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COMPLEX VARIABLES AND APPLICATIONS

Sixth Edition

James Ward Brown

*Professor of Mathematics
The University of Michigan—Dearborn*

Ruel V. Churchill

*Late Professor of Mathematics
The University of Michigan*



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APPENDIX
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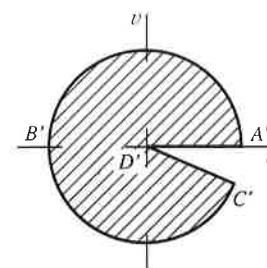
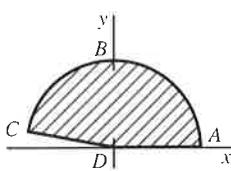
**TABLE OF TRANSFORMATIONS
OF REGIONS**
(See Chap. 8)


FIGURE 1
 $w = z^2$.

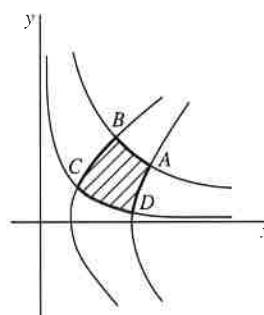


FIGURE 2
 $w = z^2$.

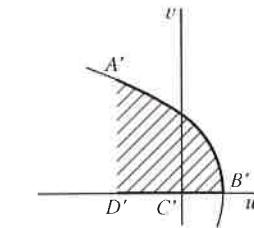
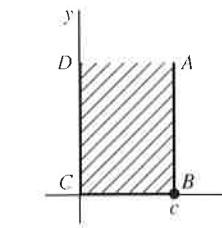
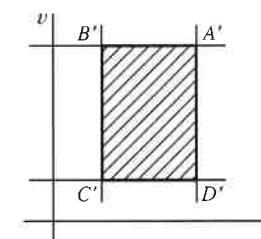


FIGURE 3
 $w = z^2$;
 $A'B'$ on parabola $v^2 = -4c^2(u - c^2)$.

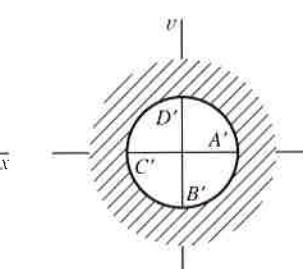
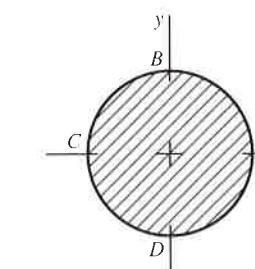


FIGURE 4
 $w = 1/z$.

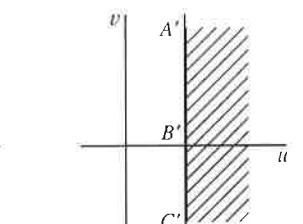
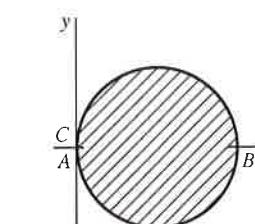


FIGURE 5
 $w = 1/z$.

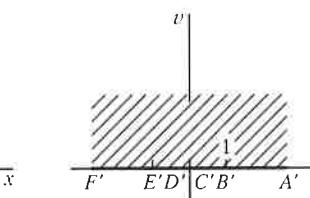
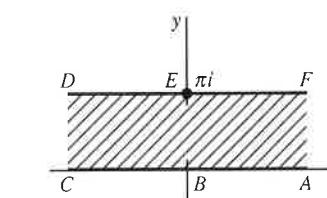
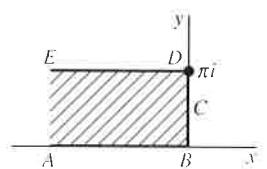
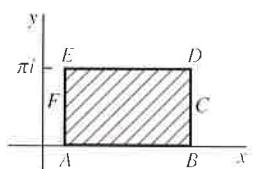
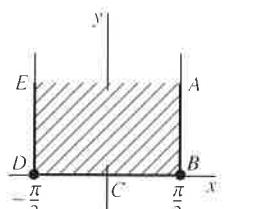
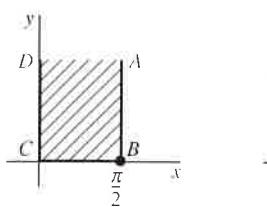
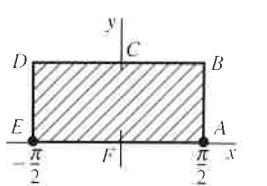
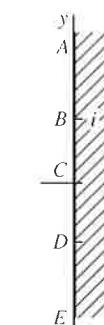
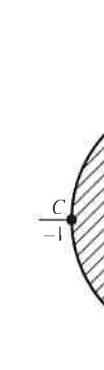


FIGURE 6
 $w = \exp z$.

