



Takeoff Performance

Texas Flying Club

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How this discussion is structured:

- 1) Start with an interesting question or observation.
- 2) Dig into the physics, TTPs, and details.
- 3) Zoom out and give actionable knowledge.

Flight Chops Video at:
<https://tiny.cc/AEM368ScaryTakeoff>

- I am not a CFI. Refer to a CFI and your POH/AFM.
- This discussion may contain simplifications or errors and thus is not appropriate or safe for your aircraft.
- Don't be this guy.



CFI: How does takeoff distance vary with _____?

POH/AFM

TAKEOFF DISTANCE MAXIMUM WEIGHT 2300 LBS

SHORT FIELD

CONDITIONS:

Flaps Up
Full Throttle Prior to Brake Release
Paved, Level, Dry Runway
Zero Wind

NOTES:

1. Short field technique as specified in Section 4.
2. Prior to takeoff from fields above 3000 feet elevation, the mixture should be leaned to give maximum RPM in a full throttle, static runup.
3. Decrease distances 10% for each 9 knots headwind. For operation with tailwinds up to 10 knots, increase distances by 10% for each 2 knots.
4. For operation on a dry, grass runway, increase distances by 15% of the "ground roll" figure.

WEIGHT LBS	TAKEOFF SPEED KIAS		PRESS ALT FT	0°C		10°C		20°C		30°C		40°C	
	LIFT OFF	AT 50 FT		GRND	TOTAL	GRND	TOTAL	GRND	TOTAL	GRND	TOTAL	GRND	TOTAL
				ROLL	TO CLEAR 50 FT OBS	ROLL	TO CLEAR 50 FT OBS	ROLL	TO CLEAR 50 FT OBS	ROLL	TO CLEAR 50 FT OBS	ROLL	TO CLEAR 50 FT OBS
2300	52	59	S.L.	720	1300	775	1390	835	1490	895	1590	960	1700
			1000	790	1420	850	1525	915	1630	980	1745	1050	1865
			2000	865	1555	930	1670	1000	1790	1075	1915	1155	2055
			3000	950	1710	1025	1835	1100	1970	1185	2115	1270	2265
			4000	1045	1880	1125	2025	1210	2175	1300	2335	1400	2510
			5000	1150	2075	1240	2240	1335	2410	1435	2595	1540	2795
			6000	1265	2305	1365	2485	1475	2680	1585	2895	1705	3125
			7000	1400	2565	1510	2770	1630	3000	1755	3245	1890	3515
			8000	1550	2870	1675	3110	1805	3375	1945	3670	2095	3990

Figure 5-4. Takeoff Distance (Sheet 1 of 2)

SECTION 5
PERFORMANCE

CESSNA
MODEL

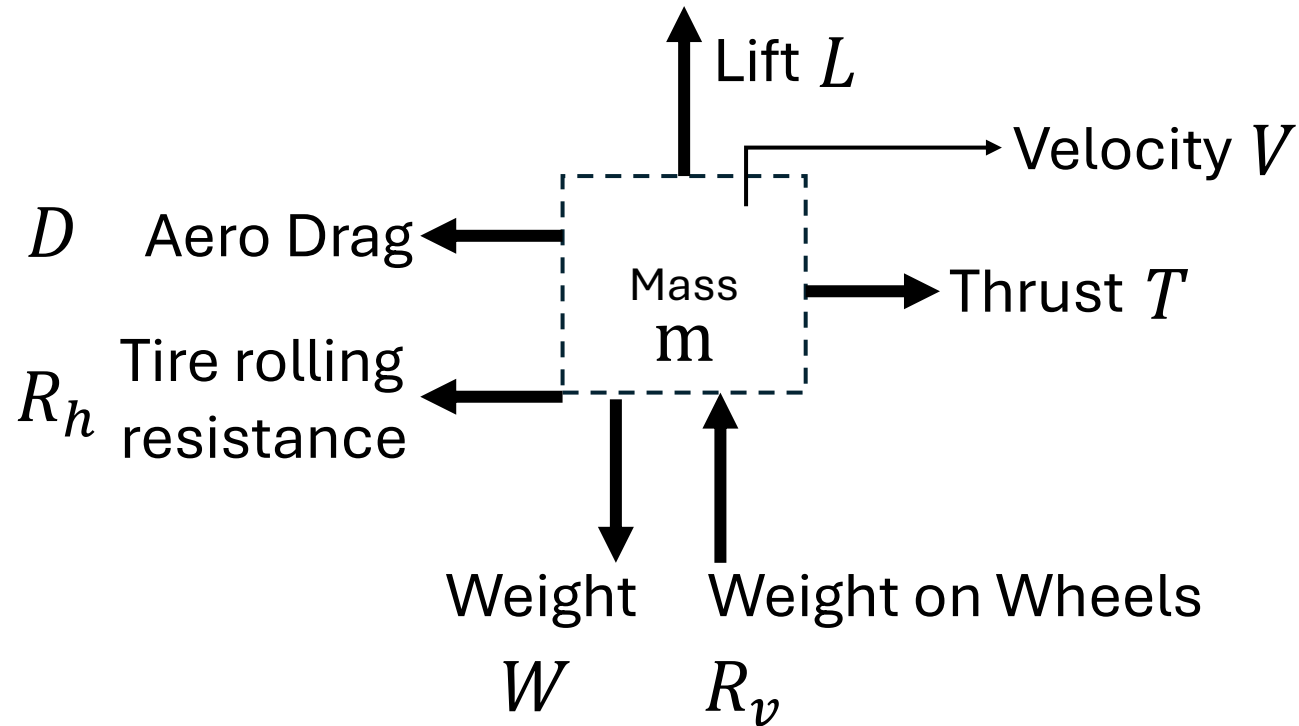
Some important questions:

