Quiz #7

1) Verify $4U_{xt} + U_{tt} + 5U_{xx} + 2U_{tx} = 0$ is hyperbolic.

Canonical form.

$$AU_{xx} + BU_{xy} + CU_{yy} + DU_x + EU_y + F = G$$

Consider $x = x$ and $t = y$

$$5U_{xx} + (4+2)U_{tx} + 1U_{tt} = 0$$

$A = 5 \qquad B = 6 \qquad C = 1$

Test

$$B^2 - 4AC = 36 - 4 \cdot 5 \cdot 1 = 36 - 20 = 16 > 0$$

Hyperbolic

2) Find and sketch characteristics

$$\frac{\xi_x}{\xi_t} = \frac{-B + \sqrt{B^2 - 4AC}}{2A} = \frac{-6 + \sqrt{16}}{2 \cdot 5} = \frac{-2}{10} = -\frac{1}{5}$$

$$\frac{\eta_x}{\eta_t} = \frac{-B - \sqrt{\cdots}}{2A} = \frac{-6 - 4}{10} = \frac{-10}{10} = -1$$

Along char'

$$d\xi = \frac{d\xi}{dx}dx + \frac{d\xi}{dt}dt \Rightarrow \xi_x\,dx = -\xi_t\,dt \Rightarrow \frac{\xi_x}{\xi_t} = -\frac{dt}{dx}$$

$$d\eta = \cdots\cdots + \cdots\cdots \Rightarrow \frac{\eta_x}{\eta_t} = -\frac{dt}{dx}$$

ξ)

$$\frac{dt}{dx} = -\frac{\xi_x}{\xi_t} = +\frac{1}{5} \Rightarrow t = \frac{1}{5}x + C_1 \Rightarrow \boxed{\xi = t - \frac{1}{5}x}$$

η)

$$\frac{dt}{dx} = -\frac{\eta_x}{\eta_t} = 1 \Rightarrow t = x + C_2 \Rightarrow \boxed{\eta = t - x}$$