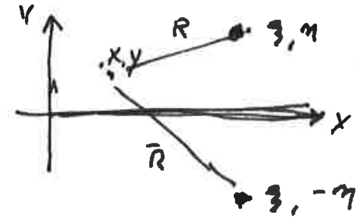
Name $O'Neill$ GES 554 HW 4 Quiz 24th Feb 2014

1) Construct a Green's function $G(x,y,\zeta,\eta)$ given the diffusion equation in the upper half domain (y>0). The potential of a point charge is provided below. **Expand G in terms of Cartesian coordinates.** [50 pts]

$$\nabla^2 u = 0 \qquad y > 0$$

$$\phi(x,y) = \frac{1}{2\pi} ln \frac{1}{R}$$



$$R = \sqrt{(x-\xi)^2 + (y-\eta)^2}$$

$$\bar{R} = \sqrt{(x-\xi)^2 + (y+\eta)^2}$$

$$\phi_{Total} = \frac{1}{2\pi} \ln \frac{1}{R} + \frac{-1}{2\pi} \ln \frac{1}{\bar{R}} = \frac{1}{2\pi} \ln \frac{\bar{R}}{R}$$

$$G(x,y,\eta,\xi) = \frac{1}{2\pi} \ln \left(\frac{\sqrt{(x-\xi)^2 + (y+\eta)^2}}{\sqrt{(x-\xi)^2 + (y-\eta)^2}} \right)$$

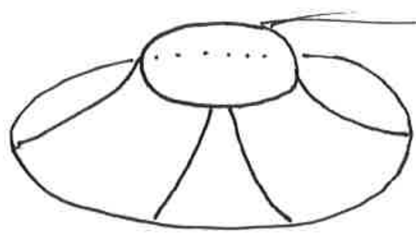
2) A soap bubble is formed between two flat, parallel rings. The inner ring is elevated 1 unit higher than the outer ring. The inner ring has a diameter of e. The outer ring has a radius of e^2. Write down the equation describing the soap bubble's surface. **Reduce to exactly two terms.** [50 pts]

$$\nabla^2 u = 0 \qquad 1 < r < 2$$

$$u(e,\theta) = 1$$

$$u(e^2,\theta) = 0.$$

Hint: $\ln(e) = 1$ and $\ln(e^2) = 2$



$$U(r,\theta) = a_0 + b_0 \ln r$$

$$a_0 + b_0 \cancel{\ln e}^{1} = 1$$

$$a_0 + b_0 \cancel{\ln e^2}^{2} = 0$$

$$\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} a_0 \\ b_0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} a_0 \\ b_0 \end{pmatrix} = \frac{\begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}}{\cancel{2\pi}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \Rightarrow \boxed{U(r,\theta) = 2 - \ln r}$$

$$\int_0^{2\pi} (1) \cos$$