- How are these generated?
- What are the physics implied in the NOTES?
- What about non-tabulated values? Is interpolation safe?
- How much does technique change the takeoff roll?

Takeoff Ground-Roll Performance Theory



Let's start with a Free Body Diagram.



Sum Forces in Vertical Direction:

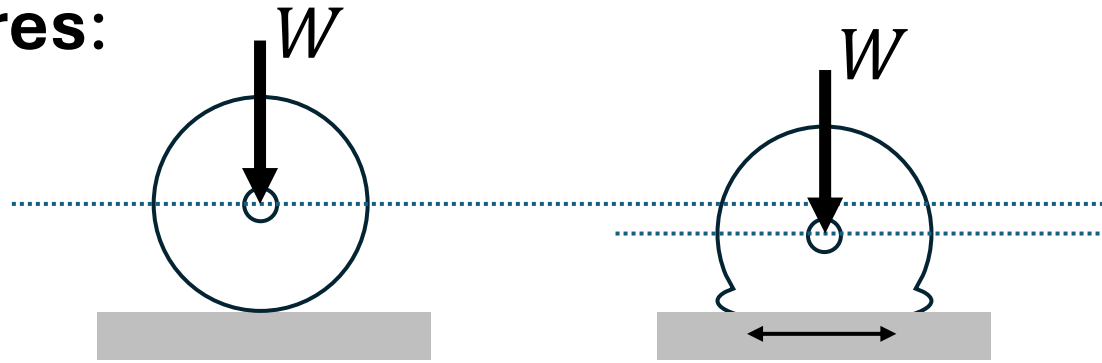
$$L + R_v = W$$

Horizontal Direction:

$$m \frac{dV}{dt} = T - D - R_h$$

All Models are wrong, but some are useful.

Tires:



Zero contact area

$$\frac{W}{A} = \infty !!!$$

$$\frac{W}{A} \approx \textit{Tire Pressure}$$

Rolling resistance

$$R_h = \mu_r (W - L)$$

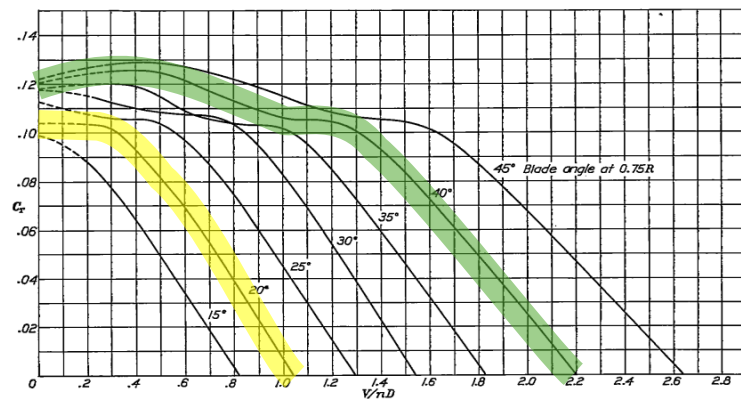
Rolling resistance coefficient μ_r depends on many parameters:

- Tire pressure & Load
- Brakes & Bearings (bad!)
- Ground conditions
 - 0.02 concrete
 - 0.05 packed dirt
 - 0.10-0.30+ soft soil

