Angles, Distances, Rates, and Rules of Thumb

Texas Flying Club
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13 Jan 2025
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How this discussion is structured:

1) Start with an interesting question or observation.
2) Dig into the physics, TTPs, and details.
3) Zoom out and give actionable knowledge.

Warning! I'm an Aerospace Engineer.

- I am not a CFI/CFII. Refer to a CFI/CFII, FARs, and your POH/AFM.
- This discussion may contain simplifications or errors that are not appropriate or safe for your aircraft.
- In the immortal words of Gary Larson from the Far Side:

"Say … what's a mountain goat doing way up here in a cloud bank?"

N60HF, O'Neill

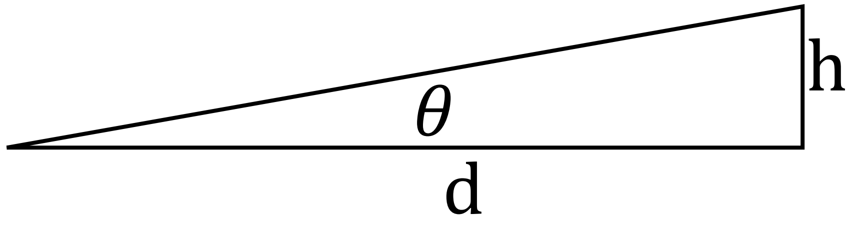
The objective of this discussion is to give you some mental math tools & insight for flying.

ATC: “Cross WAYPT at 2000’.
Q: I’m at 6000’. When should I start descending? At what rate?

Q: At 90 kts and 7.5 miles out with a 30 degree intercept, how fast does the VOR/Localizer needle move? When should I turn?

Q: Does the Earth’s curvature affect angles?

What Angle?



$$\tan\theta = \frac{height}{distance}$$

$$\theta = \tan^{-1}\left(\frac{height}{distance}\right)$$ Simplifies to $$Angle = \frac{Flight\ Level}{Nautical\ Miles}$$

Q: You are 10 miles from the airport and 5000' above. What is the angle to the airport?

$$Angle = \frac{50\ FL}{10\ miles} = 5\ degrees$$

Math: $$\theta = \tan^{-1}\left(\frac{5000}{10 \cdot 6076}\right) \cdot \frac{180}{\pi} = 4.7^{\circ}$$

Why? Freedom Units & Apple Pie (π)

1) A nautical mile is 6076 feet. A flight level is 100 ft.
2) The tangent of θ radians is approximately θ.

$$\theta = \tan^{-1}\left(\frac{h}{distance}\right) \approx \frac{FL \cdot 100}{nm \cdot 6076} = \frac{FL}{nm \cdot 60}$$

3) Conversion from radians to degrees is $\frac{180}{\pi} \approx 60$.

$$Angle_{degrees} \approx \theta \cdot 60 = \frac{FL}{nm \cdot 60} \cdot 60$$

$$Angle = \frac{Flight\ Levels}{Nautical\ Miles}$$

What Rate?

$$\theta = \tan^{-1}\left(\frac{V_h}{V_d}\right)$$ Simplifies to $$\frac{FL}{min} = Angle \cdot nm/min$$

Q: You descend at a 3-degree angle at 90 knots. What is the rate of descent?

60 kts is 1 nm/min, so 90 kts is 1.5 nm/min

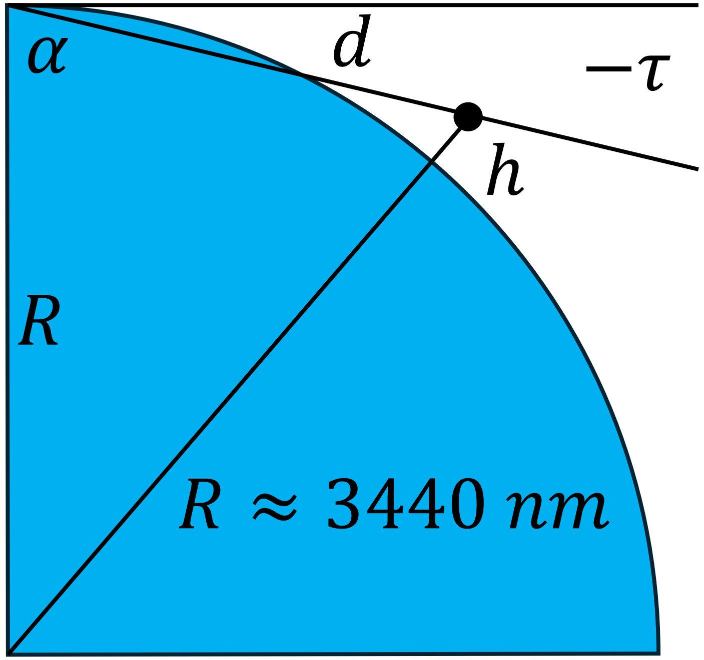
$$3 \cdot 1.5 = 4.5\frac{FL}{min} = 450\frac{ft}{min}$$

The VSI's number is the angle times nm per minute.