FIGURE 7
 $w = \exp z.$ FIGURE 8
 $w = \exp z.$ FIGURE 9
 $w = \sin z.$ FIGURE 10
 $w = \sin z.$ FIGURE 11
 $w = \sin z; BCD$ on line $y = b$ ($b > 0$),
 $B'C'D'$ on ellipse $\frac{u^2}{\cosh^2 b} + \frac{v^2}{\sinh^2 b} = 1,$ FIGURE 12
 $w = \frac{z-1}{z+1}.$ FIGURE 13
 $w = \frac{i-z}{i+z}.$ FIGURE 14
 $w = \frac{z-a}{az-1}; a = \frac{1+x_1x_2+\sqrt{(1-x_1^2)(1-x_2^2)}}{x_1+x_2},$
 $R_0 = \frac{1-x_1x_2+\sqrt{(1-x_1^2)(1-x_2^2)}}{x_1-x_2} \quad (a > 1 \text{ and } R_0 > 1 \text{ when } -1 < x_2 < x_1 < 1).$

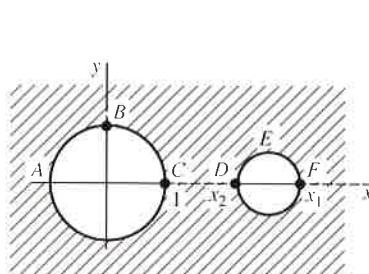


FIGURE 15

$$w = \frac{z-a}{az-1}; a = \frac{1+x_1x_2 + \sqrt{(x_1^2-1)(x_2^2-1)}}{x_1+x_2},$$

$$R_0 = \frac{x_1x_2 - 1 - \sqrt{(x_1^2-1)(x_2^2-1)}}{x_1-x_2} \quad (x_2 < a < x_1 \text{ and } 0 < R_0 < 1 \text{ when } 1 < x_2 < x_1).$$

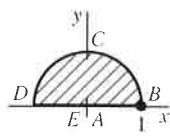
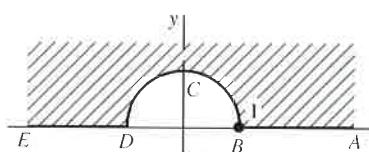
FIGURE 16
 $w = z + \frac{1}{z}$.

FIGURE 17

$$w = z + \frac{1}{z}.$$

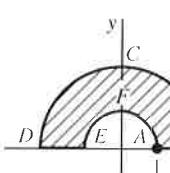


FIGURE 18

$$w = z + \frac{1}{z}; B'C'D' \text{ on ellipse } \frac{u^2}{(b+1/b)^2} + \frac{v^2}{(b-1/b)^2} = 1,$$

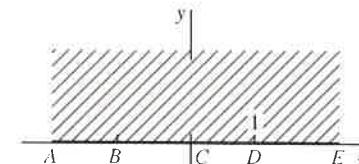
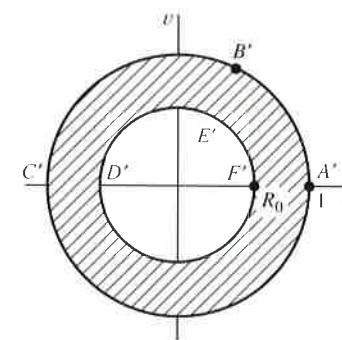


