Name Dr. O'Neill GES 554 HW 6 Quiz 14th Mar 2014

1) What is the cut-on acoustic frequency **in Hertz** for a 2d channel of height 1 foot? The speed of sound is 1100 feet per second. [50 pts]

"cut-on" at $K = \frac{\omega^2}{a^2} - \frac{n^2\pi^2}{d^2} = 0 \Rightarrow \omega = \frac{a}{d} n\pi \Rightarrow f = \frac{\omega}{2\pi} = \frac{a}{d}\frac{n\pi}{2\pi} = \frac{a}{d}\frac{n}{2} \Rightarrow f = \frac{1100\,ft}{2\,n\,s^{-1}\,1\,ft} =$ $\boxed{f = 550n}$

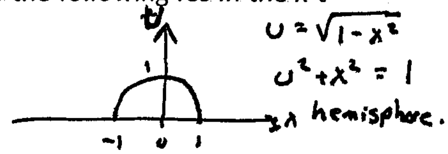
2) For the following wave equation, **find and then sketch** the solution to the following ICs in the x-t plane. [50 pts] Hit a long rope with a base ball bat.

Wave Speed $c = 1$

$$u_{tt} = u_{xx} \qquad -\infty < x < \infty$$

$$u(x,0) = \begin{cases} +\sqrt{(1-x^2)} & -1 < x < 1 \\ 0 & otherwise \end{cases}$$

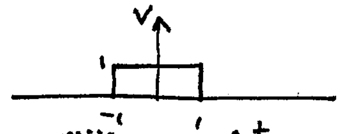
$$u_t(x,0) = \begin{cases} 1 & -1 < x < 1 \\ 0 & otherwise \end{cases}$$

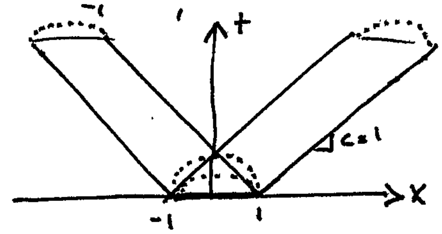


$u = \sqrt{1-x^2}$

$u^2 + x^2 = 1$

hemisphere.





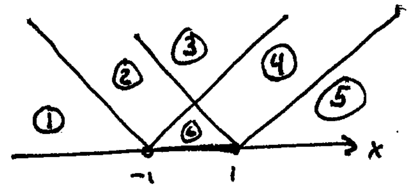
Break into displacement and velocity ICs.

- displacement. $u(x,t) = \frac{1}{2}\sqrt{1-(x+t)^2} + \frac{1}{2}\sqrt{1-(x-t)^2}$

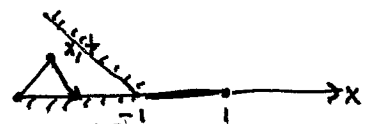
- Velocity IC. $u(x,t) = \frac{1}{2c}\int_{x-ct}^{x+ct} 1\,dx = t$? NO! This is the solution to $u_t(x,0) = 1 \quad -\infty < x < \infty$

- Velocity IC

Draw characteristics first and find regions

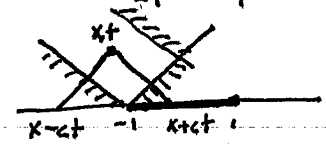


1



$$u(x,t) = \frac{1}{2c}\int_{x-ct}^{x+ct} 0\,dx = 0$$

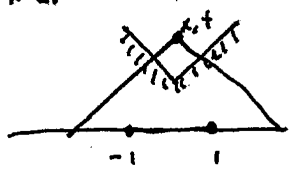
2



part of integral covers portion of IC with $u_t = 1$

$$u(x,t) = \frac{1}{2c}\int_{-1}^{x+ct} 1\,dx + \cancel{\frac{1}{2c}\int_{x-ct}^{-1} 0\,dx} = \frac{x+ct+1}{2c}$$

3



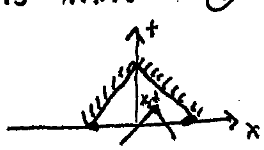
Again, only part of the integral has non-zero value.

$$u(x,t) = \frac{1}{2c}\int_{-1}^{+1} 1\,dx + \int_{x-ct}^{-1} 0 + \int_{1}^{x+ct} 0 = \frac{1}{c}$$

4 is mirrored 2

5 is mirrored 1 $= 0$

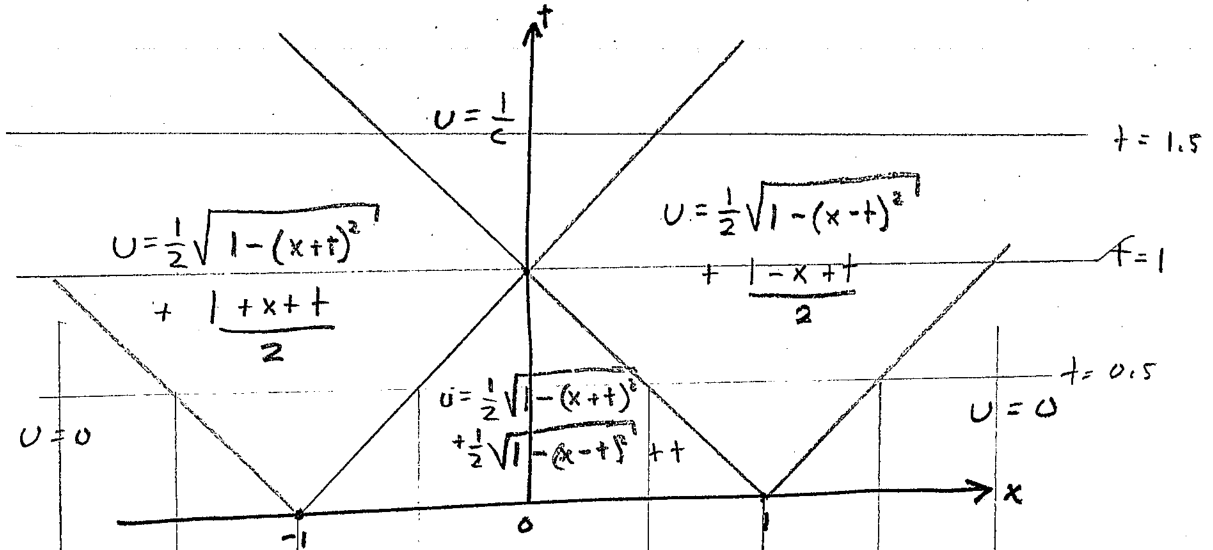
6



$$u(x,t) = \frac{1}{2c}\int_{x-ct}^{x+ct} 1\,dx = \frac{x+ct-x+ct}{2c} = t$$

Sketch:

Superposition Applies. Add solution...



Cross-Section Cuts.