This is why a CFII says:
"Small corrections"

Nose down can increase angle **and** velocity!

Curvature of the Earth



Near Exact for Spherical Earth

$$\tau = -\sin^{-1}\left(\frac{d^2 - h^2 - 2Rh}{2Rd}\right)$$

Pilot Approximation

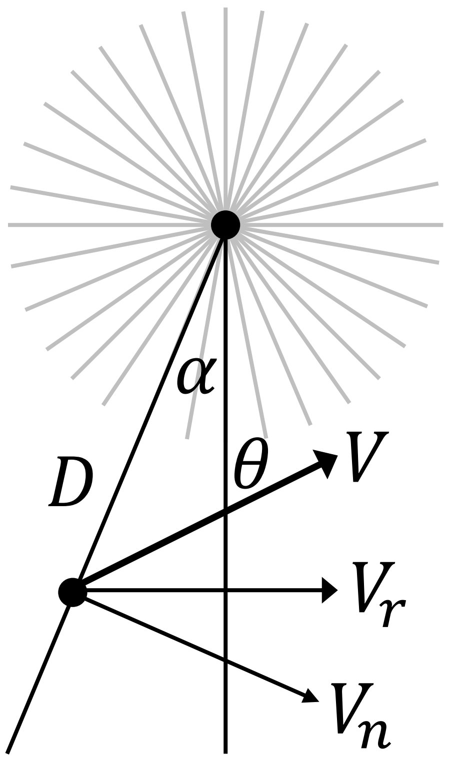
$$Angle \approx \frac{FL}{nm} - \frac{nm}{100}$$

One degree down per hundred nautical miles.

How fast am I flying through radials?

$$V_N = \frac{d\alpha}{dt} D \quad \text{And} \quad V_r = V_N \cos\alpha \quad \frac{d\alpha}{dt} = \frac{V}{D} \cdot \frac{\sin\theta}{\cos\alpha}$$

$$rate = \frac{d\alpha}{dt} \approx \frac{V}{D} \cdot \frac{\sin\theta}{\cos\alpha} \cdot \frac{180}{\pi}$$



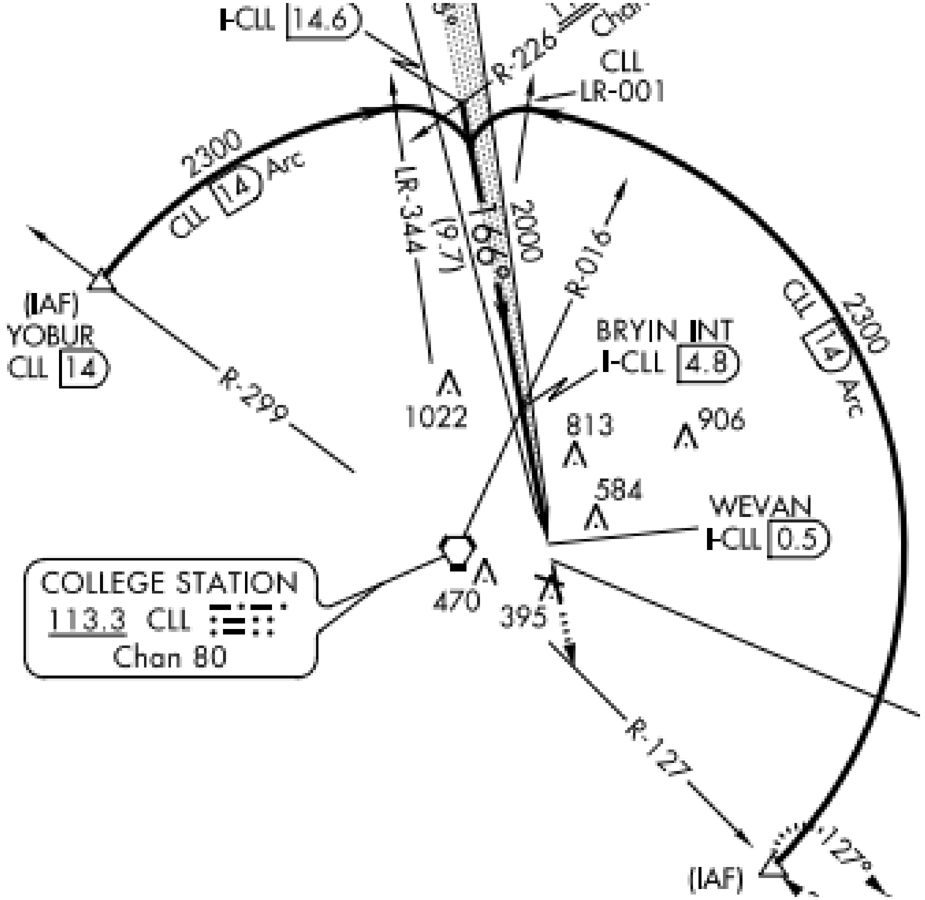
Q: You cross R180 at a heading of 090° at 60 kts and 5 nm. How fast (deg/s) does the radial change?

