

GES 554

Drumheads

# Hyperbolic Experiments + Videos

Sod Shock tube      tiny.cc / GES554 - Sod

Violin String      tiny.cc / GES554 - violinstring

notice traveling wave w/ strings  
transitions to standing wave w/o string why?

Trampoline waves      tiny.cc / GES554 - trampoline

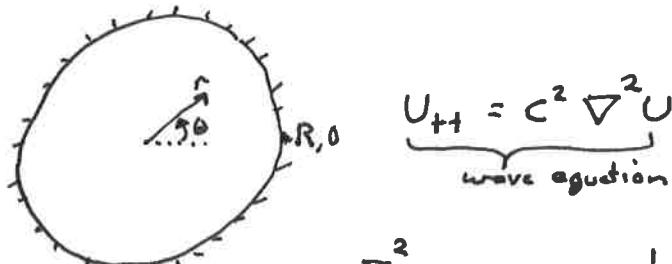
Hydraulic Jump ( $H_2O$ )      tiny.cc / GES554 - hydraulicjump

Circular Membrane      tiny.cc / GES554 - membrane

Drumhead      tiny.cc / GES554 - drumhead

# Drum head (Bessel's Eqn)

Wave equation on polar domain



$$U_{tt} = c^2 \nabla^2 U$$

wave equation

$$\nabla^2 U = U_{rr} + \frac{1}{r} U_r + \frac{1}{r^2} U_{\theta\theta}$$

Initial conditions

$$U = f(r, \theta)$$

$$U_t = g(r, \theta)$$

Boundary Condition

$$U(r=R) = 0$$

Sep of Vars (differs from Farlow slightly. He finds the Helmholtz equation.)

$$U = R(r) \Theta(\theta) T(t)$$

- pull off time term first

$$R(r) \Theta(\theta) T_{tt}(t) = c^2 \left( R_{rr}(r) \Theta(\theta) T(t) + \frac{1}{r} R_r \Theta(\theta) T(t) + \frac{1}{r^2} R \Theta_{\theta\theta} T(t) \right)$$

$$\frac{T_{tt}}{c^2 T} = \frac{R_{rr} \Theta''}{R \Theta''} + \frac{1}{r} \frac{R_r \Theta''}{R \Theta''} + \frac{1}{r^2} \frac{R \Theta_{\theta\theta}''}{R \Theta''} = -\lambda^2$$

$$\boxed{T_{tt} + \lambda^2 c^2 T = 0} \quad \text{Harmonic: sine and cosine of time}$$

- Space terms  $R, \Theta$  (premultiply by  $Rr^2\Theta$ ) (cancel terms)

$$\Theta r^2 R_{rr} + \Theta r R_r + R \Theta_{\theta\theta} = -\lambda^2 R r^2 \Theta$$

Separate  $R$  and  $\Theta$  (divide by  $\Theta R$ )

$$\frac{r^2 R_{rr}}{R} + \frac{r R_r}{R} + \frac{\Theta_{\theta\theta}}{\Theta} = -\lambda^2 r^2$$

$$\frac{r^2 R_{rr}}{R} + \frac{r R_r}{R} + \lambda^2 r^2 = -\frac{\Theta_{\theta\theta}}{\Theta} = +n^2$$

$\Theta$  term:

$$\Theta_{\theta\theta} + \Theta n^2 = 0$$

Harmonic in  $\Theta$ : sine and cosine of  $\Theta$

R.term:

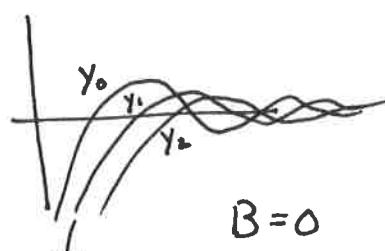
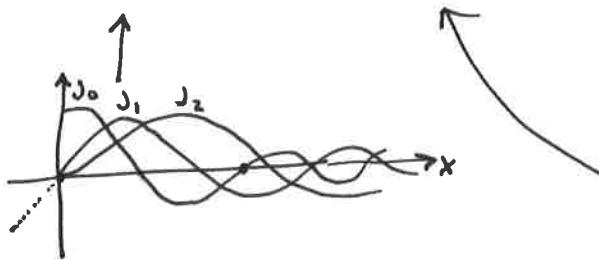
$$r^2 R_{rr} + r R_r + \lambda^2 r^2 R - R n^2 = 0$$

$$r^2 R_{rr} + r R_r + (\lambda^2 r^2 - n^2) R = 0$$

Bessel Function Equation

$$R(r) = A J_n(\lambda r) + B Y_n(\lambda r)$$

solutions to Bessel's eqn.



- As the  $\Theta$  mode increases, we move through Bessel functions of 1st kind.

$$n=0 \Rightarrow 1^{\text{st}} \text{ kind } J_0$$

$$n=1 \Rightarrow 1^{\text{st}} \text{ kind } J_1$$

$$\vdots \quad \vdots \quad \vdots$$

$$B=0$$

Useless for our drum.

Annular Drum ?!?

Bessel functions of 2nd kind.

- The radial mode is determined by roots of  $J_n(\lambda r)$   
Roots are approximately spaced by 3. ~~approximately~~

$$\text{Frequency is } \sqrt{\lambda^2 c^2} = 2c$$

Not an integer multiple of fundamental freq.

"Drums sound different from strings"

Plot

Low to High Freq (Farlow 236)

Theory

2.40

3.83

5.13

5.52

Video

82.2

158.0

217.0

227.0

mode  
 $n, m$

0, 1

1, 1

2, 1

2, 0



$$U_{tt} = c^2 \nabla^2 U$$

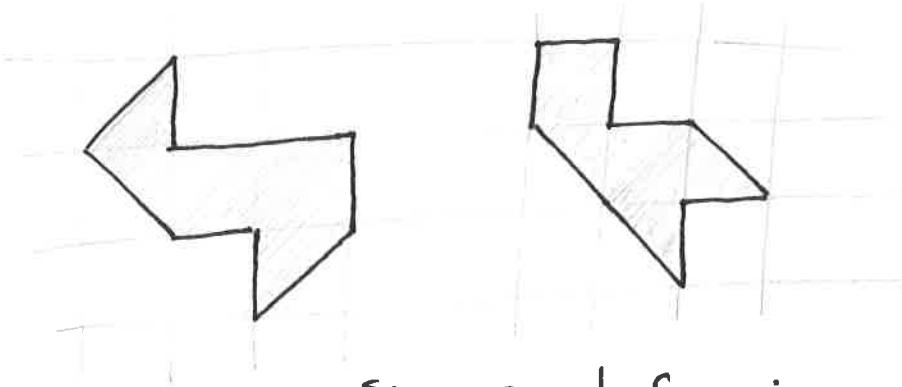
$$U(R) = 0$$

Can you hear the shape of a drum?

translation: Is the modal frequency spectrum unique?

No

Posed in 1966. Proved in 2D in 1992.



Same eigenmode frequencies

See wikipedia for more info and references,  
"Hearing the shape of a drum"

Unique spectrum does not mean no information