GES 554 Partial Differential Equations
Project 2: [100 pts]
Due: 26th Feb 2016 by 5:00 pm.

Local students: Hardcopy
Online students: Email pdf

Mathematics is the part of physics where experiments are cheap. –V. Arnold

The cantilever beam conceptually represents the most fundamental form of aircraft structures: wings, landing gear, fuselage, etc. In this project, we will determine the modeshapes and frequencies associated with the beam bending of a uniform cantilever beam. The PDE describing beam-bending motion is,

$$u_{tt} + \frac{EI}{\mu} u_{xxxx} = 0$$

with the following cantilever boundary conditions

$$u(0,t) = 0 \qquad u_x(0,t) = 0 \qquad u_{xx}(\mathrm{L},t) = 0 \qquad u_{xxx}(\mathrm{L},t) = 0$$

Include the following topics in a memo.

1. Non-dimensionalize the beam length (0<x<1) to obtain a canonical form
$$u_{tt} = \frac{1}{\beta^4} u_{xxxx}$$
2. Show separation of variables and the general X(x) and T(t) equations and solutions.
Hint: the 4th order term suggests using $-\lambda^4$ as the SoV constant.
3. Find an equation describing the vibration frequencies in **Hertz:** $f[Hz] = f(\lambda, L, E, I, \mu)$
4. Find the first 5 eigenvalues
5. Find and plot the first 5 eigenfunctions

- Bonus [20 pts]: Determine the first 5 vibration frequencies for a cantilever beam with the following properties:
    - Aluminum
    - Length 9.264 in
    - Width 1.0 in
    - Thickness 0.040 in