GES 554: Exam 1 (SAMPLE) Name:_____

2nd Feb 2015 50 minutes 5 Pages Closed book, Closed notes, No calculator.

100 total points **Read, think, plan, and then write.**

SAMPLE EXAM ONLY

University of Alabama Academic Honor Pledge:

I promise or affirm that I will not at any time be involved with cheating, plagiarism, fabrication, or misrepresentation while enrolled as a student at The University of Alabama. I have read the Academic Honor Code, which explains disciplinary procedures that will result from the aforementioned. I understand that violation of this code will result in penalties as severe as indefinite suspension from the University.

Signature:_____

Date:_____

1. Short Answer Problems [10 pts each]

- Classify $u_{xx} + u_t = 0$ as hyperbolic, parabolic, or elliptic.

- What is a Neumann boundary condition?

- Write a mathematical representation of the following boundary condition in a 1-dimensional metal bar (assume that Fourier's law models the heat flux):

  At point x_0, the heat flux always equals the temperature.

- Give 4 useful properties of the Sturm-Louisville differential equation.

2. Given the following Fourier transforms,

$f(x) = \mathrm{F}^{-1}[F]$	$F(\omega) = \mathrm{F}[f]$
u_{xx}	$-\omega^2 F(u)$
$\exp\left(-a^2 x^2\right)$	$\frac{1}{a\sqrt{2}} \exp\left(\frac{-\omega^2}{4a^2}\right)$

Obtain the solution to following diffusion equation. [20 pts]

$$\mathrm{u}_t = \mathrm{u}_{xx} \qquad -\infty < x < \infty$$
$$u(x,0) = \exp(-x^2)$$

3. Find the **steady state** solution to the following problem with insulated boundaries. Hint: insulated means that the boundaries' heat fluxes are zero. Be careful with the zero frequency coefficient (n=0). [20 pts]

$$
\begin{aligned}
&\mathrm{u}_t = \mathrm{u}_{xx} \qquad && 0 < x < 1 \\
&u_x(0,\mathrm{t}) = 0 \\
&u_x(1,\mathrm{t}) = 0 \\
&u(x,0) = x
\end{aligned}
$$

4. Obtain the solution to following diffusion equation. [20 pts]

$$u_t = u_{xx} + \sin(3\pi x) \qquad 0 < x < 1$$
$$u(0,t) = 0$$
$$u(1,t) = 0$$
$$u(x,0) = \sin(\pi x)$$