$$rate = \frac{d\alpha}{dt} \approx \frac{1\,nm/min}{5\,nm} \cdot \frac{\sin 90}{\cos 0} \cdot \frac{180}{\pi} = \frac{60}{5} = 12\,\frac{deg}{min}$$

Pilot units: $rate_{90} = \frac{V_{nm/m}}{NM} [deg/s]$

DME Arc?

On the KCLL LOC BC RWY 17, you are flying the DME arc (14 DME) at 120 kts. Using Turn 10 & Twist 10 for the arc, how many seconds is 10 degrees?



$$rate_{90} = \frac{2\ nm/min}{14\ nm}[deg/s]$$

$$= \frac{1}{7}[deg/s]$$

How long for 10 degrees?

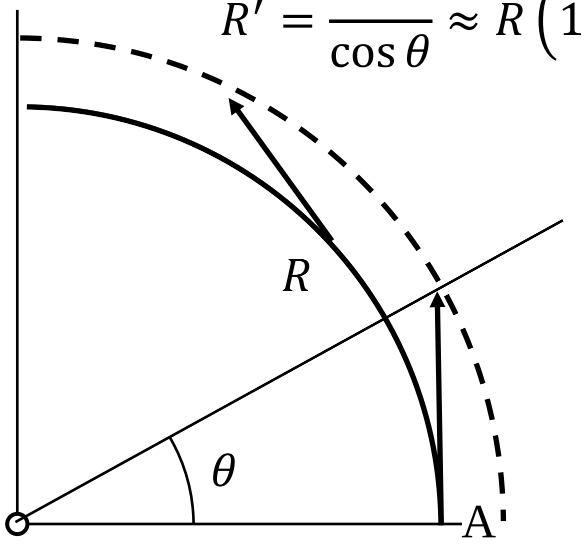
$$= 70\ seconds$$

$$rate_{90} = \frac{V_{nm/m}}{NM}[deg/s]$$

Exact Twist-Turn Flight Path

Naïve Heading Twist-Turn Arc Divergence

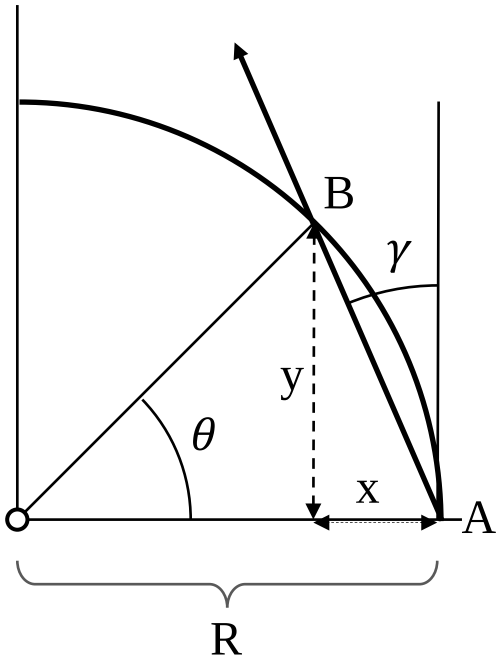
$$R' = \frac{R}{\cos\theta} \approx R\left(1 + \frac{\theta^2}{6000}\right)$$



θ deg	Cross Track
0	0%
5	0.4%
10	1.5%
15	4%
20	6.4%

$$Heading = Radial \pm 90^o$$

Twist-Turn Divergence Correction



$$\tan\gamma = \frac{x}{y}$$

$$x = R - R\cos\theta$$

$$y = R\sin\theta$$

$$\tan\gamma = \frac{R - R\cos\theta}{R\sin\theta}$$

This is a trig identity!

$$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta}$$

Thus γ is $\frac{\theta}{2}$.