FIGURE 19

$$w = \operatorname{Log} \frac{z-1}{z+1}; z = -\coth \frac{w}{2},$$

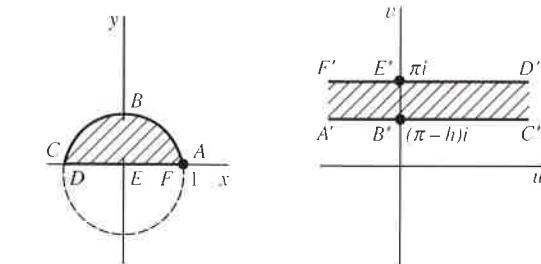
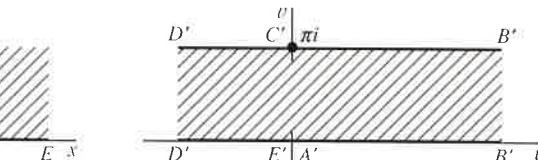


FIGURE 20

$$w = \operatorname{Log} \frac{z-1}{z+1};$$

ABC on circle $x^2 + (y + \cot h)^2 = \csc^2 h$ ($0 < h < \pi$).

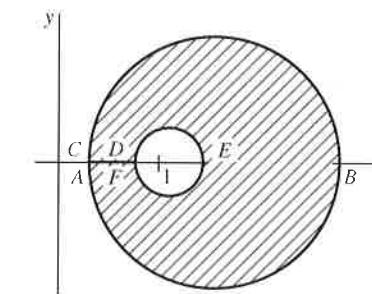


FIGURE 21

$$w = \operatorname{Log} \frac{z+1}{z-1}; \text{ centers of circles at } z = \coth c_n, \text{ radii: } \operatorname{csch} c_n \quad (n = 1, 2).$$

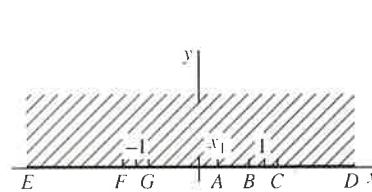


FIGURE 22

$$w = h \ln \frac{h}{1-h} + \ln 2(1-h) + i\pi - h \operatorname{Log}(z+1) - (1-h) \operatorname{Log}(z-1);$$

$$x_1 = 2h - 1.$$

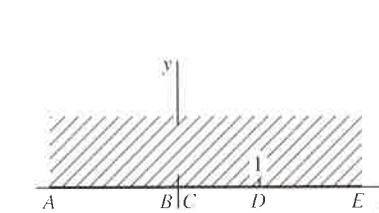
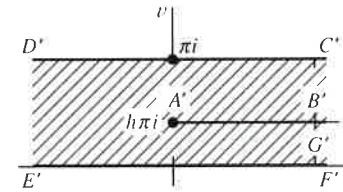
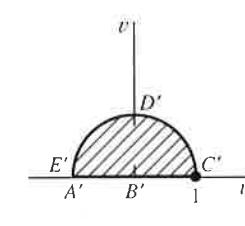
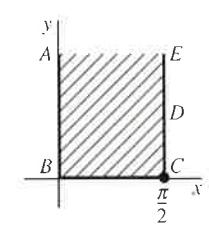
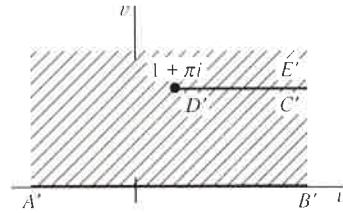
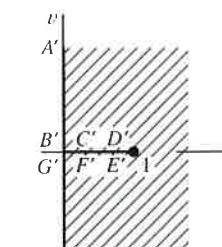
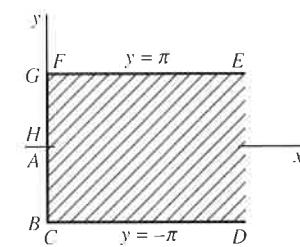


FIGURE 26

$$w = \pi i + z - \operatorname{Log} z.$$



$$w = \left(\tan \frac{z}{2}\right)^2 = \frac{1 - \cos z}{1 + \cos z}.$$



$$w = \coth \frac{z}{2} = \frac{e^z + 1}{e^z - 1}.$$

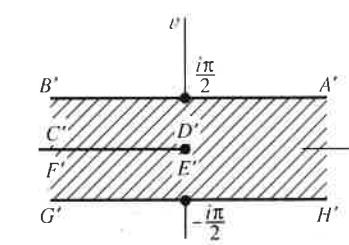
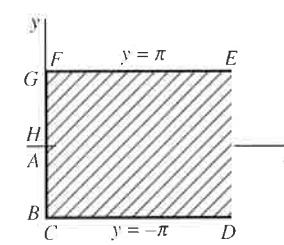


