path}}$ minimizes $\underbrace{J(y)}_{\text{time, fuel, } 1/\$, \ \vdots\ \text{etc}}$?

True when $\frac{df}{dy} - \frac{d}{dx}\left(\frac{df}{dy'}\right) = 0$

Correction for non-standard weight

$P_s = V \frac{T-D}{W}$ solve for $T = \frac{P_s W}{V} + D$ deriv wrt W $\frac{d}{dW}(\cancel{T})^{\nearrow 0} = \frac{d}{dW}\left(\frac{P_s W}{V} + D\right)$

$$0 = \frac{dP_s}{dW}\frac{W}{V} + \frac{P_s}{V} + \frac{dD}{dW}$$

solve for $\frac{dP_s}{dW} = -\frac{V}{W}\frac{P_s}{V} - \frac{V}{W}\frac{dD}{dW} = -\frac{P_s}{W} - \frac{V}{W}\frac{dD}{dW}$

multiply by ΔW (std - test)

$$\Delta P_s = \underbrace{-\frac{P_s}{W}\Delta W}_{\text{inertia}} - \underbrace{\frac{V}{W}\Delta D}_{\text{induced drag}}$$

(and since $\Delta D = \cancel{\Delta D_{profile}}^{\approx 0} + \Delta D_{induced}$)

From Aerodynamics, we can estimate drag (induced) as $D = \frac{2n^2W^2\cos^2\theta}{b^2\pi e M p_o \delta}$

$$\Delta D = \frac{2\cos^2\theta}{b^2\pi e M^2 p_o}\left[\left(\frac{(nW)^2}{\delta}\right)_{std} - \left(\frac{(nW)^2}{\delta}\right)_{test}\right]$$

ΔP_s:

$$\Delta P_s = -\frac{P_s}{W}\Delta W - \frac{V}{W}\frac{2\cos^2\theta}{b^2\pi e M^2 p_o}\left[\frac{(nW)^2}{\delta}\bigg|_{std} - \frac{(nW)^2}{\delta}\bigg|_{test}\right]$$

For a level acceleration

$$\boxed{\Delta P_s = P_{s_{std}} - P_{s_{test}} = -\frac{P_{s_{test}}}{W_{test}}\left(W_{std} - W_{test}\right) - \frac{2V_a}{W_{test}\, b^2\pi e M^2 p_o}\left[\frac{(nW)^2}{\delta}\bigg|_{std} - \frac{(nW)^2}{\delta}\bigg|_{test}\right]}$$

Example: An A5M is at 200 kts at $h_p =$ 10000 ft. If the standard weight is 3700 lbf and the test is at 3500 lbf, determine the corrected specific excess power. The wing span is 36 ft. The aircraft is accelerating at 2 kt/s.

200 kt @ 10 kft
M 0.31
$\delta =$ 0.688

$$P_{s_{test}} = \cancel{\dot{h}}^{\nearrow 0} + \frac{V}{g}\frac{dV}{dt} = \frac{200\,kt}{} \left|\frac{s^2}{32.174\,ft}\right| \frac{2\,kt}{s} \left|\frac{1.688^2\,ft^2}{kt^2\,s^2}\right| = 35\,\frac{ft}{s}$$

$$\Delta P_s = \frac{-35\,ft}{s}\left|\frac{200\,lbf}{3500\,lbf}\right| - 2\,\frac{200\,kt}{}\left|\frac{1.688\,ft}{kt\,s}\right|\frac{1}{3500\,lbf}\left|\frac{1}{36^2\,ft^2}\right|\frac{1}{\pi}\left|\frac{1}{1}\right|\frac{1}{0.31^2}\left|\frac{ft^2}{2116\,lbf}\right|$$

$$\cdot\left(\frac{3700^2\,lbf^2}{0.688} - \frac{3500^2\,lbf^2}{0.688}\right)$$

elliptical wing!

$= -2.0 - 0.488$

$$P_{s_{dw}} = P_{s_{test}} + \Delta P_s = 35\,\frac{ft}{s} - 2.488\,\frac{ft}{s} = \underline{\underline{33.5\,\frac{ft}{s}}}$$

Altitude Correction

From our atmosphere $dp = -\rho g_0 dh \Rightarrow \Delta h = -\dfrac{\Delta p}{\rho g_0}$

For a standard atmosphere, $\Delta h_i = -\dfrac{\Delta p}{\rho_{std} g_0}$

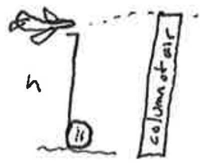
For our "test" atmosphere, $\Delta h_t = -\dfrac{\Delta p}{\rho_t g_0}$

