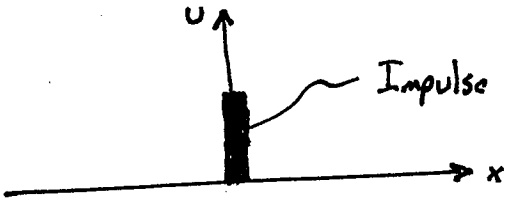
Quiz #6.5 (+50pts to quiz #6)

$$u_{tt} = u_{xx} \qquad -\infty < x < \infty$$

$$u_t(x,0) = \delta(x)$$

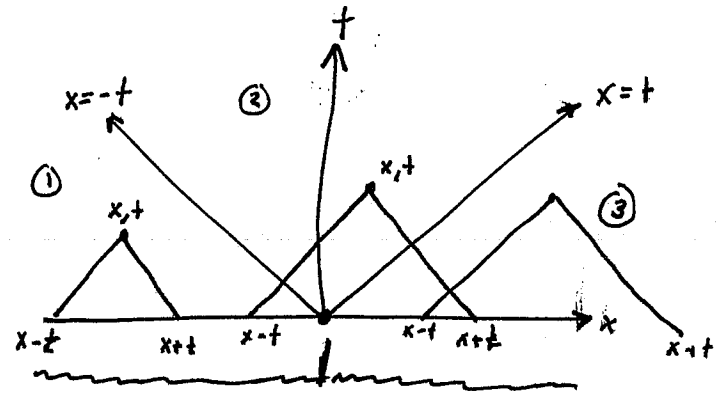



Impulse function $\delta = \begin{cases} 1 & \text{when } x = 0 \\ 0 & \text{otherwise} \end{cases}$

$$\int \delta = H(x) = \begin{cases} 1 & \text{when } x > 0 \\ 0 & \text{otherwise} \end{cases}$$

Find $u(x,t)$

Solution




Region ①:

$$u(x,t) = \frac{1}{2c}\int_{x-t}^{x+t} 0\, d\alpha = 0$$

Region ②:

$$u(x,t) = \frac{1}{2c}\int_{x-t}^{x+t} \delta(x)\, dx = \frac{1}{2c} H(x)\Big|_{x-t}^{x+t} = \frac{1}{2c} H(x+t) - \frac{1}{2c} H(x-t)$$

Region ③:

$$u(x,t) = \frac{1}{2c}\int_{x-t}^{x+t} 0\, d\alpha = 0$$

$$\boxed{u(x,t) = \frac{1}{2c}\left(H(x+t) - H(x-t)\right)} = \boxed{\begin{cases} 0 & x < -t \\ 1/2c & -t < x < t \\ 0 & x > t \end{cases}}$$