PART 4: TWO-DIMENSIONAL SEPARATION REVISITED
WITH THE THREE-DIMENSIONAL CONCEPTS
OR A CASE APPARENTLY SIMPLE!

Separation in three-dimensional steady flow
Separation in two-dimensional flow
Classical definition

Skin friction distribution
Generalisation of the notion of two-dimensional separation

Bubble inside an axisymmetric wake: the toroidal vortex is closed on itself
Generalisation of the notion of two-dimensional separation

Structure of a burst vortex. Meridian flow
Separation in two-dimensional flows

There are many experimental evidences of the existence of three-dimensional perturbations in two-dimensional separated flows.

- Ginoux - 1960
- Roshko and Thomke - 1965
- Settles et al. - 1978
- others...

Question: is the word "perturbation" appropriate as meaning a defect relatively to a real configuration?

In reality, the so-called real configuration does not exist, the perturbed case having only a true existence.
Separation in two-dimensional flows

- In the framework of the critical point theory, in a two-dimensional flow a separation line--of detachment or attachment--bears an infinite string of saddle points.

- Such a circumstance is very unlikely in the real world where the three-dimensional character must manifest itself:

  - either a the microscopic scale by existence of sub-structures superimposed on an organisation globally two-dimensional,

  - or at the macroscopic scale by an overall organisation having lost any two-dimensional character.
Separation in two-dimensional flow
Reattachment behind an axisymmetric step

Roshko & Thomke
Separation in two-dimensional flow
Reattachment on an axisymmetric flare

Wind tunnel R2Ch. © ONERA
The critical point theory and two-dimensional flow

Examination of the case \( q = 0 \)

\[
q = \left( \frac{\partial \tau_x}{\partial x} \frac{\partial \tau_z}{\partial z} - \frac{\partial \tau_z}{\partial x} \frac{\partial \tau_x}{\partial z} \right)
\]

The eigenvalues are given by:

\[
S^2 + pS = 0 \quad \text{ou} \quad S(S + p) = 0
\]

Roots \( \rightarrow S_1 = 0, S_2 = -p \)

Skin friction line equations:

\[
x(t) = \frac{A_1 \mu_2}{\lambda_1 \mu_2 - \lambda_2 \mu_1} - \frac{A_2 \mu_1}{\lambda_1 \mu_2 - \lambda_2 \mu_1} \exp(pt)
\]

\[
z(t) = -\frac{A_1 \lambda_2}{\lambda_1 \mu_2 - \lambda_2 \mu_1} + \frac{A_2 \lambda_1}{\lambda_1 \mu_2 - \lambda_2 \mu_1} \exp(pt)
\]
Let us write:

\[ B_1 = \frac{A_1}{\lambda_1 \mu_2 - \lambda_2 \mu_1}, \quad B_2 = \frac{A_2}{\lambda_1 \mu_2 - \lambda_2 \mu_1} \]

\[
\begin{align*}
  x(t) &= B_1 \mu_2 - B_2 \mu_1 \exp(pt) \\
  z(t) &= -B_1 \lambda_2 + B_2 \lambda_1 \exp(pt)
\end{align*}
\]

Skin friction lines → straight lines of equation:

\[ \lambda_1 x + \mu_1 z = B_1 \left( \lambda_1 \mu_1 - \lambda_2 \mu_2 \right) \]

with slope:

\[ \frac{dz}{dx} = -\frac{\lambda_1}{\mu_1} \]
The critical point theory and two-dimensional flow

If \( \lambda_1 > 0 \) and \( \mu_1 > 0 \) and if \( p > 0 \)

When \( t \) varies from \( +\infty \) to \( -\infty \) the solution curve is run from infinity to the critical point of coordinates:

\[
\begin{align*}
  x(\pm \infty) &= -\infty \quad \rightarrow \quad x(-\infty) = x^* = B_1\mu_2 \\
  z(\pm \infty) &= \pm\infty \quad \rightarrow \quad z(-\infty) = z^* = -B_1\lambda_2
\end{align*}
\]

Points \( x^* \) and \( z^* \) are on the straight line of equation:

\[
\lambda_2 x^* + \mu_2 z^* = 0
\]

This line bears an infinity of critical points obtained by varying \( B_1 \)

When \( t \) varies from \( -\infty \) to \( +\infty \) the skin friction lines are run according to a motion starting from the critical point attachment behaviour
The critical point theory and two-dimensional flow

Case \( p > 0 \)  
Motion away from the critical point (attachment)

\[
\lambda_1 x + \mu_1 z = B_1(\lambda_1 \mu_1 - \lambda_2 \mu_2)
\]

\[
\lambda_2 x^* + \mu_2 z^* = 0
\]
Case $p < 0$  Motion towards the critical point (detachment)

\[ \lambda_1 x + \mu_1 z = B_1 (\lambda_1 \mu_1 - \lambda_2 \mu_2) \]

\[ \lambda_2 x^* + \mu_2 z^* = 0 \]
Topological structure of planar reattachment

Plane of the flow

Skin friction line

Body surface

x

z

X
Topological structure of planar detachment (separation)

- Plane of the flow
- Skin friction line
- Body surface
Topological structure of planar separation

Critical point in the plane of the flow

Attachment

Detachment
Special situations in the \([p,q]\) plane

- \(q = \frac{p^2}{4}\)
- Isotropic node
- Axisymmetric attachment
- Axisymmetric detachment
- Linear string of an infinity of saddle points
- Two-dimensional detachment
- Two-dimensional attachment

$p=0$ and $q=0$
Special situations in the \([p,q]\) plane

\[ q = \frac{p^2}{4} \]

Side of the nodes

Side of the saddle points
Special situations in the \([p,q]\) plane

Tendency to axis \( q = 0 \) or when nodes and saddle points meet

Attachment in a axisymmetric flow
Separation in two-dimensional flow

Detachment  Reattachment

Infinite string of node-saddle point combinations in correspondence
Separation in two-dimensional flow

Detachment region

Detachment surface

Skin friction line pattern

Detachment line
Separation in two-dimensional flow

Flow topology and contra-rotating vortices
Detachment and reattachment in two-dimensional flow
Detachment and reattachment in two-dimensional flow

Flow organisation in the separation bubble
Configuration rarely observed. The spanwise limitation of the geometry imposes a three-dimensional large scale structure.