Correction is half the twist!

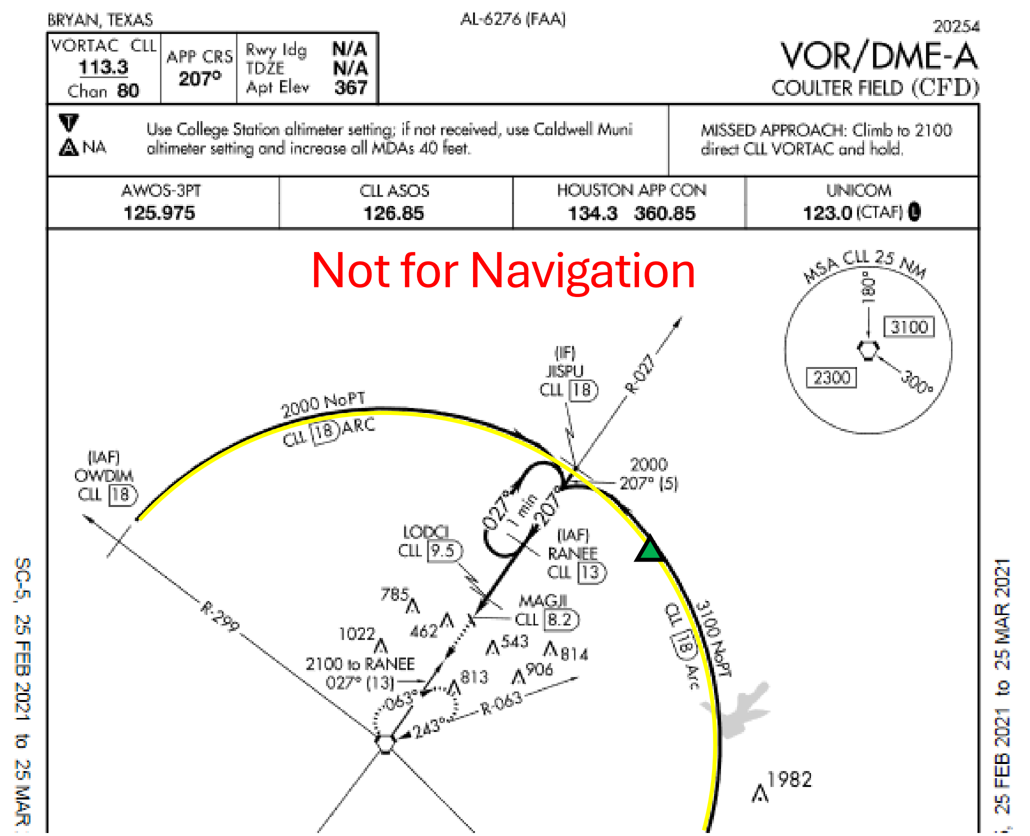
$$Heading = Radial \pm 90^o \pm \frac{1}{2}\theta$$

Turn ½ of the Twist Angle into the Arc

Exact Twist-Turn Flight Path

Pilot Rule of Thumb: Lead by ½ of the Twist

For the a DME arc, let’s fly a turn 10 and twist 10 strategy. On the 040 radial for a left arc, turn an extra 5 degrees left for a perfect arc (no wind).



Turn to here

Twist to 030

R-036

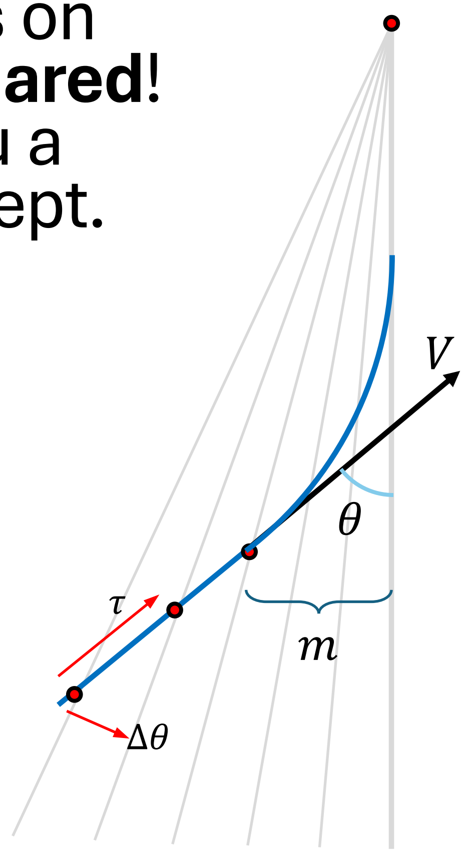
Arrived at R-030

Repeat
-or-
Prepare to Intercept 207 inbound

When to turn for an exact intercept?

