15p2

$$U_t = U_{xx} - 2U_x$$
$$U(x,0) = e^x \sin(x)$$

$U_t = U_{xx}$ diffusion and $\underbrace{U_t = -2U_x}$ convection

Solution $\approx$ $U = e^{k_1(x-ct)}$

pull this out of $u(x,t)$

$$U(x,t) = e^{k_1(x-ct)} w(x,t)$$

Subst. into gov equ.

$$\underbrace{-k_1 c\, e^{k_1(x-ct)} w + e^{k_1(x-ct)} w_t}_{U_t} = \underbrace{k_1^2 e^{k_1(x-ct)} w + 2k_1 e^{k_1(x-ct)} w_x + e^{k_1(x-ct)} w_{xx}}_{U_{xx}} \underbrace{- 2k_1 e^{k_1(x-ct)} w + -2 e^{k_1(x-ct)} w_x}_{-2U_x}$$

pull out $e^{k_1(x-ct)}$ term

$-c = \cancel{1}\ \cancel{-1} \Rightarrow c = 1$

$$-k_1 c w + w_t = k_1^2 w + 2k_1 w_x + w_{xx} - 2k_1 w - 2w_x .$$

0 if $k_1 = 1$

If $c = 1$ and $k_1 = 1$

$$w_t = w_{xx}$$

IC

$$U(x,0) = e^x \sin(x) = e^{k_1(x - \cancel{ct})} w(x,0) = e^x w(x,0) \Rightarrow w(x,0) = \sin(x)$$

Solution to w

$$w = e^{-t} \sin(x)$$

Combine back to $u = e^{(x-t)} w$

$$U = e^{(x-t)} e^{-t} \sin(x)$$

$$\boxed{u = e^x e^{-2t} \sin(x)}$$