Thrust:

From our last conversation, the thrust at low speeds is relatively constant.

$$T = C_T \rho n^2 D^4$$



Low Speed J High Speed

Simple Physics Model of the Takeoff Roll

Assumptions for our calculus and algebra elves:

- Level attitude (not a tailwheel ac) $\rightarrow C_L \approx 0$ until rotation
- Aero drag is small. (meh!)

$$a = \frac{dV}{dt} = g \left(\frac{T}{W} - \frac{D}{W} - \mu_r \left(1 - \frac{L}{W} \right) \right) \approx g \frac{T}{W} - g\mu_r$$

The takeoff distance, s , is

$$s_{to} = \frac{1}{\rho} \left(\frac{W}{S} \right) \frac{1}{C_{L_{to}}} \frac{1}{g \left(\frac{T}{W} - \mu_r \right)}$$

Simplify to:

$$s_{to} \cong \frac{1}{\rho^2} \left(\frac{W^2}{S} \right) \frac{1}{C_{L_{to}}} \frac{\rho_{ssl}}{gT_{ssl}}$$

- Inverse of Lift Coefficient (AOA!)
- Inverse of density **squared**! Why? This is about 10% per 1000 feet DA.
- **Square** of mass! Why?
- Rolling resistance directly reduces T/W ratio.
- Inversely with wing area

Q: Does a headwind reduce takeoff roll by the **same** increment as a tailwind increases it?

Herrington's Semi-Empirical Takeoff Estimation

or

How did the Cessna engineers “reduce” limited takeoff data for the POH?

“A low cost method for generating takeoff ground roll charts from flight test data”,
Erb, SFTE, 1996 & Herrington's original AFTR 6273, 1966 work.

Tabulated values for the exponents

Density:

$$\frac{S}{S_0} = \left(\frac{\rho}{\rho_0} \right)^a$$

Weight:

$$\frac{S}{S_0} = \left(\frac{W}{W_0} \right)^b$$

Wind:

$$\frac{S}{S_0} = \left(\frac{GS}{TAS} \right)^c$$

Headwind Exponent	Tailwind Exponent	Weight Exponent	Density Exponent	Source
0.987	1.44	1.37	-2.34	Beechcraft Sierra, Sundowner
1.07	2.46	2.12	-3.73	Piper Tomahawk, Archer II
1.88	---	2.39	-2.40	Cessna T-41C/D
1.85	1.85	2.40	-2.40	Herrington
2.00	2.00	2.00	-2.00	Analytical

from Erb, 1996

Runway Slope (iterative solve!):

$$\frac{S_{slope}}{S_{level}} = 1 + \frac{2gS_{slope} \sin \theta}{V_{to}^2}$$

- Answer: A tailwind is worse.
- Are you surprised that ac are different?!
- Cessna: 10% degradation per 1000 ft DA.
- Pipers: 15-18% degradation per 1000 ft DA. Why?

Examples:

Q1: Your C-172 claims a takeoff distance of 860 ft at 0 C, what is the distance at 40 C?

Ideal gas laws say density scales with the inverse of temperature.

$$\frac{S}{S_o} = \left(\frac{\rho}{\rho_o}\right)^{-2.40} = \left(\frac{0 + 273.15}{40 + 273.15}\right)^{-2.40} = 0.87^{-2.4} = 1.4 \quad S = 1.4S_o = 1.4 \cdot 860 = 1200 \text{ ft}$$

The POH says 1150 ft.

Q2: The club's 172N takeoff NOTES say the following. Verified?

3. Decrease distances 10% for each 9 knots headwind. For operation with tailwinds up to 10 knots, increase distances by 10% for each 2 knots.

$$\frac{S}{S_o} = \left(\frac{59 - 9}{59}\right)^{1.85} = 0.74 \quad \frac{S}{S_o} = \left(\frac{59 + 2}{59}\right)^{1.85} = 1.06$$

Cessna is being conservative. Gusts are a thing.

Data Reduction Example:

Reduce the following flight test data for a $V_r=60$ kt aircraft at 2400 lbs.

Given					
TO dist	temp	slope	Weight	Alt	Wind (kt)
1157	80	0.2°	2450	2000	7
1509	82	0.2°	2440	2000	-2
1181	86	-0.2°	2395	2000	1
1145	87	-0.2°	2390	2000	2

Data Reduction Example:

to SSL
 Reduce the following Data for a $V_{to} = 60^{kt}$ airplane at 2400^{lb} .