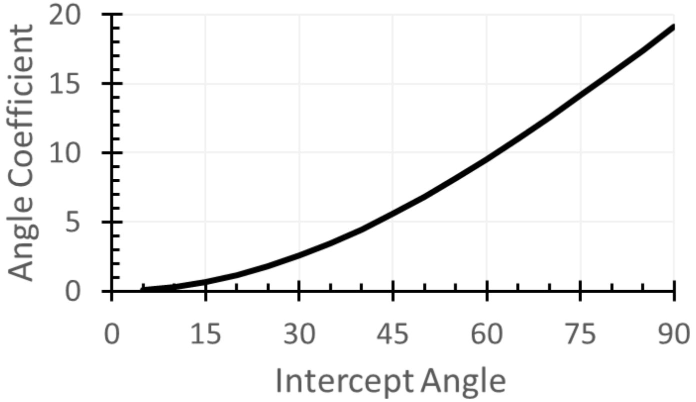
$$m = \frac{V}{D} \cdot \frac{\sin\theta}{\cos\alpha} \cdot \frac{1}{\omega} \cdot \tan\frac{\theta}{2}$$

Note: The angle depends on the intercept angle θ **squared**! This is why ATC gives you a nice 20-30 degree intercept.



Pilot Approximation

$$Angle = \frac{V_{nm/min}}{D} \cdot C$$



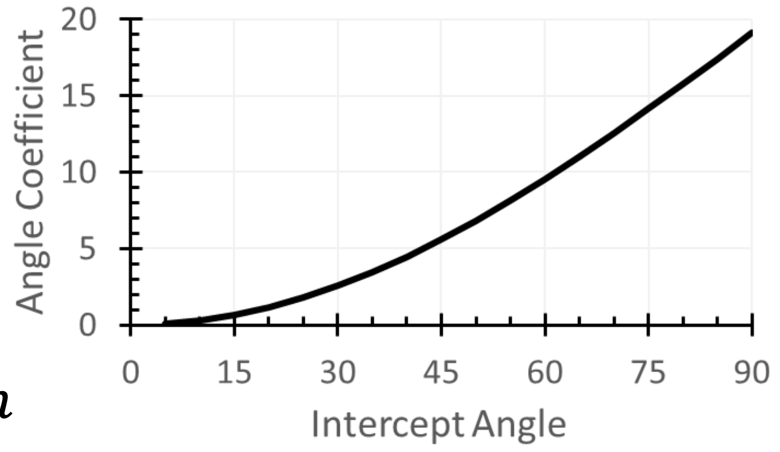
For the previous DME Arc at 90 kts, when should the turn to 207 inbound be started?

90 degrees in $C = 20$

$$Angle = \frac{V}{D} \cdot C = \frac{1.5}{14} \cdot 20 \approx \frac{1}{10} \cdot 20 = 2 \text{ deg}$$

Start turn inbound at R029

Pilot Rule of Thumb: “Lead by GS/200 miles”



Our Exact Intercept Equation

$$Angle = \frac{V_{nm/min}}{D_{nm}} \cdot C$$

$$Lead = Angle_{radians} \cdot D = \frac{V}{D} \cdot \frac{C}{60} \cdot D = \frac{C}{60} \cdot V_{nm/min}$$

Rewrite in term of Kts and C=18 for 90 deg intercept

$$Lead = \frac{18}{60} \cdot \frac{V_{kts}}{60} \approx \frac{1}{3} \cdot \frac{V_{kts}}{60} = \frac{V_{kts}}{180}$$

Confirmed! The pilot rule of thumb is:
Lead a 90 degree turn by GS/200 miles.

Lead vs Intercept:

90 deg → GS/200

60 deg → GS/400

30 deg → GS/1500

10 deg → GS/4500

Angles, Distances, Rates, and Rules of Thumb Takeaways:

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1

$$Angle = \frac{Flight\ Levels}{Nautical\ Miles}$$

2

The VSI's number is the angle times nm per minute.

3

Turn ½ of the Twist Angle into the Arc

4

Lead a 90 deg turn by GS/200 miles.