FIGURE 25

$$w = \operatorname{Log} \left(\coth \frac{z}{2} \right).$$

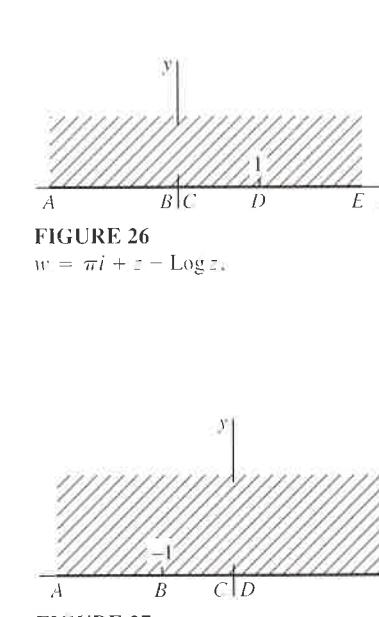


FIGURE 27

$$w = 2(z+1)^{1/2} + \operatorname{Log} \frac{(z+1)^{1/2} - 1}{(z+1)^{1/2} + 1}.$$

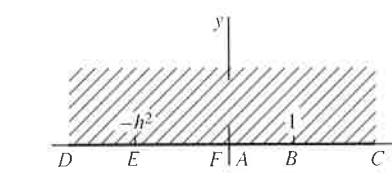
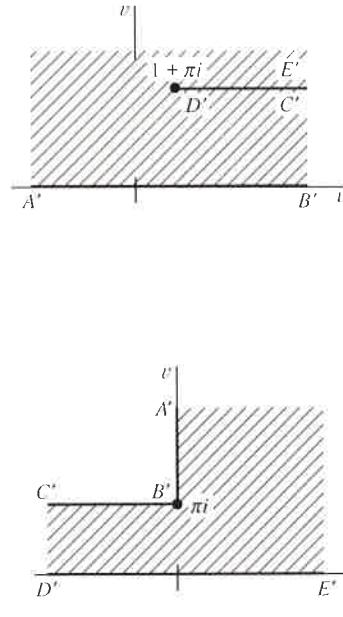
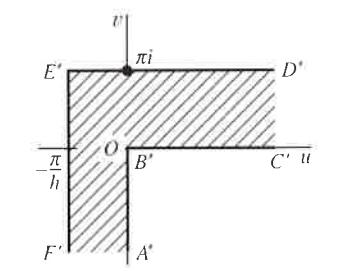


FIGURE 28

$$w = \frac{i}{h} \operatorname{Log} \frac{1 + iht}{1 - iht} + \operatorname{Log} \frac{1+t}{1-t}; t = \left(\frac{z-1}{z+h^2} \right)^{1/2}.$$



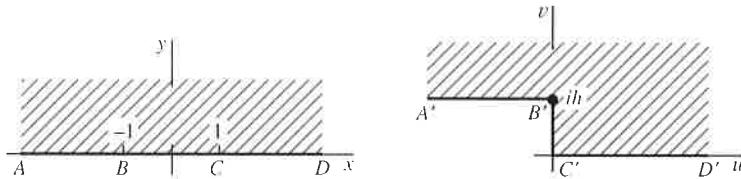


FIGURE 29

$$w = \frac{h}{\pi}[(z^2 - 1)^{1/2} + \cosh^{-1} z].^*$$

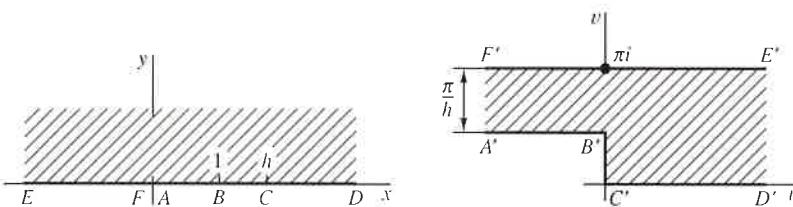


FIGURE 30

$$w = \cosh^{-1}\left(\frac{2z-h-1}{h-1}\right) - \frac{1}{h}\cosh^{-1}\left[\frac{(h+1)z-2h}{(h-1)z}\right].$$

* See Exercise 3, Sec. 99.

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