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at 2400^{lb} calculated

Slope($\frac{\Sigma}{S_0}$)	Weight	Density	Wind
1.06	1.05	1.266	0.81
1.06	1.04	1.277	1.06
0.94	0.995	1.300	0.969
0.94	0.99	1.306	0.94

Reduced TO dist
 best fit
 1000 ft

product of ratios

- 1.15
- 1.51
- 1.18
- 1.15

Q: Does the proper takeoff configuration and procedure affect takeoff roll distance?

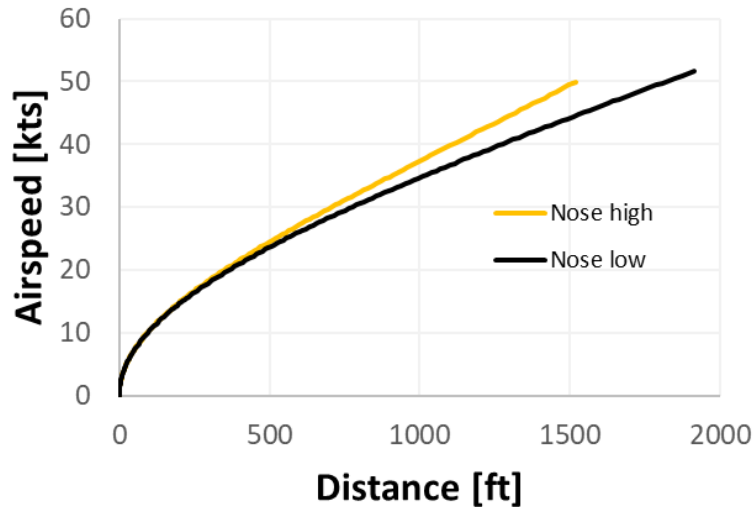
Takeoff Roll (Directly Integrate the ODE)

$$\frac{dV}{dt} = g \left(\frac{T}{W} - \frac{D}{W} - \mu_r \left(1 - \frac{L}{W} \right) \right)$$

Choose your favorite technique for controlling Lift during the TO roll.

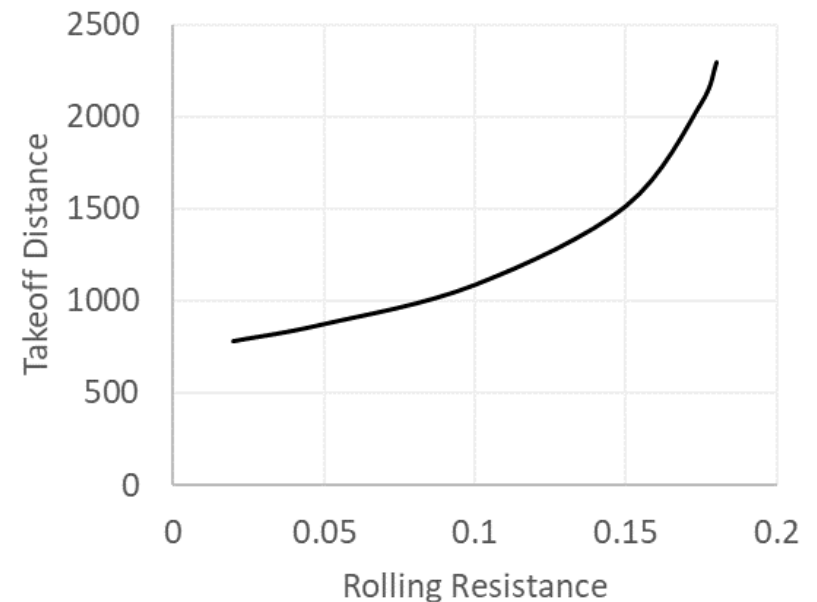
Q: Does keeping the nose high during a soft field TO help?

Yes. For soft dirt ($\mu=0.15$) this reduces the distance by about 25%. The gain is mainly above 25 kts.



Yet, for a normal TO on concrete, a nose high attitude increases the TO distance by at least 10%.

Small changes in surface resistance create huge changes in TO distances. When μ is greater than T/W , you are officially stuck.



Q: Is the 70% airspeed at 50% distance true?

Yes. This seems to be a good rule of thumb.

Q: Could this tool introduce some dangerous yet physically feasible results?

Yes, and so I'll stop here.

Remember that the takeoff is only part of the challenge.

