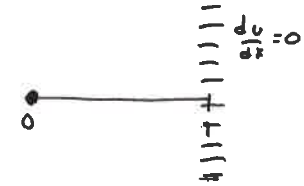
L7P1

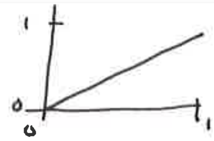
$U_t = U_{xx}$

$U(0,t) = 0$

$U_x(1,t) = 0$

$U(x,0) = x$





SoV:

$U = XT$

Gov Equ:

$$T_t X = T X_{xx} \Rightarrow \frac{T_t}{T} = \frac{X_{xx}}{X} = -\lambda^2$$

2 ODEs

$$T_t + \lambda^2 T = 0 \quad \text{and} \quad X_{xx} + \lambda^2 X = 0$$

Eigenfunctions

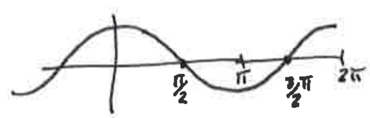
$$X = A\sin(\lambda x) + B\cos(\lambda x)$$

Fit BCs

1) $\underbrace{X(0)=0}_{BC\ \#1} = A\sin(\lambda\cdot 0) + B\cos(\cancelto{1}{\lambda\cdot 0}) \Rightarrow B = 0$

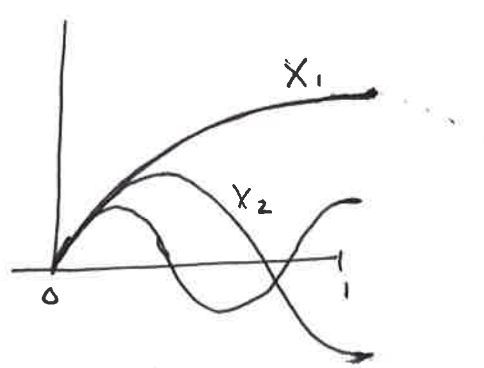
2) $X_x(1)=0 = A\lambda\cos(\lambda\cdot 1) + -\lambda \cancelto{0}{B}\sin(\lambda\cdot 1) \Rightarrow$ either $A=0 \leftarrow$ boring
$\lambda = 0 \leftarrow$ we know $\lambda \to \infty$
or
$\cos(\lambda) = 0$ ✓

$\cos(\lambda) = 0$



$$\lambda = \frac{(2n-1)\pi}{2} = (n-\tfrac{1}{2})\pi$$

$$X = A\sin\left((n-\tfrac{1}{2})\pi x\right)$$



Write IC in terms of $X(x)$

$u(x,0) = x$ written as $u = a_n \sin\left((n-\tfrac{1}{2})\pi x\right)$

premultiply by $\sin\left((m-\tfrac{1}{2})\pi x\right)$ and integrate

$$\int_0^1 x \sin\left((m-\tfrac{1}{2})\pi x\right) dx = \underbrace{\int_0^1 a_n \sin\left((n-\tfrac{1}{2})\pi x\right) \sin\left((m-\tfrac{1}{2})\pi x\right) dx}$$

What do we know about this?

S-L problem ✓ orthogonal ✓

$$\int_0^1 x \sin\left((m-\tfrac{1}{2})\pi x\right) dx = a_n \frac{1}{2} \Rightarrow a_n = 2\int_0^1 x \sin\left((m-\tfrac{1}{2})\pi x\right) dx$$

$$= \frac{-4\cos(n\pi)}{(2n-1)^2\pi^2} - \frac{2\sin(n\pi)}{(2n-1)\pi}$$

Solution $u = XT$

$$\boxed{u = a_n \sin\left((n-\tfrac{1}{2})\pi x\right) e^{-(n-\frac{1}{2})^2\pi^2 t}}$$

$$a_n = \frac{-4\cos(n\pi)}{(2n-1)^2\pi^2}$$

$a_1 = \frac{4}{\pi^2}$ $a_2 = -\frac{4}{9}\frac{1}{\pi^2}$

$a_3 = \frac{4}{25}\frac{1}{\pi^2}$

$$u = \frac{4}{\pi^2}\sin\left((n-\tfrac{1}{2})\pi x\right) e^{-\frac{\pi^2}{4}t}$$

$$+ -\frac{4}{9}\frac{1}{\pi^2}\sin(1.5\pi x)\, e^{-2.25\pi^2 t}$$

$$+ \frac{4}{25}\frac{1}{\pi^2}\sin(2.5\pi x)\, e^{-6.25\pi^2 t}$$