

1) Please write down the location, date and time of our final exam? [10 pts]

251 Hardaway, Thursday May 1st, 3:30 - 6:00 pm.

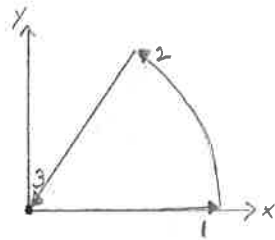
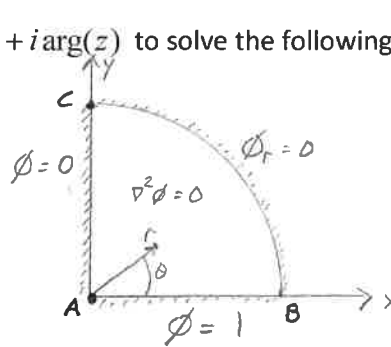
2) Use conformal mapping with $w = \log(z) = \log|z| + i \arg(z)$ to solve the following PDE [90 pts]

$$\nabla^2 \phi = 0 \quad 0 < r < 1 \quad 0 < \theta < \frac{\pi}{2}$$

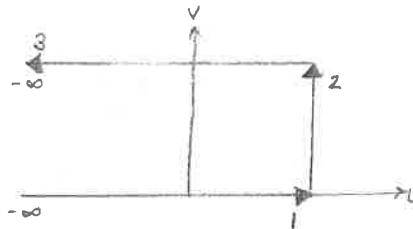
$$\phi(r, 0) = 1$$

$$\phi(r, \pi/2) = 0$$

$$\phi_r(1, \theta) = 0$$



$$w = \log(z) \Rightarrow z = e^w$$

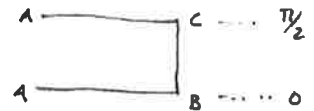


$$w = \log|z| + i \arg z = u + i v$$

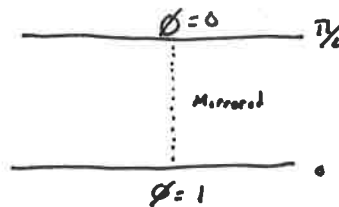
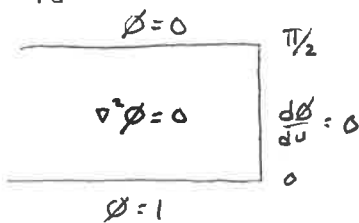
$$pt A = (0, 0)_z \Rightarrow |z|=0 \quad \arg(z)=0 \Rightarrow (0, 0)_w = 0 + i 0$$

$$pt B = (1, 0)_z \Rightarrow |z|=1 \quad \arg(z)=0 \Rightarrow 1 + i 0$$

$$pt C = (0, 1)_z \Rightarrow |z|=1 \quad \arg(z)=\pi/2 \Rightarrow 1 + i \pi/2$$



Mapped to



Solution is

$$\phi = Av + B \quad \text{fit to } \phi(0, \pi/2) = 0 \quad \text{and } \phi(0, 0) = 1$$

$$\phi = -\frac{2}{\pi} v + 1$$

Substitute for $v = \arg(z) = \theta$

$$\phi = -\frac{2}{\pi} \theta + 1$$

$$\boxed{\phi(r, \theta) = -\frac{2\theta}{\pi} + 1}$$

$$\text{Verify: } \nabla^2 \phi = \phi_{rr} + \frac{1}{r} \phi_r + \frac{1}{r^2} \phi_{\theta\theta} = 0$$

$$= 0 + 0 + \frac{1}{r^2} 0 = 0 \quad \checkmark$$

Fits BCs:

$$\phi(r, 0) = 1 \quad \checkmark$$

$$\phi(r, \pi/2) = 0 \quad \checkmark$$

$$\phi_r = 0 \quad \checkmark$$