Coulter Runway Challenge

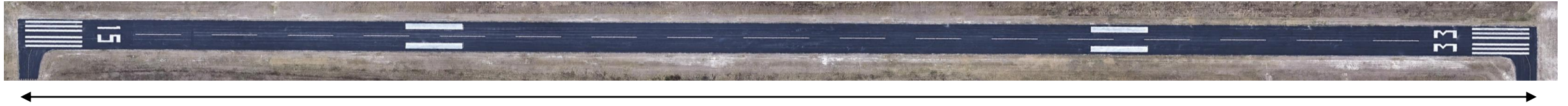


Distances

1. Runway length & Width
2. 1000' Marker & Length
3. Centerline Spacing & Painted Length
4. Numbers
5. Threshold Offset and Length
6. Where is $\frac{1}{2}$? Where is $\frac{1}{2}$ marking?
7. How deep is the drainage ditch?

Coulter Runway Solution (Google Earth)

1. Runway Length = 4000 ft



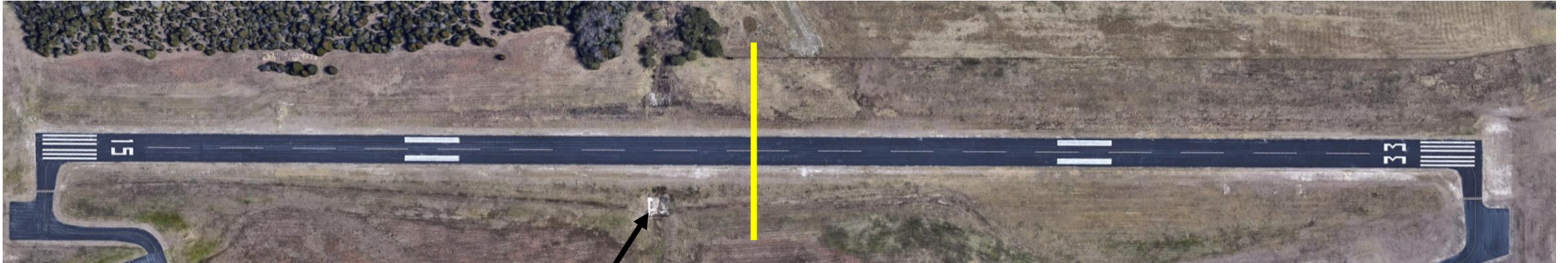
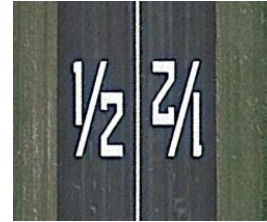
Runway Width = 6 Stripes = 75 ft

2. The 1000' Markers start at 1020' and are 150' in length.



Coulter Runway Solution

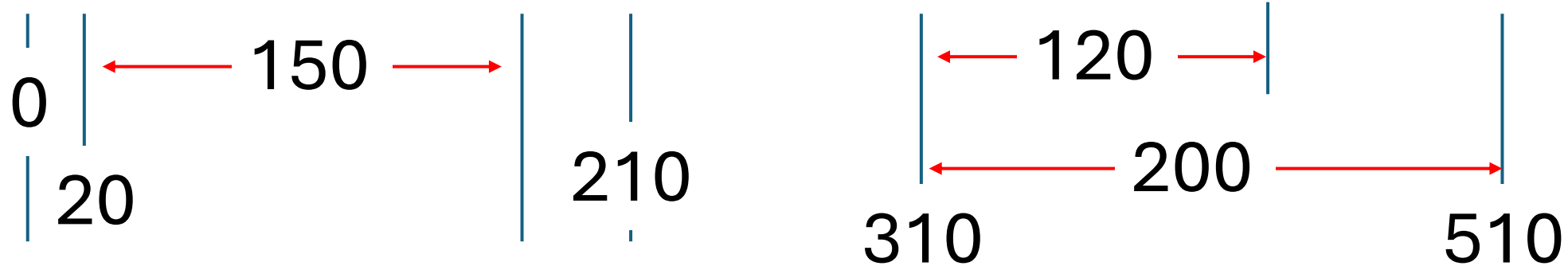
6. The $\frac{1}{2}$ point is near the tree line.
The painted $\frac{1}{2}$ marker is gone.



7. The drainage ditch is 25 feet deep!

Coulter Runway Solution

3. Centerline Spacing 200' & Painted Length 120'
4. Numbers start at 210'
5. Threshold 20' Offset and 150' Length



Q: Quick Coulter Distances?

Number 200, Stripes 300, 500, 700, 900, 1000 foot.

Q: Did this change my (CO's) takeoff procedure?

Yes, previously I briefed "*If not off by 1/2, abort takeoff*".

Now, my limit is **much** shorter.

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