$\Bigg\}$ divide $\dfrac{\Delta h_i}{\Delta h_t} = \dfrac{-\Delta p}{-\rho_{std} g_0}\dfrac{\rho_t g_0}{\Delta p} = \dfrac{\rho_t}{\rho_{std}}$

At the same pressure (using the altimeter!) $\qquad = \dfrac{P_t}{RT_t}\dfrac{R\,T_{std}}{P_{std}}$

$$\underset{\text{"tapeline"}}{\Delta h_i} = \frac{T_{std}}{T_t}\Delta h_t \Rightarrow \frac{dh_i}{dt} = \frac{T_{std}}{T_t}\frac{dh_t}{dt}$$

The tapeline conversion from a test atmosphere to a standard atmosphere is



$$\boxed{\dot h_{std_{tape}} = \frac{\dot h_{test}}{\theta} = \dot h_{test}\frac{T_{std}}{T_t} \\ P_{s_{std}} =}$$

aka $\left(\dfrac{dh}{dt}\right)_2$

This only corrects the measured altitude term!!

Ex:

An AV8r aircraft climbs at 1500 fpm at sea level on a 90°F day, what is the tapeline correction?

$$\dot h = \left(\frac{dh}{dt}\right)_2 = 1500\ \text{fpm}\cdot\underbrace{\frac{518.67^R}{549.67^R}}_{1/\theta} = \underline{1415\ \text{fpm}}$$

Thrust and Drag Correction

Thrust horsepower ≡ THP $\qquad \dot h = \dfrac{THP_{available} - THP_{required}}{W} = \dfrac{TV_\infty - DV_\infty}{W}$

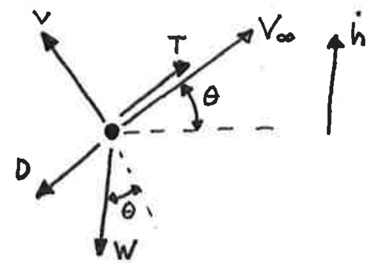
$$\underset{\text{thrust corrected}}{\dot h} = \left(\frac{dh}{dt}\right)_3 = \frac{THP_{A_{std}} - THP_{R_{std}}}{W} \xrightarrow{\text{after some work}} \left(\frac{dh}{dt}\right)_3 = \underbrace{\sqrt{\frac{T_{std}}{T_t}}}_{\sqrt{\frac{1}{\theta}}}\left(\left(\frac{dh}{dt}\right)_2 + \underbrace{\frac{\overbrace{THP_{A_t}}^{TV_\infty}(T_{n_{std}} - T_{n_t})}{W\,\underbrace{T_{n_t}}_{\text{"Test" thrust}}}}_{\text{Hard to measure!!!}}\right)$$

Ex: The same AV8r aircraft above at 2000 lbs and 200 ft/s V_∞ was measured to provide 500 lbs of thrust. It should provide 525 lbs. Compute the thrust corrected ROC.

$$\left(\frac{dh}{dt}\right)_3 = \sqrt{\frac{518.67}{549.67}}\left(1415\ \text{fpm} + \underbrace{\frac{500\ \text{lbf}}{}\Big|\frac{200\ \text{ft}}{s}\Big|\frac{(525-500)\ \text{lbf}}{}\Big|\frac{}{2000\ \text{lbf}}\Big|\frac{}{500\ \text{lbf}}\Big|\frac{60\ s}{\text{min}}}_{150\ \text{fpm}}\right)$$

$$= \underline{\underline{1520\ \text{fpm}}}$$

Accelerated Climb



Along $\perp V_\infty$: $\cancel{ma}^{\,0} = \Sigma F = L - W\cos\theta$

Along V_∞ : $\underbrace{ma}_{\frac{W}{g}\frac{dV}{dt}} = \Sigma F = T - D - W\sin\theta$

$$\underbrace{\frac{V}{g}\frac{dV}{dt}}_{\text{Acceleration power}} = \underbrace{V\frac{T-D}{W}}_{\text{excess power}} - \underbrace{V\sin\theta}_{\dot h} \qquad \frac{dh}{dt} = V\frac{T-D}{W} - \frac{V}{g}\frac{dV}{dt}$$

Also, $\frac{dV}{dt} = \frac{dV}{dh}\frac{dh}{dt}$ so $\frac{V}{g}\frac{dV}{dh}\frac{dh}{dt} = V\frac{T-D}{W} - \cancel{V\sin\theta}^{\,\frac{dh}{dt}}$

