

- 1) Does a solution exist to the following Laplacian PDE? If the solution exists, find the solution. Otherwise, explain why a solution cannot exist. [50 pts]

$$\begin{aligned}\nabla^2 u &= 0 & 0 \leq r \leq 1 & \quad 0 \leq \theta < 2\pi \\ u_r(1, \theta) &= \sin(1.5\theta)\end{aligned}$$

- 2) Solve the following interior Dirichlet problem.[50 pts]

$$\begin{aligned}\nabla^2 u &= 0 & 0 \leq r \leq 5 & \quad 0 \leq \theta < 2\pi \\ u(5, \theta) &= \sin(2\theta) + \cos(3\theta) + 2\end{aligned}$$