

For the following wave equation, find and sketch the solution to the following ICs in the x-t plane. There are 6 regions of interest.

$$u_{tt} = u_{xx} \quad -\infty < x < \infty$$

$$u(x, 0) = 0$$

$$u_t(x, 0) = \begin{cases} 1 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

D'Alembert's solution is

$$u(x, t) = \frac{1}{2} f(x+ct) + \frac{1}{2} f(x-ct) + \frac{1}{2c} \int_{x-ct}^{x+ct} g(\zeta) d\zeta$$