Solve for dh/dt :

$$\frac{dh}{dt}\left(1 + \frac{V}{g}\frac{dV}{dh}\right) = V\frac{T-D}{W} \Rightarrow \boxed{\frac{dh}{dt} = \frac{V(T-D)}{W}\cdot\left(\frac{1}{1+\frac{V}{g}\frac{dV}{dh}}\right)}$$

Nonstandard Lapse Rates

$$\left(\frac{dh}{dt}\right)_t = \frac{V_t\left(T_{n_t} - D_t\right)}{W_t}\left(\frac{1}{1+\frac{V_t}{g}\frac{dV_t}{dh_t}}\right) \text{ and } \left(\frac{dh}{dt}\right)_{std} = \frac{V_{std}\left(T_{n_{std}} - D_{std}\right)}{W_{std}}\left(\frac{1}{1+\frac{V_{std}}{g}\frac{dV_{std}}{dh_{std}}}\right)$$

Ratio of

$$\frac{\left(\frac{dh}{dt}\right)_{std}}{\left(\frac{dh}{dt}\right)_t} = \text{1st order term} + \text{2nd order terms} + \cdots \approx 1 + \frac{V_t}{g}\frac{\Delta V_t}{\Delta h_t} - \frac{V_{std}}{g}\frac{\Delta V_{std}}{\Delta h_{std}} + \text{higher order terms}$$

$$\left(\frac{dh}{dt}\right)_4 = \left(\frac{dh}{dt}\right)_3\left(1 - \frac{V_{std}}{g\,\Delta h}\left(\Delta V_{std} - \Delta V_t\right)\right)$$

with $V = Ma = M\sqrt{\gamma R T}$

$\Delta V = M\underbrace{\sqrt{\gamma R}}_{\sqrt{2403.1\,\frac{ft\,lbf}{R\,slug}} = 49.02}\left(\sqrt{T_2} - \sqrt{T_1}\right)$

Ex: The AV8r aircraft above shows a temperature of 30°F at 10000 ft and 28 at 11000 ft.

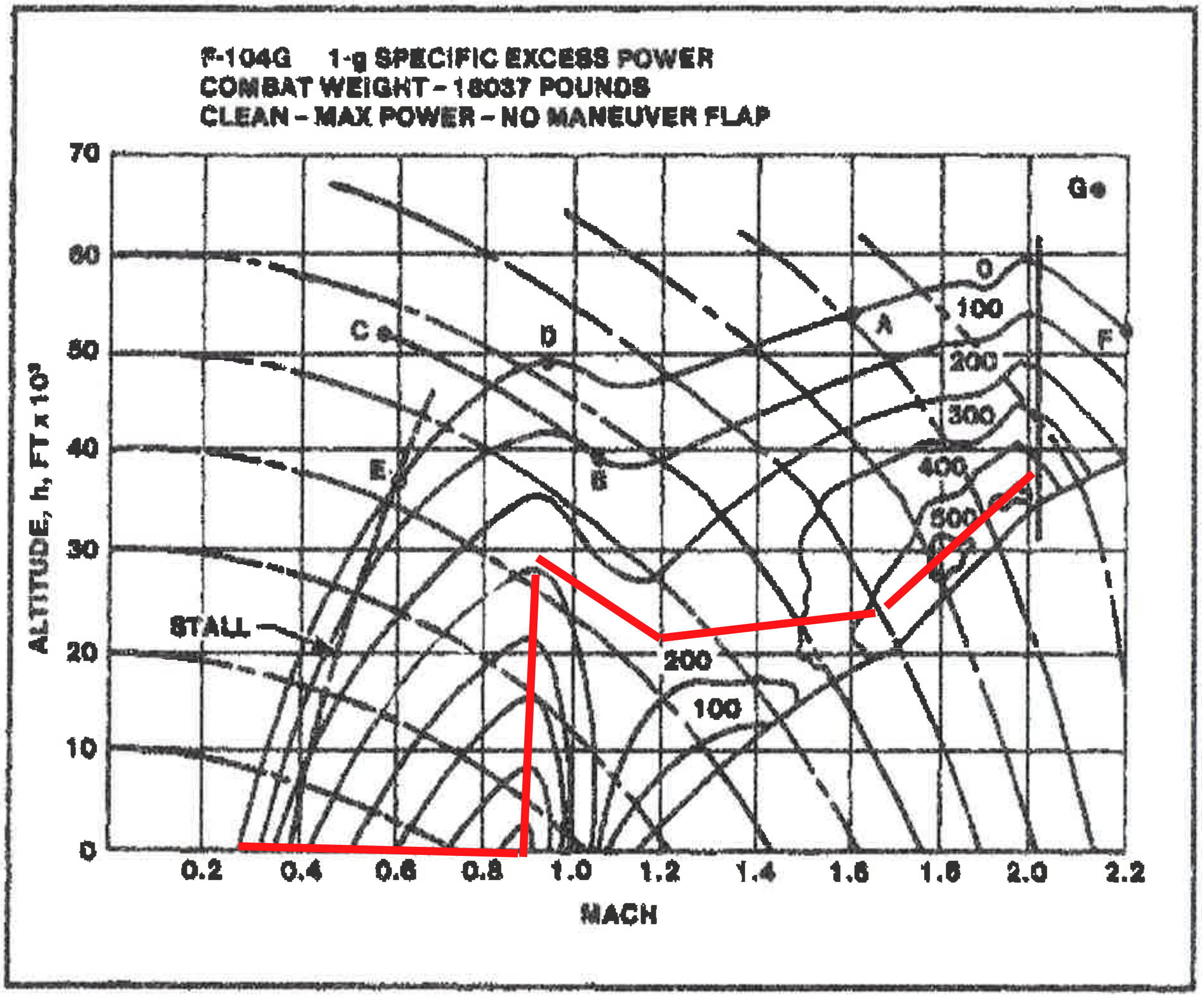
Mach $= \frac{200\text{ fps}}{1077\text{ fps}} = 0.185$