The axisymmetry allows a periodic organisation of this type.
Topology of flows in planar two-dimensional geometries

Shock wave reflection at Mach 2
Reflection of an oblique planar shock wave

Surface flow visualisation

J. Green, 1969
Reflection of an oblique planar shock wave

Skin friction line pattern showing side effects
Topology of flows in planar two-dimensional geometries

Transonic channel in the Onera wind tunnel S8Ch
Detachment and attachment in a transonic channel

Surface flow visualisation
Detachment and attachment in a transonic channel

Skin friction line pattern topology
Flow past a two-dimensional profile in a transonic wind tunnel
Two-dimensional profile in a transonic wind tunnel

Skin friction line pattern on the suction side

Side effects

Side effects
Two-dimensional profile in a transonic wind tunnel

Skin friction line pattern on the pressure side

Attachment node

Detachment node
Wind tunnel side wall

Leading edge

Skin friction line pattern at the profile-side wall junction
Two-dimensional profile in a transonic wind tunnel

Skin friction line pattern at the profile-side wall junction

Tunnel side wall

Trailing edge
Two-dimensional profile in a transonic wind tunnel

Detachment surface $\Sigma_1$
Two-dimensional profile in a transonic wind tunnel

Detachment surface $\Sigma_2$

Trailing edge
Two-dimensional profile in a transonic wind tunnel

Detachment surface $\Sigma_3$
Two-dimensional profile in a transonic wind tunnel

Detachment surface $\Sigma_3$. Other view

Trailing edge
Two-dimensional profile in a transonic wind tunnel

Detachment surface starting from the truncated trailing edge
Flow in a schematic compressor cascade

© ONERA
Flow in a schematic compressor cascade

Skin friction line pattern on a blade suction side
Flow in a schematic compressor cascade

Navier-Stokes computation

Topological interpretation

Skin friction line pattern on a blade suction side
Flow in a schematic compressor cascade

Reconstruction of the toroidal vortex
Flow in a schematic compressor cascade

Cut of the vortex by a median plane
Flow in a schematic compressor cascade

- Leading edge horseshoe vortex
- Lateral tornado-like vortex
- Toroidal vortex

Flow reconstruction past the blade
Axisymmetric flows

Propelled afterbody in the Onera-Meudon wind tunnel R1Ch
Axisymmetric afterbody at zero incidence

Ideal axisymmetric case (improbable)

*The base sharp shoulder bears an infinite string of saddle points*

Cellular organisation

*The base sharp shoulder bears a finite number of saddle points and nodes*

Two-vortex organisation

Possible skin friction line pattern
Flow in the meridian plane. Ideal axisymmetric flow

Non-propelled axisymmetric afterbody at zero incidence
Non-propelled axisymmetric afterbody at zero incidence

Flow in the meridian plane. Effective nearly axisymmetric flow
Non-propelled axisymmetric afterbody at zero incidence
Flow behind a circular disk

H. Werlé, © Onera
Non-propelled axisymmetric afterbody at zero incidence

Ideal axisymmetric case (improbable)

Cellular organisation

Two-vortex organisation

Skin friction lines and detachment surface at the base
Non-propelled axisymmetric afterbody at zero incidence

Two-vortex system. Field in a vertical downstream plane
Propelled axisymmetric afterbody at zero incidence

H. Werlé. © Onera
Propelled axisymmetric afterbody at zero incidence

Meridian field. Ideal axisymmetric flow
Cellular organisation. Separation surfaces

Propelled axisymmetric afterbody at zero incidence
Propelled axisymmetric afterbody at zero incidence

Cellular organisation. Skin friction line pattern on the base
Unsteady flow

- The critical point theory and the resulting topological concepts can be applied to any vector field.

- If the flow is time dependant, the previous considerations are applied to the field at a given instant.

- Taking the time into consideration allows a still more faithful description of reality, the steady case being as inexistent as the two-dimensional case.

- The resulting analysis avoids the use of an average flow concept which is often acrobatic.
Karman vortex street behind a cylinder
Karman vortex street behind a cylinder

Instantaneous field topology
Application of two-dimensional concepts to three-dimensional flows can be highly misleading. In 3D it is necessary to introduce a new terminology more precise and more accurate.

The critical point theory provides a tool – or grammar – allowing a rational description of three dimensional fields.

The skin friction line patterns are the imprint on the surface of the outer flow. Their close examination and analysis are indispensable.
Modern investigation techniques – multi-hole pressure probes, LDV, PIV – coupled with powerful computing capacity may produce a huge quantity of results.

Construction of a topologically consistent picture of these results is a prerequisite to any attempt to understand the physics or separated three-dimensional flows.

The above remarks also apply to the results produced by computer codes solving the Navier-Stokes equations.