GES 554: Exam 2 Name:___________________________

4th Mar 2016 60 minutes 6 Pages Closed book, Closed notes, No calculator.

100 total points **Read, think, plan, and then write.**

This exam is open between 3rd March 2016 and 11th March 2016.

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University of Alabama Academic Honor Pledge:

I promise or affirm that I will not at any time be involved with cheating, plagiarism, fabrication, or misrepresentation while enrolled as a student at The University of Alabama. I have read the Academic Honor Code, which explains disciplinary procedures that will result from the aforementioned. I understand that violation of this code will result in penalties as severe as indefinite suspension from the University.

Signature:____________________________________

Date:____

1. [30 pts] What is the value of $u(42,\infty)$ when x=42 and time is infinity?

$$u_{tt} = u_{xx} \qquad -\infty < x < \infty$$

$$u(\mathrm{x},0) = \begin{cases} 1 & -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$u_t(x,0) = \begin{cases} 1 & -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

2. Given the following hyperbolic PDE

$$u_{xx} + 4u_{xy} + u_{yy} = 0$$

- Determine the solution characteristics ζ and η. [20 pts]
- Determine the canonical PDE form $u_{\zeta\eta} = \Phi$. [20 pts]

$$\bar{A} = A\zeta_x^2 + B\zeta_x\zeta_y + C\zeta_y^2 = 0$$
$$\bar{B} = 2A\zeta_x\eta_x + B\left(\zeta_x\eta_y + \zeta_y\eta_x\right) + 2C\zeta_y\eta_y$$
$$\bar{C} = A\eta_x^2 + B\eta_x\eta_y + C\eta_y^2 = 0$$
$$\bar{D} = A\zeta_{xx} + B\zeta_{xy} + C\zeta_{yy} + D\zeta_x + E\zeta_y$$
$$\bar{E} = A\eta_{xx} + B\eta_{xy} + C\eta_{yy} + D\eta_x + E\eta_y$$
$$\bar{F} = F$$
$$\bar{G} = G$$

3. [30 pts] Solve the following convection-diffusion problem with a coordinate transform and a Fourier transform. Tables are attached.

$$u_t = u_{xx} - 2u_x \qquad -\infty < x < \infty$$
$$u(\mathrm{x},0) = \sin(x)$$

TABLE A Exponential Fourier Transform

$f(x) = \mathcal{F}^{-1}[F] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(\omega) e^{i\omega x}\, d\omega$		$F(\omega) = \mathcal{F}[f] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-i\omega x}\, dx$
1. $f'(x)$		$i\omega F(\omega)$
2. $f''(x)$		$-\omega^2 F(\omega)$
3. $f^{n}(x)$	(nth derivative)	$(i\omega)^n F(\omega)$
4. $f(ax)$	$a > 0$	$\frac{1}{a} F\left(\frac{\omega}{a}\right)$

…

18. $\begin{matrix} 1 - \lvert x \rvert \\ 0 \end{matrix}$	$\begin{matrix} \lvert x \rvert < 1 \\ \lvert x \rvert > 1 \end{matrix}$	$2\sqrt{\frac{2}{\pi}} \left[\frac{\sin(\omega/2)}{\omega} \right]^2$
19. $\cos(ax)$		$\sqrt{\frac{\pi}{2}} [\delta(\omega + a) + \delta(\omega - a)]$
20. $\sin(ax)$		$i\sqrt{\frac{\pi}{2}} [\delta(\omega + a) - \delta(\omega - a)]$