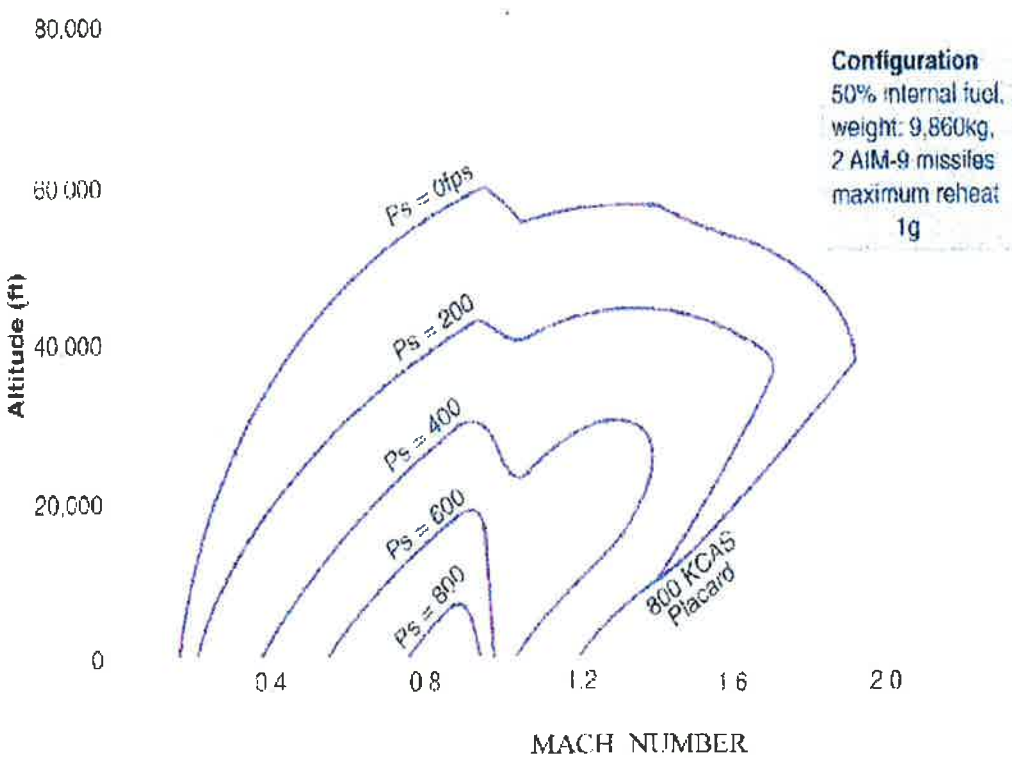
$\Delta V_t = 0.185\cdot 49.02\cdot\left(\sqrt{28+459} - \sqrt{30+459}\right) = -0.412\ ft/s$

$\Delta V_{std} = 0.185\cdot 49.02\cdot\left(\sqrt{19+459} - \sqrt{23+459}\right) = -0.74\ ft/s$

$$\left(\frac{dh}{dt}\right)_4 = 1520\text{ fpm}\left(1 - \frac{200\ ft}{s}\cdot\frac{s^2}{32.174\ ft}\cdot\frac{(-0.74+0.412)\ ft/s}{1000\ ft}\right) = 1520\cdot 1.002 =$$

$$= \underline{1523\text{ fpm}}$$





F-16C SEP datas, from air international.